

① Curve sketch: exponential function

AP Calculus Logs and Exponentials Test Review WS #3

Name K E Y Date _____

1. Let f be the function defined by $f(x) = \frac{x^3}{e^x}$

- a. State the domain of $f(x)$.

$$(-\infty, \infty)$$

- c. Find each relative maximum and relative minimum. (ordered pairs) Justify Answer

$$f'(x) = \frac{3x^2 e^x - x^3 e^x}{e^{2x}} = \frac{3x^2 - x^3}{e^x} = \boxed{\frac{x^2(3-x)}{e^x}} \quad x=0, 3$$

$\leftarrow + \begin{matrix} + & + \\ 0 & 3 \end{matrix} \rightarrow$ Rel. max at $(3, \frac{27}{e^3})$

b/c $f'(x)$ changes from + to -

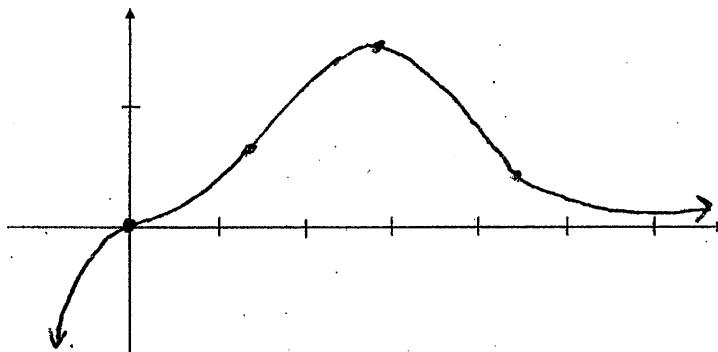
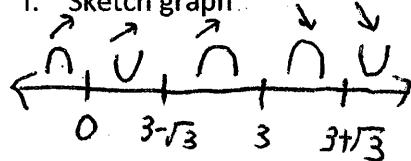
Comparative growth rate

$$\lim_{x \rightarrow \infty} \frac{x^3}{e^x} = 0$$

- d. Find $\lim_{x \rightarrow \infty} f(x)$ and $\lim_{x \rightarrow -\infty} f(x)$

$$\lim_{x \rightarrow -\infty} \frac{x^3}{e^x} \rightarrow \frac{(-\infty)^3}{e^{-\infty}} = (-\infty)^3 e^{\infty} = \boxed{-\infty}$$

- f. Sketch graph.



2. If $y = (e^{-x})(\ln x)$, then dy/dx when $x = 1$ is

$$y' = (e^{-x})(-1)\ln x + e^{-x}\left(\frac{1}{x}\right)$$

$$y' = \frac{-\ln x}{e^x} + \frac{1}{xe^x}$$

$$y'(1) = \frac{-\ln 1}{e^1} + \frac{1}{e^1} = \boxed{\frac{1}{e}}$$

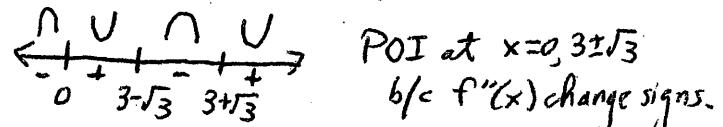
- b. Find the range of f . (find this after sketching your curve)

$$\left(-\infty, \frac{27}{e^3}\right)$$

- Find each point of inflection on graph of f . (Provide only x value) Justify answer.

$$f''(x) = \frac{(6x-3x^2)e^x - (3x^2-x^3)e^x}{e^{2x}} = \frac{6x-3x^2-3x^2+x^3}{e^x}$$

$$f''(x) = \frac{x^3-6x^2+6x}{e^x} = \frac{x(x^2-6x+6)}{e^x} \quad x=0, 3+\sqrt{3}, 3-\sqrt{3}$$



POI at $x=0, 3 \pm \sqrt{3}$

b/c $f''(x)$ change signs.

- e. Find x-intercept(s) $(0,0)$

② Curve sketching polynomial

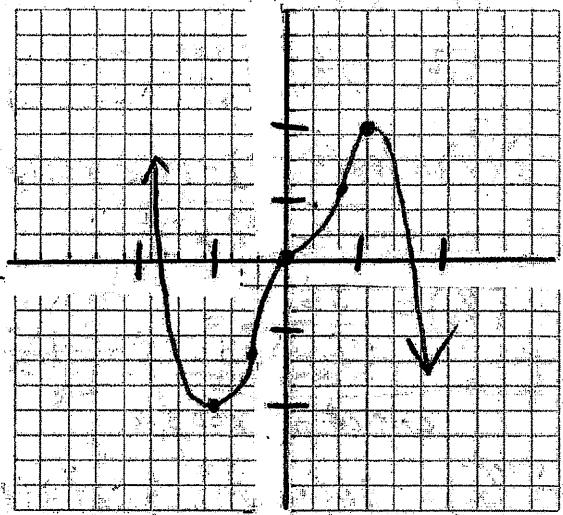
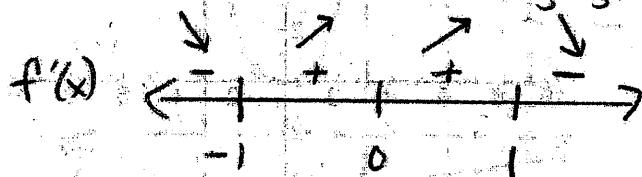
$$Ch. 3.6 \text{ Curve Sketching } O = x^3(-3x^2 + 5) \quad x = 0, \pm\sqrt{\frac{5}{3}}$$

1. Sketch the graph of the function and find the below information: $f(x) = -3x^5 + 5x^3$

$$f'(x) = -15x^4 + 15x^2 = -15x^2(x^2 - 1)$$

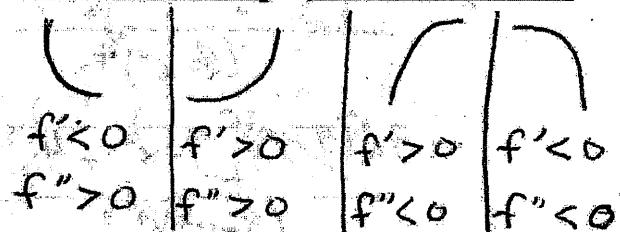
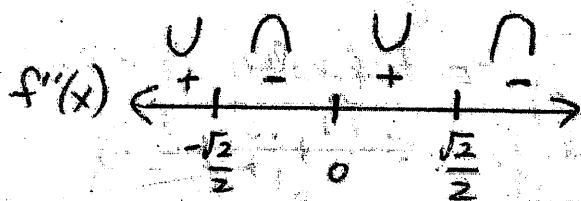
$$O = -15x^2(x+1)(x-1)$$

critical values: $x = 0, -1, 1$



$$f''(x) = -60x^3 + 30x = -30x(2x^2 - 1)$$

critical values: $x = 0, \pm\sqrt{\frac{1}{2}}$



Rel. min at $(-1, -2)$ b/c $f'(x)$ changes from - to +

Rel. max at $(1, 2)$ b/c $f'(x)$ changes from + to -

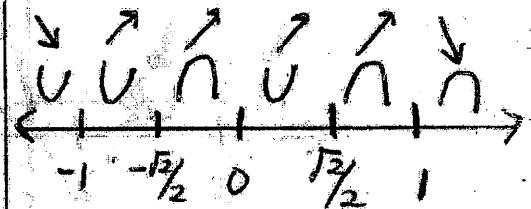
$f(x)$ increasing $(-1, 0) \cup (0, 1)$ b/c $f'(x) > 0$

$f(x)$ decreasing $(-\infty, -1) \cup (1, \infty)$ b/c $f'(x) < 0$

$f(x)$ concave up $(-\infty, -\frac{\sqrt{2}}{2}) \cup (0, \frac{\sqrt{2}}{2})$ b/c $f''(x) > 0$

$f(x)$ concave down $(-\frac{\sqrt{2}}{2}, 0) \cup (\frac{\sqrt{2}}{2}, \infty)$ b/c $f''(x) < 0$

POI at $x = -\frac{\sqrt{2}}{2}, 0, \frac{\sqrt{2}}{2}$ b/c $f''(x)$ change signs.



x-ints: $(0, 0)$ $(\sqrt{\frac{5}{3}}, 0)$ $(-\sqrt{\frac{5}{3}}, 0)$ y-ints: $(0, 0)$

V.A. none

H.A. none

Domain: $(-\infty, \infty)$

Interval Increasing $(-1, 0) \cup (0, 1)$

Interval Decreasing $(-\infty, -1) \cup (1, \infty)$

Relative Maximum $(1, 2)$

Relative Minimum: $(-1, -2)$

Points of Inflection: $(-\frac{\sqrt{2}}{2}, -1.24)$, $(\frac{\sqrt{2}}{2}, 1.24)$

Interval Concave Up: $(-\infty, -\frac{\sqrt{2}}{2}) \cup (0, \frac{\sqrt{2}}{2})$

Interval Concave Down: $(-\frac{\sqrt{2}}{2}, 0) \cup (\frac{\sqrt{2}}{2}, \infty)$

③ curve sketch rational function

2. Sketch the graph of the function and find the below information: $f(x) = \frac{2x^2}{9-x^2}$

$$VA: x = 3, -3 \quad HA: y = \frac{2}{-1} = -2$$

$$x\text{-int: } 0 = 2x^2 \quad \boxed{x=0} \quad y\text{-int: } y = \frac{0}{9-0} = 0$$

$$f'(x) = \frac{4x(9-x^2) - (2x^2)(-2x)}{(9-x^2)^2} = \frac{36x - 4x^3 + 4x^3}{(9-x^2)^2}$$

$$f'(x) = \frac{36x}{(9-x^2)^2}$$

\leftarrow	\downarrow	\downarrow	\rightarrow
-	+	+	+

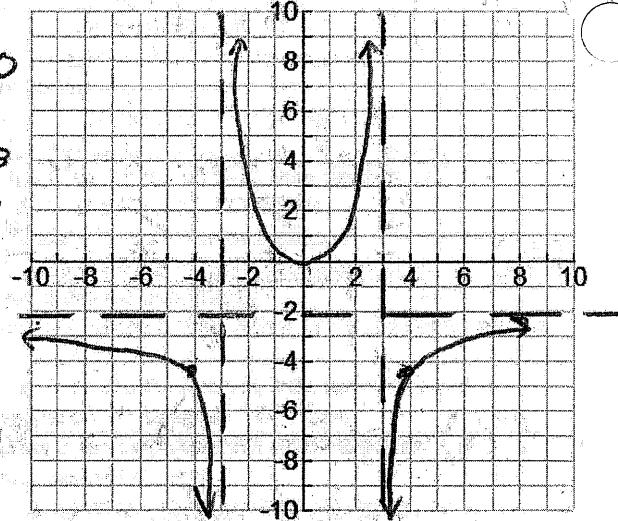
critical pts: $x = 0, 3, -3$

$$f''(x) = \frac{36(9-x^2)^2 - 36x[2(9-x^2)(-2x)]}{(9-x^2)^4}$$

$$= \frac{36(9-x^2)[9-x^2 + 4x^2]}{(9-x^2)^4} = \boxed{\frac{36(9+3x^2)}{(9-x^2)^3}}$$

\downarrow	\cap	\cup	\downarrow	\uparrow	\cap	\uparrow
-	+	+	+	+	+	-

-3 0 3



critical values: $x = 3, -3$

\leftarrow	\cap	\cup	\cap	\rightarrow
-	+	+	+	-

x-ints: (0, 0)

y-ints: (0, 0)

V.A. $x = 3, x = -3$

H.A. $y = -2$

Domain: $(-\infty, -3) \cup (-3, 3) \cup (3, \infty)$

Interval Increasing

$(0, 3) \cup (3, \infty)$

Interval Decreasing $(-\infty, -3) \cup (-3, 0)$

Relative Maximum

None

Relative Minimum: $(0, 0)$

Points of Inflection: None

Interval Concave Up: $(-3, 3)$

Interval Concave Down: $(-\infty, -3) \cup (3, \infty)$

④ Curve sketch ~~for~~ Trig function

4. a) Given the function $y = x - 2\cos x$ on the interval $[-\pi, \pi]$, find intervals increasing, decreasing, relative extrema (ordered pairs!), Points of Inflection (ordered pairs!), intervals of concave up/down. Justify your answers!

b) Create one sign line with all critical points from $f'(x)$ and $f''(x)$

c) Sketch graph from information above.

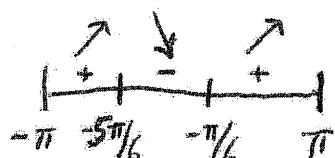
$$y' = 1 - 2(-\sin x) = 1 + 2\sin x$$

$$0 = 1 + 2\sin x$$

$$1 + 2\sin x = 0$$

$$\sin x = -\frac{1}{2}$$

$$x = -\frac{\pi}{6}, \frac{5\pi}{6}$$



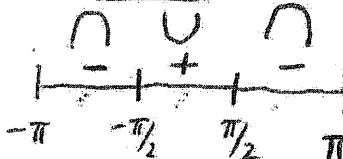
$$y'' = 0 + 2\cos x = 2\cos x$$

$$0 = 2\cos x$$

$$2\cos x = 0$$

$$\cos x = 0$$

$$x = -\frac{\pi}{2}, \frac{\pi}{2}$$



$$f(-\frac{5\pi}{6}) = -\frac{5\pi}{6} - 2\cos(-\frac{5\pi}{6}) = -\frac{5\pi}{6} - 2(\frac{-\sqrt{3}}{2}) \approx -1$$

$$f(-\frac{\pi}{6}) = -\frac{\pi}{6} - 2\cos(-\frac{\pi}{6}) = -\frac{\pi}{6} - 2(\frac{\sqrt{3}}{2}) \approx -2$$

$$f(-\frac{\pi}{2}) = -\frac{\pi}{2} - 2\cos(-\frac{\pi}{2}) = -\frac{\pi}{2} - 2(0) \approx -1.5$$

$$f(\frac{\pi}{2}) = \frac{\pi}{2} - 2\cos(\frac{\pi}{2}) = \frac{\pi}{2} - 2(0) \approx 1.5$$

Justify your answers with "Because" statements!

Interval(s) increasing:

$$(-\pi, -\frac{5\pi}{6}) \cup (-\frac{\pi}{6}, \pi) \text{ b/c } f'(x) > 0$$

Interval(s) Decreasing:

$$(-\frac{5\pi}{6}, -\frac{\pi}{6}) \text{ b/c } f'(x) < 0$$

Relative Maximum(s): $(-\frac{5\pi}{6}, -\frac{5\pi}{6} + \sqrt{3})$

b/c $f'(x)$ changes from + to -

Relative Minimum(s): at $(-\frac{\pi}{6}, -\frac{\pi}{6} - \sqrt{3})$

b/c $f'(x)$ changes from - to +

Interval(s) concave up:

$$(-\pi, -\frac{\pi}{2}) \cup (\frac{\pi}{2}, \pi) \text{ b/c } f''(x) > 0$$

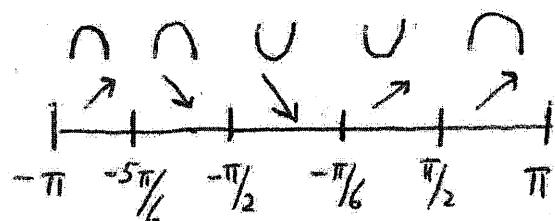
Interval(s) concave down:

$$(-\pi, -\frac{\pi}{2}) \cup (\frac{\pi}{2}, \pi) \text{ b/c } f''(x) < 0$$

Point(s) of inflection:

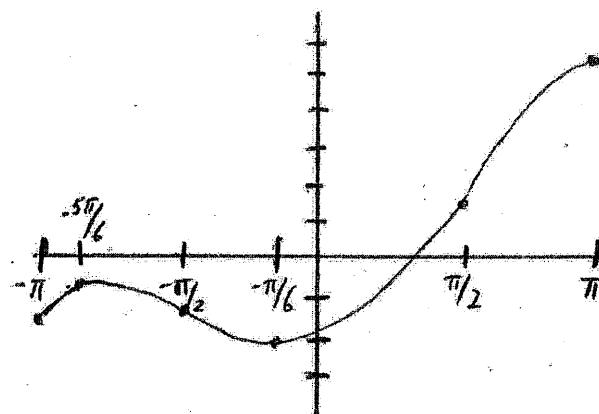
at $(-\frac{\pi}{2}, -\frac{\pi}{2})$ and $(\frac{\pi}{2}, \frac{\pi}{2})$ b/c $f''(x)$ changes signs.

Sign Line:



$$\text{Sketch Graph: } f(-\pi) = -\pi - 2\cos(-\pi) = -\pi + 2 = -1.14$$

$$f(\pi) = \pi - 2\cos(\pi) = \pi + 2 = 5.14$$



STATION #1 Given the function $h(t) = \frac{\ln t}{t}$ find the following

(5) Curve sketch Log function

a. Find $h'(t)$ and find extrema. Justify your answer

b) Find $h''(t)$ and POI. Justify your answer

c) Find end behavior: $\lim_{x \rightarrow 0^+} h(t)$ and $\lim_{x \rightarrow \infty} h(t)$

d) Find Domain and Range (you can find this after sketching your graph)

e) Sketch graph using info (a - d) above. Let $e \approx 3$, and $e^{\frac{3}{2}} \approx 4.5$

STATION #1 Given the function $h(t) = \frac{\ln t}{t}$ find the following

Domain: $t > 0$

a) Find $h'(t)$ and find extrema. Justify your answer

$$h'(t) = \frac{\frac{1}{t}(t) - \ln t(1)}{t^2} = \frac{1 - \ln t}{t^2}$$

$1 - \ln t = 0$ $\begin{array}{c|cc} + & 0 & e \\ \hline & + & - \end{array}$ Rel. max at $(e, \frac{1}{e})$
 $1 = \ln t$ b/c $h'(t)$ changes from + to -.
 $1 = \log_e t \quad /e^t = t$

b) Find $h''(t)$ and POI. Justify your answer

$$h''(t) = \frac{-\frac{1}{t}(t^2) - (1 - \ln t)(2t)}{t^4} = \frac{-t - 2t + 2t \ln t}{t^4}$$

$$-3 + 2\ln t = 0 \quad \begin{array}{c|cc} + & e^{\frac{3}{2}} & \infty \\ \hline & + & + \end{array} \quad = \frac{-3t + 2t \ln t}{t^4}$$

$$\ln t = \frac{3}{2} \quad \text{POI at } (e^{\frac{3}{2}}, \frac{3}{2e^{\frac{3}{2}}}) \quad b/c h''(t) \text{ change signs}$$

$$e^{\frac{3}{2}} = t \quad h''(t) = \frac{-3 + 2\ln t}{t^3}$$

c) Find end behavior: $\lim_{t \rightarrow 0^+} h(t)$ and $\lim_{t \rightarrow \infty} h(t)$

$$\lim_{t \rightarrow 0^+} \frac{\ln t}{t} = \frac{-}{+} = -\infty$$

$$\lim_{t \rightarrow \infty} \frac{\ln t}{t} = 0 \quad \leftarrow \text{use comparative growth rate}$$

$L < R < P < E$

d) Find Domain and Range (you can find this after sketching your graph)

$$D: (0, \infty)$$

$$R: (-\infty, \frac{1}{e}]$$

e) Sketch graph using info (a - d) above. Let $e \approx 3$, and $e^{\frac{3}{2}} \approx 4.5$

