Solutions to the Mini-Mathletes Competition 2014

Written Test

1. Heads:Tails = 2:1. With a total of 9 coins, we get Heads = 2(3) = **6**.
2. 5 days is 5(24) hours. Every four hours, the monkey eats one banana, for a total of 5(24)/4 = 5(6) = **30** bananas.
3. The perimeter of the rectangle can be expressed in terms of the width:
   $$(2w + w) + (2w + w) = 6w = 60 \rightarrow w = 10$$
   The area of the rectangle is (length)(width) = (10)(20) = **200**.
4. The median (middle number is 11). The mean, or average, is:
   $$\frac{2 + 5 + 11 + 17 + 20}{5} = 11$$
   The difference between the median and the mean is 11 – 11 = **0**.
5. Approximate 30 days for a month. 199 ÷ 30 = 6 remainder 19. This indicates the completion of 6 entire months, and about halfway into the 7th month, **July**.
6. Use logic to solve this problem – you don't actually need to use any equations. Neither Jo nor Bo has as much money as Flo. So Flo clearly does not have the least amount of money. Rule out Flo. Both Bo and Coe have more than Moe. Rule out Bo and Coe; they clearly do not have the least amount of money. Jo has more than Moe. Rule out Jo. The only person who has not been ruled out is **Moe**.
7. First we find the fraction of white bows in the bag:
   $$1 - \left(\frac{1}{5} + \frac{1}{2} + \frac{1}{10}\right) = 1 - \frac{8}{10} = \frac{1}{5}$$
   1/5 of the bag is 30 bows, which means the whole bag has 5(30) = 150 bows. The number of green bows is (1/10)(150) = **15**.
8. We have to rearrange the digits of 7005 to form as many different, valid four-digit numbers as we can. Note that a four-digit number can't start with a 0, or else it would be a three-digit number! Thus, a number could only start with 7 or 5. We can list out the following:

7005	5007
7050	5070
7500	5700

   That's **6** four-digit numbers.
9. Simplify out each of the expressions in the parentheses to get:
   $$2\left(\frac{1}{2}\right) + 3\left(\frac{2}{3}\right) + 4\left(\frac{3}{4}\right) + \cdots + 10\left(\frac{9}{10}\right) = 1 + 2 + 3 + \cdots + 9 = \boxed{45}.$$

10. The area of the rectangular garden is (60)(20) = 1200 ft^2. We calculate the length of the fence to be 2(60+20) = 160 feet. If this length of fence is used to encompass a square garden, the side length of the square would be 160/4 = 40 feet. The area of the square is 40^2 = 1600 ft^2. The area has increased by 1600 – 1200 = **400** ft^2.

Ciphering

Practice: Simplify the fraction into (10/8) ÷ (4/5) = (10/8)(5/4) = **25/16**.

1. 7 out of 10 pieces of the pie are left. This is (7/10)(100) = **70%** of the pie.
2. There are 3 small triangles, 1 big triangle encompassing the three small triangles, and 1 medium-sized triangle encompassing 2 small triangles – **5** triangles in total.
3. Brianna is 40/2 = 20 years old. Since Caroline is 6 years younger than Brianna, she is 20 – 6 = **14** years old.
4. After 24 days, (0.5)(24) = 12 gallons of water have evaporated. 200 – 12 = **188** gallons remain in the pool.
5. Notice that the 1st term is 1(5) = 5, the 3rd term is 3(5) = 15, the 5th term is 5(5) = 25, etc. We can conclude that the 19th term is 19(5) = 95, so the 20th term is 95 + 2 = **97**.
6. The sum of the reciprocals of 4 and 5 is:
$$\frac{1}{4}+\frac{1}{5}=\frac{5+4}{20}=\frac{9}{20}$$
The reciprocal of that is **20/9**.
7. 12 paper clips cost 48¢, so 1 paper clip costs 4¢. For a dollar, we can buy 100/4 = **25** paper clips.
8. Since the sum of the three numbers ends in 4, the expression (X + X) in the ones place must end in 0, which means X could be 0 or 5. However, X can't be 0 since that would cause the second two-digit number to be 00, which is invalid. Thus, X = **5**. It works too – (25 + 55 + 24) = 104.
9. Base angles are equivalent in an isosceles triangle. Thus,
$$\angle B=\angle C=\frac{180-\angle A}{2}=\frac{180-40}{2}=\frac{140}{2}=\mathbf{70^\circ}.$$
10. If the surface are of a cube is 600 square units, on face is 600/6 = 100 square units. Therefore, the side length of the cube is 10 units. The volume is 10^3 = **1000** $units^3$.