**Calculus AB Optimization Practice Problems**

1. A rectangular field is to be fenced off along the bank of a river; no fence is required along the river. If the material for the fence costs \$8 per running foot for the two ends, and \$12 per foot for the side parallel to the river, find the dimensions of the field of largest possible area that can be enclosed with \$3600 worth of fence.

2. A rectangular storage container with an open top is to have a Volume of 10 $m^3$. The length of its base is twice its length. Material for the base costs \$10/ $m^2$. Material for the sides cost \$6/$m^2$. Find the cost of material for the cheapest container. (Hint: Minimize surface area)

3. A piece of cardboard measures 10 by 15 in. For equal squares are removed from corners of all sides. Find the maximum volume.

4. 1988 multiple choice problem #45
The volume of a cylindrical tin can with a top and a bottom is to be $16\pi$ cubic inches. If a minimum amount of tin is to be used to construct the can, what must be the height, in inches, of the can? (A cylinder with radius $r$ and height $h$ has a volume of $V = \pi r^2 h$ and a surface area of $S = 2\pi r^2 + 2\pi rh$.)

5. Highway 400 averaged 54,000 cars per day for its first 5 years charging \$0.50 per car. A scientific research study concludes that for every \$0.05 increase in the toll, the number of cars will be reduced by 500. In order to maximize revenue, what toll should Highway 400 charge?

6. The manager of a department store wants to build a 600 square foot rectangular enclosure on the store's parking lot in order to display some equipment. Three sides of the enclosure will be built of redwood fencing, at a cost of \$14 per running foot. The forth side will be built of cement blocks, at a cost of \$28 per running foot. What dimensions will minimize the total cost of the building materials? What will this minimum cost be?

7. A 150-room resort hotel is filled at a room rate of \$125 per day. For each \$5 increase in the room rate, three fewer rooms are rented. What room rate will result in maximum daily revenue? How many rooms will be rented at that rate?

8. A farmer has 400 feet of fencing to make three rectangular pens. What dimensions x and y will maximize the total area?

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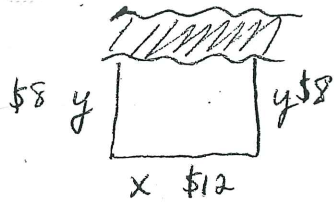
Solution Key

1. A rectangular field is to be fenced off along the bank of a river; no fence is required along the river. If the material for the fence costs $8 per running foot for the two ends, and $12 per foot for the side parallel to the river, find the dimensions of the field of largest possible area that can be enclosed with $3600 worth of fence.




$P = 12x + 16y$

$3600 = 12x + 16y$

$\frac{3600-12x}{16} = y$

(* Optimize Area)

$A = xy$

$A = x\left[\frac{3600-12x}{16}\right]$

$A = 225x - \frac{3}{4}x^2$

$A'(x) = 225 - \frac{6}{4}x$

$0 = 225 - \frac{3}{2}x$

$\frac{3}{2}x = 225$

$x = 150\text{ ft}$

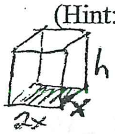
$3600 = 12(150) + 16y$

$1800 = 16y$

$y = 112.5\text{ ft}$

$\boxed{\text{Dimensions: } 150\text{ ft by } 112.5\text{ ft.}}$

2. A rectangular storage container with an open top is to have a Volume of 10 m$^3$. The length of its base is twice its length. Material for the base costs \$10/ m$^2$. Material for the sides cost \$6/m$^2$. Find the cost of material for the cheapest container. (Hint: Minimize surface area)




$S = 2x^2 + 2xh + 2xh + xh + xh$

$S = 2x^2 + 2xh + 4xh$

$S = 2x^2 + 6xh$

Cost

$\rightarrow C(x) = (\$10)2x^2 + (\$6)6xh$

$C(x) = 20x^2 + 36xh$

(Minimize Cost)

$V = 2x^2h$

$10 = 2x^2h$

$\frac{10}{2x^2} = h$

$C(x) = 20x^2 + 36x\left(\frac{10}{2x^2}\right)$

$C(x) = 20x^2 + 180x^{-1}$

$C'(x) = 40x - 180x^{-2}$

$0 = 40x - \frac{180}{x^2}$

$\frac{180}{x^2} = 40x$

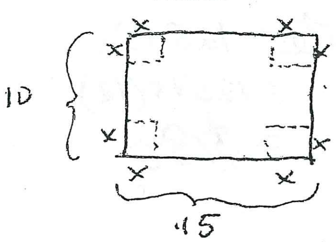
$40x^3 = 180$

$x = \sqrt[3]{9/2}$

$x \approx 1.65\text{ m}$

$\boxed{C(1.65) = \$163.54}$

3. A piece of cardboard measures 10 by 15 in. For equal squares are removed from corners of all sides. Find the maximum volume.




$V = (x)(10-2x)(15-2x)$

$V = x(150 - 50x + 4x^2)$

$V = 150x - 50x^2 + 4x^3$

$V'(x) = 150 - 100x + 12x^2$

$0 = 2(6x^2 - 50x + 75)$

$\frac{50 \pm \sqrt{50^2 - 4(6)(75)}}{2(6)}$

$x = \cancel{6.3715}$

$x = 1.962$

$\boxed{V(1.962) = 132.038\text{ in}^3}$

4. 1988 multiple choice problem #45
The volume of a cylindrical tin can with a top and a bottom is to be $16\pi$ cubic inches. If a minimum amount of tin is to be used to construct the can, what must be the height, in inches, of the can? (A cylinder with radius $r$ and height $h$ has a volume of $V = \pi r^2 h$ and a surface area of $S = 2\pi r^2 + 2\pi rh$.)

(Optimize Surface Area)

$V = \pi r^2 h$

$16\pi = \pi r^2 h$

$\frac{16\pi}{\pi r^2} = h$

$\frac{16}{r^2} = h$

$S = 2\pi r^2 + 2\pi r\left(\frac{16}{r^2}\right)$

$S = 2\pi r^2 + \frac{32\pi}{r}$

$S = 2\pi r^2 + 32\pi r^{-1}$

$S' = 4\pi r - 32\pi r^{-2}$

$0 = 4\pi r - \frac{32\pi}{r^2}$

$0 = 4\pi r - \frac{32\pi}{r^2}$

$\frac{32\pi}{r^2} = 4\pi r$

$32\pi = 4\pi r^3$

$2 = r$

$h = \frac{16}{r^2}$

$h = \frac{16}{4} = 4\text{ in.}$

$h = 4\text{ in.}$

5. Highway 400 averaged 54,000 cars per day for its first 5 years charging \$0.50 per car. A scientific research study concludes that for every \$0.05 increase in the toll, the number of cars will be reduced by 500. In order to maximize revenue, what toll should Highway 400 charge?

*Rate = (Change in revenue)(change in # of cars

$R(x) = (0.50 + 0.05x)(54{,}000 - 500x)$

$R(x) = 27000 - 250x + 2700x - 25x^2$

$R(x) = -25x^2 + 2450x + 27000$

$R'(x) = -50x + 2450$

$0 = -50x + 2450$

$50x = 2450$

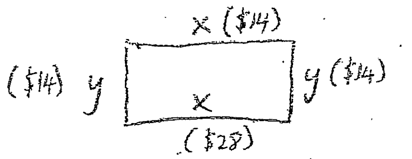
$x = 49$

Toll $= 0.50 + 0.05x$

$= 0.50 + 0.05(49)$

$= \boxed{\$2.95}$

6) The manager of a department store wants to build a 600 square foot rectangular enclosure on the store's parking lot in order to display some equipment. Three sides of the enclosure will be built of redwood fencing, at a cost of \$14 per running foot. The forth side will be built of cement blocks, at a cost of \$28 per running foot. What dimensions will minimize the total cost of the building materials? What will this minimum cost be?




$C(x) = 28x + 14x + 14y + 14y$

$C(x) = 42x + 28y$

$A = xy$

$600 = xy$

$\frac{600}{x} = y$

$C(x) = 42x + 28\left(\frac{600}{x}\right)$

$C(x) = 42x + 16800x^{-1}$

$C'(x) = 42 - 16800x^{-2}$

$0 = 42 - \frac{16800}{x^2}$

$42 = \frac{16800}{x^2}$

$x^2 = \frac{16800}{42}$

$x = 20$

20

$x = 20 \quad y = 30$

$C(x) = 42(20) + 28(30)$

$\boxed{C(x) = \$1680}$

7) A 150-room resort hotel is filled at a room rate of \$125 per day. For each \$5 increase in the room rate, three fewer rooms are rented. What room rate will result in maximum daily revenue? How many rooms will be rented at that rate? Revenue = (room rate)(price rate)

(change in # rooms)

(change in price)

$R(x) = (125 + 5x)(150 - 3x)$

$= -15x^2 + 375x + 18750$

$R'(x) = -30x + 375$

$0 = -30x + 375$

$x = 12.5$

12.5

$R(12) = \$21{,}090$

$R(13) = \$21{,}090$

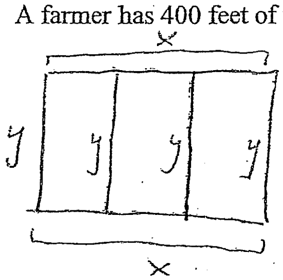
$\boxed{12 \text{ or } 13 \text{ ~~rate~~ increases}}$

Room rate $= 125 + 5x$

$= 125 + 5(12)$

$= \$185$

8. A farmer has 400 feet of fencing to make three rectangular pens. What dimensions x and y will maximize the total area?




$P = 2x + 4y$

$400 = 2x + 4y$

$\frac{400 - 2x}{4} = y$

$A = xy$

$A = \left[\frac{400 - 2x}{4}\right]x$

$A = 100x - \frac{1}{2}x^2$

$A'(x) = 100 - x$

$0 = 100 - x$

$x = 100$

$400 = 2(100) + 4y$

$200 = 4y$

$50 = y$

Dimensions: 100 ft by 50 ft