Calculus AB Ch. 3 Test Review WS #1

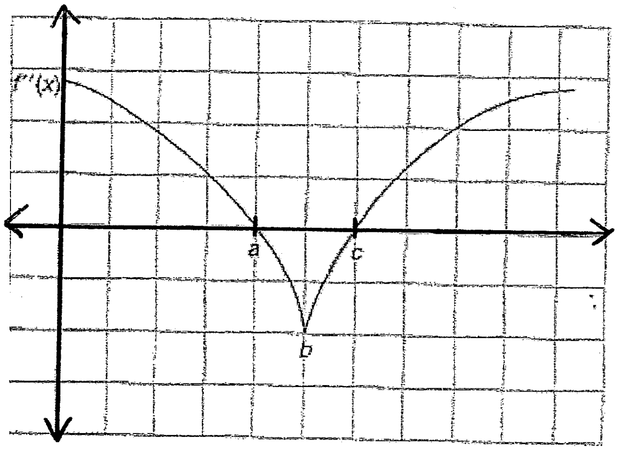
1. A t-shirt maker estimates that the weekly cost of making x shirts is $C(x) = 50 + 2x + \frac{x^2}{20}$

   The weekly revenue from selling x shirts is given by the function $R(x) = 20x + \frac{x^2}{200}$

   a) What is the profit if all the shirts made are sold? (Profit = Revenue – Cost)

   b) What is the maximum weekly profit?

2. The second derivative of $f(x)$ has zeros at $x = a$ and $x = c$ and a minimum at $x = b$ as shown. The function $f(x)$ is concave up



(A) when $0x < a$
(B) when $0x < b$
(C) when $x > b$
(D) when $0x < a$ and $x > c$
(E) nowhere

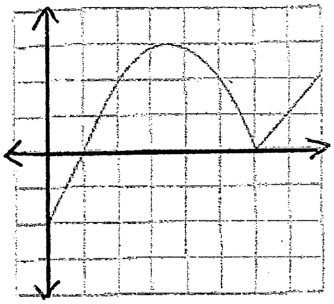
3. Verify whether $f(x) = 3x^2 - 12x + 1$ satisfies Rolle's theorem on the interval [0, 4] and find all numbers c that satisfy $f'(c) = 0$
   A) $c = 0$
   B) $c = 1$
   C) $c = 2$
   D) $c = 4$
   E) f(x) does not satisfy Rolle's theorem on interval [0, 4]

4. Which of the following statements is true of the function $f(x) = x^{2/3}$

I.There is a critical point at (0, 0
II. f '(0) and f''(0) are undefined
III. The curve is concave up over the interval $(0, \infty)$
IV. The curve is concave down over interval $(-\infty, 0)$

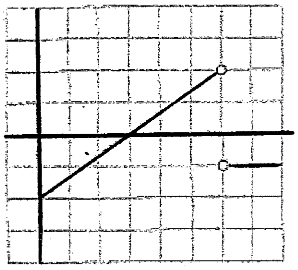
A. I and III only
B. I, II, IV only
C. I, II, III
D. I, III, and IV
E. I, II, III, and IV

The graph of a function $g(x)$ is given.

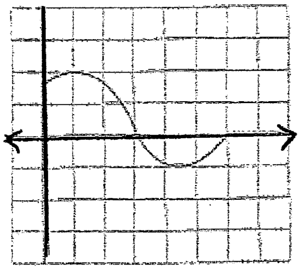


Which of the following could be the graph of $g'(x)$?

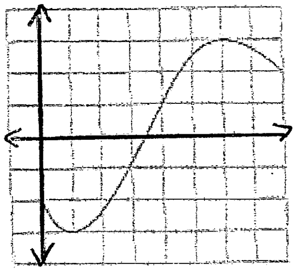
(A)



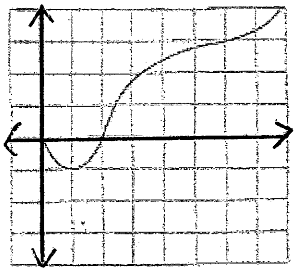
(B)



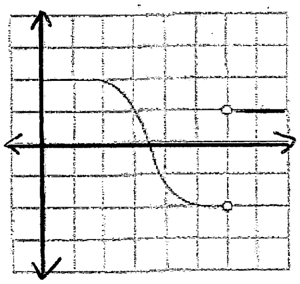
(C)



(D)



(E)



6. The height of an object t seconds after it is dropped from a height of 500 meters is $x(t) = -4.9t^2 + 500$

a) Find the avg velocity of the object during the first 4 seconds (Think avg slope)

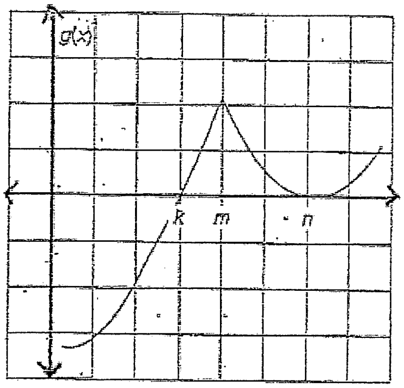
b) Use the Mean Value Theorem to verify that , at some point during the first 4 seconds of the fall, the instantaneous velocity equals the avg velocity. Find that time and height.

7. Max plans to build two side-by-side identical rectangular pens for his pigs that will enclose a total area of 216 ft^2. What is the minimum length of fencing he will need?

8. A manufacturer wants to design an open-top box having a square base and a surface area of 80 square inches.
    a. What dimensions will provide a box with maximum volume?
    b. Find maximum volume

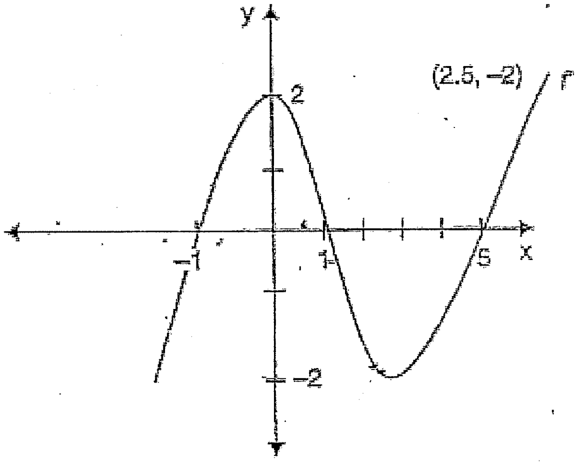
Calculus AB Ch. 3 Test Review WS #2

1. The graph of g(x) has zeros at x = k, x = n, and a relative maximum at m as shown. Based on the graph, which of the following is true?



a) g'(x) has a relative maximum at x = k
b) g'(x) has a zero at x = m
c) g''(x) has a zero at x = n
d) g'(x) is continuous everywhere
e) g''(x) is never negative

2. Given the graph of f', find the following properties of the function f:



a) The intervals on which f is increasing or decreasing

b) The location of the relative maxima and minima

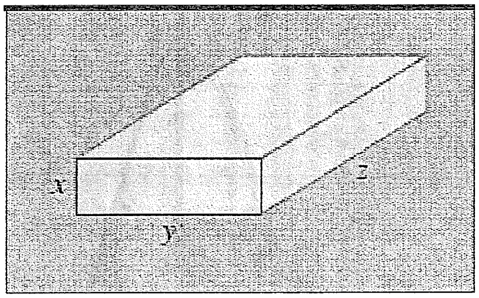
c) The points of inflection and concavity of f

d) Draw a sketch of f, given that f(-1) = – 5
f(1) = 5, f(0) = 0, and f(5) = –5

3. A landscape architect wishes to enclose a rectangular garden on one side by a brick wall costing $30/foot and on the other 3 sides by a metal fence costing $10/foot. If the area of the garden is 1000 square feet, find the dimensions of the garden that minimze cost. Round dimensions to 3 decimal places.

4.

A jewel box is to be constructed of material that costs $1 per square inch for the bottom, $2 per square inch for the sides, and $5 per square inch for the top. If the total volume is to be 96 in.^{3} what dimensions will minimize the total cost of construction?



5. A 300-room hotel in Las Vegas is filled to capacity every night at $80 a room. For each $1 increase in rent, 3 fewer rooms are rented. If each rented room costs $10 to service per day, how much should the management charge for each room to maximize gross profit? What is the maximum gross profit?

Calculus AB Ch. 3 Test Review WS #1

$\boxed{\text{Solutions}}$

1. A t-shirt maker estimates that the weekly cost of making x shirts is $C(x) = 50 + 2x + \frac{x^2}{20}$

The weekly revenue from selling x shirts is given by the function $R(x) = 20x + \frac{x^2}{200}$

a) What is the profit if all the shirts made are sold? (Profit = Revenue – Cost)

$$P(x) = 20x + \frac{x^2}{200} - \left(50 + 2x + \frac{x^2}{20}\right)$$

$$= 18x - 50 - \frac{9x^2}{200}$$

b) What is the maximum weekly profit?

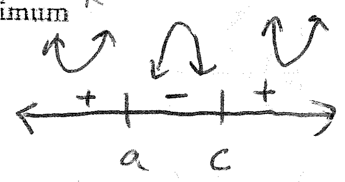
$$P'(x) = 18 - 2\left(\frac{9}{200}\right)x$$

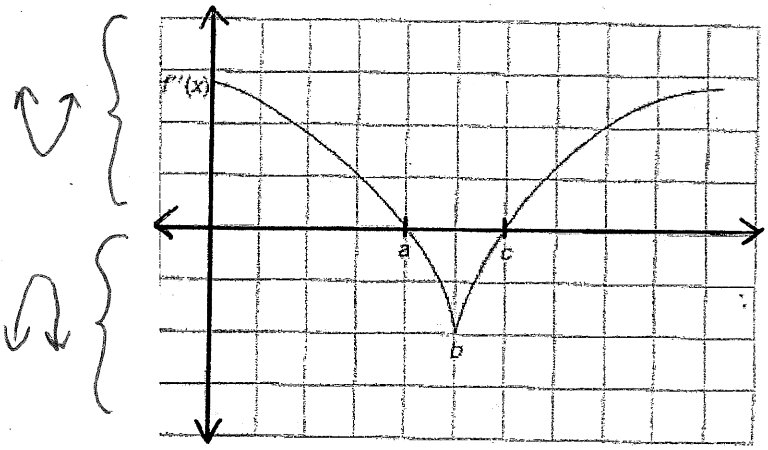
$$0 = 18 - \frac{18}{200}x$$

$$18 = 0.09x$$

$$\boxed{\begin{array}{l} x = 200 \text{ shirts} \\ P(200) = \$1750 \end{array}}$$

2. The second derivative of $f(x)$ has zeros at $x = a$ and $x = c$ and a minimum at $x = b$ as shown. The function $f(x)$ is concave up





(A) when $x < a$

(B) when $x < b$

(C) when $x > b$

$\boxed{\text{(D) when } x < a \text{ and } x > c}$

(E) nowhere

3. Verify whether $f(x) = 3x^2 - 12x + 1$ satisfies Rolle's theorem on the interval [0, 4] and find all numbers c that satisfy $f'(c) = 0$

A) c = 0

B) c = 1

$\boxed{\text{C) } c = 2}$

D) c = 4

E) f(x) does not satisfy Rolle's theorem on interval [0, 4]

f(x) continuous on [0,4] differentiable on (0,4).

$f(0) = 1$
$f(4) = 1$] $f(0) = f(4) = 1$

*set $f'(x) = 0$

$f'(x) = 6x - 12$

$6x - 12 = 0$

$x = 2$

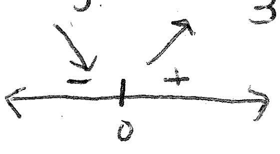
$\boxed{c = 2}$

4. Which of the following statements is true of the function $f(x) = x^{2/3}$

I. There is a critical point at (0, 0) True
II. f '(0) and f ''(0) are undefined True
III. The curve is concave up over the interval (0, ∞) False
IV. The curve is concave down over interval (−∞, 0) True

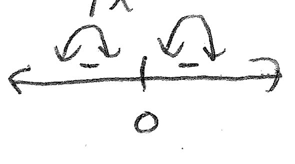
A. I and III only
$\boxed{\text{B. I, II, IV only}}$
C. I, II, III
D. I, III, and IV
E. I, II, III, and IV

$$f'(x) = \frac{2}{3}x^{-1/3} = \frac{2}{3x^{1/3}}$$



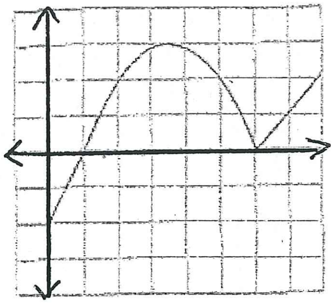
$$f''(x) = \frac{2}{3}\left(-\frac{1}{3}\right)x^{-4/3}$$

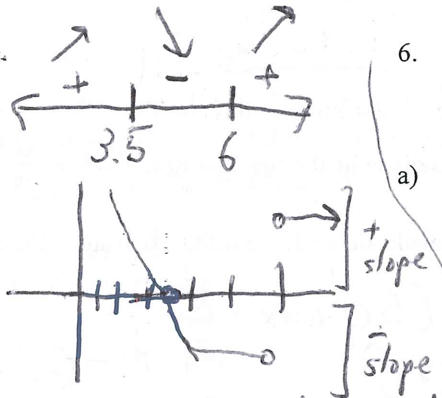
$$= \frac{-2}{9x^{4/3}}$$



$\boxed{B}$

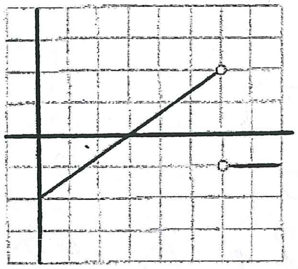
5) The graph of a function $g(x)$ is given.



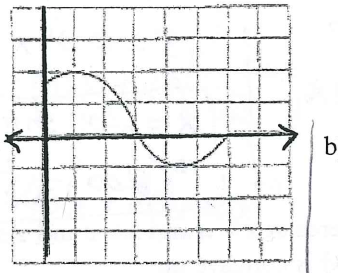


Which of the following could be the graph of $g'(x)$? $*f'(6)$ undefined

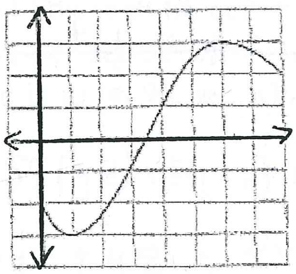
(A)



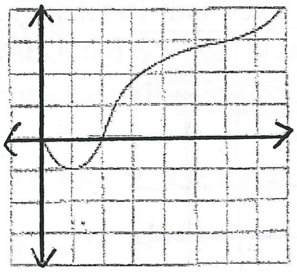
(B)



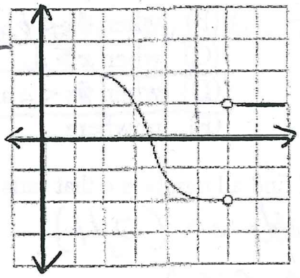
(C)



(D)



[✓ (E)]



6. The height of an object t seconds after it is dropped from a height of 500 meters is $x(t) = -4.9t^2 + 500$

a) Find the avg velocity of the object during the first 4 seconds (Think avg slope)

$s(0) = 500$

$s(4) = 421.6$

$m_{Avg} = \frac{500-421.6}{0-4}$

$= -19.6 \text{ m/s}$

b) Use the Mean Value Theorem to verify that , at some point during the first 4 seconds of the fall, the instantaneous velocity equals the avg velocity. Find that time and height.

$*$set $x'(t) = m_{Avg}$

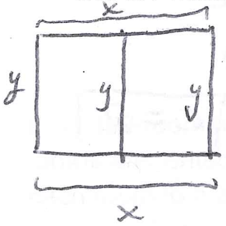
$x'(t) = -9.8t$

$-9.8t = -19.6$

$t = 2$ seconds

$x(2) = 480.4$ ft.

7. Max plans to build two side-by-side identical rectangular pens for his pigs that will enclose a total area of 216 ft^2. What is the minimum length of fencing he will need?



$*P = 2x + 3y$

$A = xy$

$216 = xy$ $\quad y = \frac{216}{x}$

$P = 2x + 3\left(\frac{216}{x}\right)$

$P = 2x + 648x^{-1}$

$P'(x) = 2 - 648x^{-2}$

$0 = 2 - \frac{648}{x^2}$

$x^2 = 324$

$x = 18$ ft.

$216 = xy$

$216 = 18y$

$y = 12$ ft.

$P = 2x + 3y$

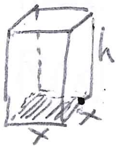
$2(18) + 3(12)$

$P_{min} = 72$ ft. of fencing

8. A manufacturer wants to design an open-top box having a square base and a surface area of 80 square inches.

a. What dimensions will provide a box with maximum volume?

b. Find maximum volume



$S = x^2 + 4xh$

$80 = x^2 + 4xh$

$\frac{80-x^2}{4x} = h$

$V = x^2 h$

$V = x^2\left[\frac{80-x^2}{4x}\right]$

$V = 20x - \frac{1}{4}x^3$

$V'(x) = 20 - \frac{3}{4}x^2$

$0 = 20 - \frac{3}{4}x^2$

$\frac{3}{4}x^2 = 20$

$x^2 = \frac{80}{3}$

$x = \sqrt{80/3} \approx 5.164$ in.

$h = \frac{80-(5.164)^2}{4(5.164)}$

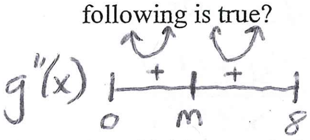
$h = 2.582$ in.

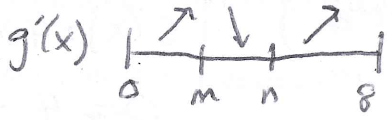
$V_{max} = 68.853 \text{ in}^3$

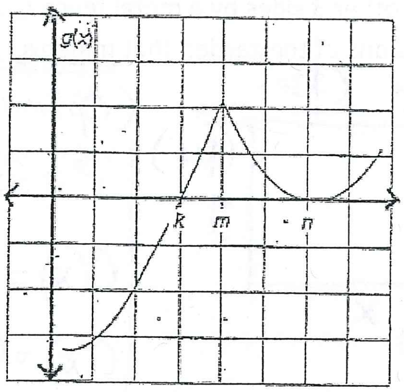
Calculus AB Ch. 3 Test Review WS #2

Solutions

1. The graph of g(x) has zeros at x = k, x = n, and a relative maximum at m as shown. Based on the graph, which of the following is true?







a) g'(x) has a relative maximum at x = k false

b) g'(x) has a zero at x = m slope undefined, false

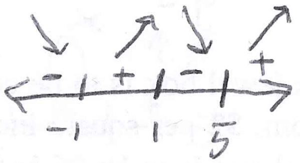
c) g''(x) has a zero at x = n. concave up, $g''>0$, false

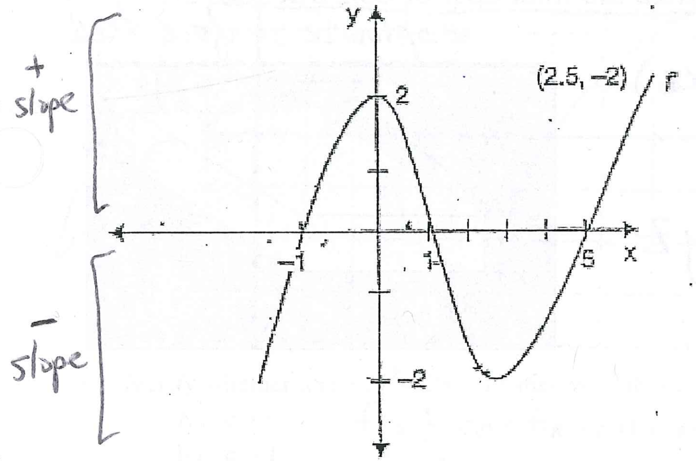
d) g'(x) is continuous everywhere false, $g'(m)$ undefined

[✓ e)] g''(x) is never negative

True, $f''>0$, except when f'' undefined at $x=m$

2. Given the graph of f', find the following properties of the function f:





a) The intervals on which f is increasing or decreasing

f(x) increasing $(-1,1)\cup(5,\infty)$ b/c $f'(x)>0$

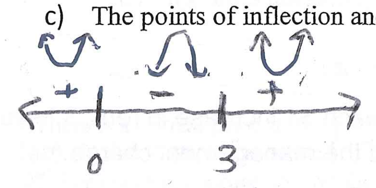
f(x) decreasing $(-\infty,-1)\cup(1,5)$ b/c $f'(x)<0$

b) The location of the relative maxima and minima

Rel. max at x=1 b/c $f'(x)$ changes from + to −

Rel. min at x=−1, x=5 b/c $f'(x)$ changes from − to +.

c) The points of inflection and concavity of f

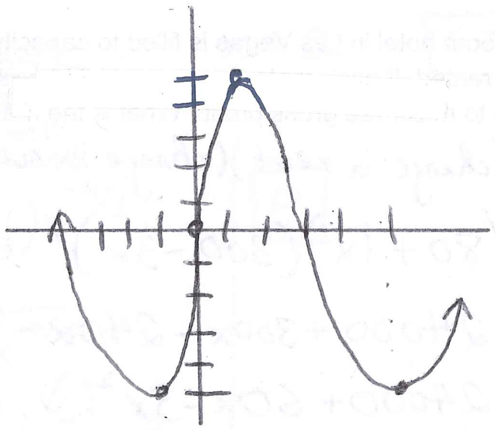


Concave up $(-\infty,0)\cup(3,\infty)$ b/c $f''(x)>0$

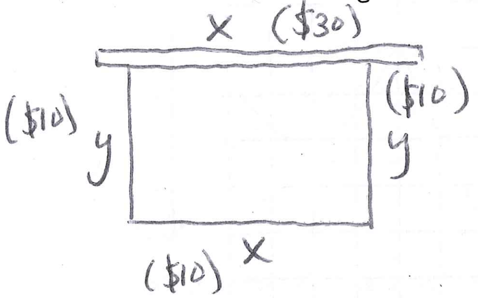
Concave down $(0,3)$ b/c $f''(x)<0$

POI at x=0, 3 b/c $f''(x)$ change signs.

d) Draw a sketch of f, given that f(-1) = – 5
f(1) = 5, f(0) = 0, and f(5) = –5



3. A landscape architect wishes to enclose a rectangular garden on one side by a brick wall costing \$30/foot and on the other 3 sides by a metal fence costing \$10/foot. If the area of the garden is 1000 square feet, find the dimensions of the garden that minimze cost. Round dimensions to 3 decimal places.



$A = xy$

$1000 = xy$

$y = \frac{1000}{x}$

(Optimize perimeter)

$P = 2x + 2y$

$C(x) = 30x + 10x + 10y + 10y$

$C(x) = 40x + 20y$

$C(x) = 40x + 20\left(\frac{1000}{x}\right)$

$C(x) = 40x + 20000x^{-1}$

$C'(x) = 40 - 20000x^{-2}$

$0 = 40 - \frac{20000}{x^2}$

$40 = \frac{20000}{x^2}$

$x = \sqrt{500} = 10\sqrt{5} \approx \underline{\underline{22.361}}$

$1000 = xy$

$1000 = 10\sqrt{5}(y)$

$y \approx \underline{44.721}$ ft.

$\boxed{\text{22.361 ft by 44.721 ft.}}$

4. A jewel box is to be constructed of material that costs \$1 per square inch for the bottom, \$2 per square inch for the sides, and \$5 per square inch for the top. If the total volume is to be 96 in.^{3} what dimensions will minimize the total cost of construction?

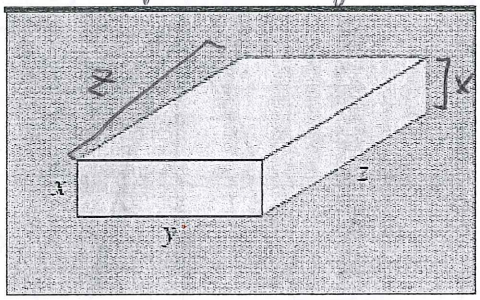
*optimize surface area

$C(x) = \$5(yz) + \$1(yz) + \$2(xy + xy + xz + xz)$

$= 5yz + yz + 4xy + 4xz$

$C(x) = 6yz + 4xy + 4xz$

$V = xyz$



5. A 300-room hotel in Las Vegas is filled to capacity every night at \$80 a room. For each \$1 increase in rent, 3 fewer rooms are rented. ~~If each rented room costs \$10 to service per day,~~ how much should the management charge for each room to maximize gross profit? What is the maximum gross profit?

Rate = (change in rent)(change in rooms rented)

$R(x) = (80 + 1x)(300 - 3x)$

$= 24000 + 300x - 240x - 3x^2$

$= 24000 + 60x - 3x^2$

$R'(x) = 60 - 6x$

$0 = 60 - 6x$

$6x = 60$

$\underline{\underline{x = 10}}$ rent increases

Rent = $80 + 1x$

$= 80 + 1(10)$

$= \boxed{\$90}$

Max profit

$R(10) = \$24{,}300$