Mini-Mathletes 2017: Ciphering Solutions

Problem 1: 150 minutes is 2 hours and 30 minutes. Counting backwards, 2 hours and 30 minutes before 4:00 PM is **1:30 (PM)**.

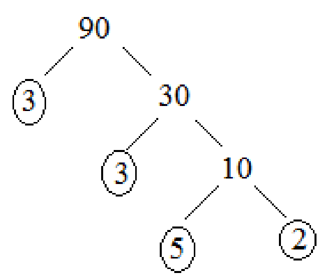
Problem 2: If two of my shoes weigh 10 kg together, one shoe alone weighs 5 kg. The total weight of twelve of my shoes is $12(5) =$ **60**.

Problem 3: List out the days of the week and the number of rabbits in the cage:

Sunday: 1 rabbit
Monday: 2 rabbits
Tuesday: 4 rabbits
Wednesday: 8 rabbits
Thursday: 16 rabbits
Friday: 32 rabbits

Friday is the first day where the rabbits in the cage exceeds 20.

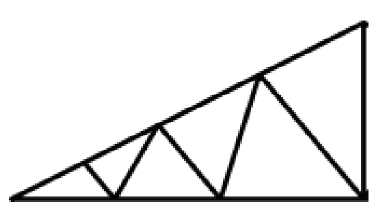
Problem 4: We can use one of the many methods available to find the prime factorization of 90. For example:



The *distinct* prime factors of 90 are 2, 3, and 5. The sum of these numbers is **10**.

Problem 5: If a square has one side length of 12, then it has area 144. Subtracting 12 (the area of the second square) from 144 gives us an answer of **132**.

Problem 6: There are triangles of many different sizes here.



First, we count 6 individual triangles. Next, there is exactly one triangle made from each of 2, 3, 4, 5, and 6 smaller triangles. Therefore, there is a total of $6 + 5(1) =$ **11 (triangles)**.

<u>Problem 7:</u> Subtracting the area of the circle from the area of the square will give us the total area of the shaded regions. Because the circle is inscribed in the square, the highest point to the lowest point of the circle is the diameter of the circle, and also the side length of the square. So, the area of the square is $10^2 = 100$. The area of the circle is $\pi(5)^2 = 25\pi$. Thus, the answer to this problem is **100 - 25π**.

<u>Problem 8:</u> Since non-negative also includes 0, the first five non-negative powers of ten include $10^0, 10^1, 10^2, 10^3$, and 10^4. To find the average of these numbers, we must find their sum and then divide by five: $\frac{1+10+100+1000+10000}{5} =$ **2222.2**.

<u>Problem 9:</u> The goal here is to set up an equation. To find the average, we must find the sum of all the terms in the set and then divide by the number of terms (5). We can express the average in two ways, x or $\frac{9+5+4+10+x}{5}$. Setting these quantities equal to each other gives us $\frac{9+5+4+10+x}{5} = x \rightarrow x =$ **7**.

<u>Problem 10:</u> Since all integers in our triples have to be positive, we can't have any triples containing 0 or any number greater than or equal to 5. In order for the numbers to add to 6, we can have a 4 in a triple, with two other 1's. Note we can't have a triple in the form (4, 2, ___) because we will have a 0. We can also have a triple with one (but not two) 3. This triple would have to be in some order of (3, 2, 1). Lastly, we can have (2, 2, 2) in exactly one way.

- (4, 1, 1) can be ordered in 3 ways
- (3, 2, 1) can be ordered in 6 ways
- (2, 2, 2) can be ordered in 1 way

$3 + 6 + 1 =$ **10** possible triples.