

### I. Algebraic Steps Evaluating Limits Approaching a Real Number $\lim_{x \rightarrow c} f(x)$

1. Plug in argument x-value (Ignore one-sided limit for now)
2. Find the Limit (plug in/ reduce if  $\frac{0}{0}$  , re-evaluate)
3. If Limit DNE (does not exist), then evaluate further ONLY IF one-sided limit)
4. Choose between  $+\infty$  and  $-\infty$
5. Plug in the appropriate decimal value to determine  $+\infty$  or  $-\infty$

### II. Algebraic Steps Evaluating Limits Approaching Infinity $\lim_{x \rightarrow \infty} f(x)$ or $\lim_{x \rightarrow -\infty} f(x)$

1. Compare degrees between numerator vs. denominator
  - a. If Numerator < Denominator  $\lim_{x \rightarrow \infty} f(x) = \lim_{x \rightarrow -\infty} f(x) = 0$
  - b. If Numerator = Denominator  $\lim_{x \rightarrow \infty} f(x) = \lim_{x \rightarrow -\infty} f(x) = \text{ratio of coefficients}$
  - c. If Numerator > Denominator, then  $\lim_{x \rightarrow \infty} f(x) = \pm\infty$  or  $\lim_{x \rightarrow -\infty} f(x) = \pm\infty$

(Plug in a large positive or large negative value to help you determine the sign at infinity)

### III. Continuity Conditions

1.  $f(c)$  is defined (point exists on the graph)
  2. The  $\lim_{x \rightarrow c} f(x)$  exists  $\left[ \lim_{x \rightarrow c^+} f(x) = \lim_{x \rightarrow c^-} f(x) \right]$
  3.  $f(c) = \lim_{x \rightarrow c} f(x)$
- If function passes all 3 conditions , the function has continuity at  $x = c$
  - If condition #2 FAILS, the function has **nonremovable** discontinuity at  $x = c$
  - If function PASSES condition #2 and FAILS condition #3, the function has **removable** discontinuity at  $x = c$

### IV. Intermediate Value Theorem (IVT) Steps

1. Test and determine continuity on closed interval  $[a, b]$
2. Find the y-value at the endpoints ,  $f(a)$  and  $f(b)$
3. Confirm that  $f(c)$  is between  $f(a)$  and  $f(b)$  [ example:  $f(a) < f(c) < f(b)$  ]
4. Find the c-value (find the x-value by plugging the y-value given at the start of problem into the function)

\*Make sure c-value(s) are inside the interval  $[a,b]$ . c-values that are outside the interval  $[a,b]$  are excluded.