

**AP FRQ Review: "Curve Sketching"**

1) Non-Calculator

Consider a differentiable function  $f$  having domain all positive real numbers, and for which it is known that

$$f'(x) = (4 - x)x^{-3} \text{ for } x > 0.$$

- (a) Find the  $x$ -coordinate of the critical point of  $f$ . Determine whether the point is a relative maximum, a relative minimum, or neither for the function  $f$ . Justify your answer.
- (b) Find all intervals on which the graph of  $f$  is concave down. Justify your answer.
- (c) Given that  $f(1) = 2$ , determine the function  $f$ .

2) Calculator

The function  $g$  is defined for  $x > 0$  with  $g(1) = 2$ ,  $g'(x) = \sin\left(x + \frac{1}{x}\right)$ , and  $g''(x) = \left(1 - \frac{1}{x^2}\right)\cos\left(x + \frac{1}{x}\right)$ .

- (a) Find all values of  $x$  in the interval  $0.12 \leq x \leq 1$  at which the graph of  $g$  has a horizontal tangent line.
- (b) On what subintervals of  $(0.12, 1)$ , if any, is the graph of  $g$  concave down? Justify your answer.
- (c) Write an equation for the line tangent to the graph of  $g$  at  $x = 0.3$ .
- (d) Does the line tangent to the graph of  $g$  at  $x = 0.3$  lie above or below the graph of  $g$  for  $0.3 < x < 1$ ? Why?

3) Non-Calculator

Let  $f$  be the function given by  $f(x) = \frac{\ln x}{x}$  for all  $x > 0$ . The derivative of  $f$  is given by

$$f'(x) = \frac{1 - \ln x}{x^2}.$$

- (a) Write an equation for the line tangent to the graph of  $f$  at  $x = e^2$ .
- (b) Find the  $x$ -coordinate of the critical point of  $f$ . Determine whether this point is a relative minimum, a relative maximum, or neither for the function  $f$ . Justify your answer.
- (c) The graph of the function  $f$  has exactly one point of inflection. Find the  $x$ -coordinate of this point.
- (d) Find  $\lim_{x \rightarrow 0^+} f(x)$ .

4) Non-Calculator

Let  $f$  be the function defined by  $f(x) = k\sqrt{x} - \ln x$  for  $x > 0$ , where  $k$  is a positive constant.

- (a) Find  $f'(x)$  and  $f''(x)$ .
- (b) For what value of the constant  $k$  does  $f$  have a critical point at  $x = 1$ ? For this value of  $k$ , determine whether  $f$  has a relative minimum, relative maximum, or neither at  $x = 1$ . Justify your answer.
- (c) For a certain value of the constant  $k$ , the graph of  $f$  has a point of inflection on the  $x$ -axis. Find this value of  $k$ .