Key

Ch. 4 Test Review WS #4 Riemann Sums Practice Worksheet

(Determining Units of Measure and interpreting Definite Integrals!)

***Important Key Point*: When applying(or approximating) a Calculus process(derivatives or integrals), your units of measure will change!**

1)

t (minutes)	0	1	3	6	9	10
c(t) (ounces per minute)	0	5.1	4.2	3.3	1.2	2.3

Hot water is dripping through a coffeemaker, filling a large cup with coffee. The rate that water in the cup at time t is changing, $0 \le t \le 10$, is given by a differential function c(t) , where t is measured in minutes. Select values if c(t), measured in ounces per minute are given in the table above.

a) Interpret the meaning of $c'(6)$ *and* indicate the units of measure.

$c'(6)$ tells us how fast the rate of water added to the cup is changing. (units is ounces/min²)

b) Approximate the value of $c'(6)$ and indicate the units of measure.

$c'(6) \approx \dfrac{1.2-3.3}{9-6} = -0.7$ ounces/min²

* choosing any ordered pairs close to t=6 and finding slope would be acceptable.

c) Intepret the meaning of $\int_1^{10} c(t)dt$ and indicate the units of measure.

* using 1st Theorem, $\int_1^{10} c(t)dt = C(10)-C(1)$. This represents the change in the amount of coffee in the cup between the 1st minute and the 10th minute. (units is ounces)

d) Approximate the value of $\int_1^{10} c(t)dt$ using 2 middle rectangles and indicate the units of measure.

$\int_1^{10} c(t)dt \approx 5(4.2) + 4(1.2) = 21 + 4.8 = \boxed{25.8 \text{ ounces}}$

e) Approximate the average rate of water being added on time interval [1, 10] using result from part d)

* Avg. value theorem

$\dfrac{1}{b-a}\int_a^b f(x)dx$

$\dfrac{1}{10-1}\int_1^{10} c(t)dt = \frac{1}{9}(25.8) = \boxed{2.867 \text{ ounces/minute}}$

2)

t (minutes)	0	12	20	24	40
$v(t)$ (meters per minute)	0	200	240	−220	150

Johanna jogs along a straight path. For $0 \le t \le 40$, Johanna's velocity is given by a differentiable function v. Selected values of $v(t)$, where t is measured in minutes and $v(t)$ is measured in meters per minute, are given in the table above.

a) Interpret the meaning of $v'(20)$ *and* indicate the units of measure.

$v'(20) = a(20)$ is the rate of change of velocity at $t=20$, (or acceleration)
units is $\text{meters}/\text{min}^2$

b) Approximate the value of $v'(18)$ and indicate the units of measure.

$$v'(18) = a(18) = \frac{v(20)-v(12)}{20-12} = \frac{240-200}{20-12} = \frac{40}{8} = \boxed{5 \text{ meters}/\text{min}^2}$$

c) Inteprete the meaning of $\int_{20}^{40} v(t)dt$ and indicate the units of measure.

*FFTC:

$$\int_a^b f(x) = F(b) - F(a)$$

$\int_{20}^{40} v(t)dt = x(40) - x(20)$ is the change in distance between 20 and 40 minutes. (or displacement).
Units is meters

d) Approximate the value of $\int_{20}^{40} v(t)dt$ using 2 trapezoids and indicate the units of measure.

$$\int_{20}^{40} v(t)dt = \frac{1}{2}(4)[240+-220] + \frac{1}{2}(16)[-220+150] = 40 - 560 = \boxed{-520 \text{ meters}}$$

Trapezoid Area is $\frac{1}{2}w[h_1+h_2]$

e) Approximate Johanna's average velocity on [20, 40] using the results from part d)

*Avg. value theorem:

$$\frac{1}{b-a}\int_a^b f(x)dx$$

$$\frac{1}{40-20}\int_{20}^{40} v(t)dt = \frac{1}{20}\int_{20}^{40} v(t)dt$$

$$= \frac{1}{20}[-520] = \boxed{-26 \text{ meters}/\text{minute}}$$