

I. Limits:

1. Using the graph $f(x)$, find the below values.

a. $\lim_{x \rightarrow -\infty} f(x) =$

b. $\lim_{x \rightarrow -2} f(x) =$

c. $\lim_{x \rightarrow -2^-} f(x) =$

d. $\lim_{x \rightarrow -2^+} f(x) =$

e. $\lim_{x \rightarrow 0} f(x) =$

f. $\lim_{x \rightarrow 0^+} f(x) =$

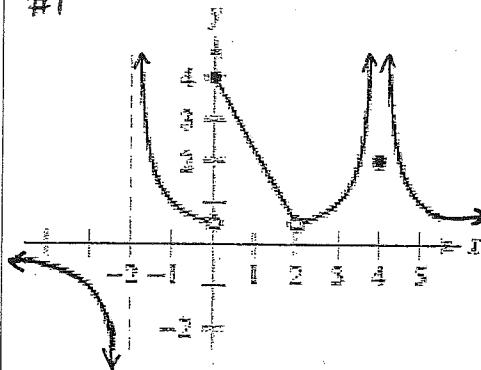
g. $\lim_{x \rightarrow 0^-} f(x) =$

h. $\lim_{x \rightarrow 2} f(x) =$

i. $\lim_{x \rightarrow 4} f(x) =$

j. $\lim_{x \rightarrow \infty} f(x) =$

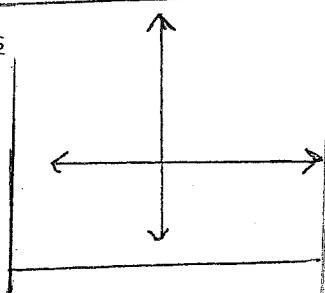
#1

1b. Draw a graph with the given characteristics

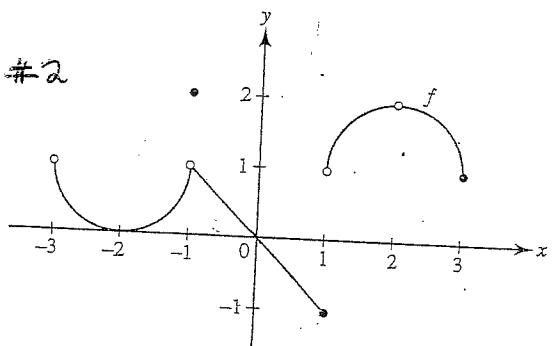
a) $f(-2)$ undefined d) $\lim_{x \rightarrow 3} f(x) = -\infty$

b) $\lim_{x \rightarrow -2^-} f(x) = 6$ e) $f(3) = 0$

c) $\lim_{x \rightarrow -2^+} f(x) = 1$



#2



2.

- a. For what value(s) of a is it true that $\lim_{x \rightarrow a} f(x)$ exists and $f(a)$ exists, but $\lim_{x \rightarrow a} f(x) \neq f(a)$
- A) -1 only B) 1 only C) 2 only
D) -1 or 1 only E) -1 or 2 only

b. $\lim_{x \rightarrow a} f(x)$ does not exist for $a =$

- A) -1 only B) 1 only C) 2 only
D) 1 and 2 only E) -1, 1, and 2

c. Which statements about limits at $x = 1$ are true?

- I. $\lim_{x \rightarrow 1^-} f(x)$ exists
II. $\lim_{x \rightarrow 1^+} f(x)$ exists
III. $\lim_{x \rightarrow 1} f(x)$ exists
A) none B) I only C) II only
D) I and II only E) all 3

II. Continuity Conditions:

3. Step through continuity conditions to determine if function is continuous or discontinuous. If discontinuous, state whether removable or nonremovable discontinuity. Use continuity condition to justify continuity or discontinuity

a.
$$f(x) = \begin{cases} x + 1, & x \leq 2 \\ -x, & x > 2 \end{cases}$$

b.
$$f(x) = \begin{cases} \tan \frac{\pi x}{4}, & -1 < x < 1 \\ x, & x \leq -1, x \geq 1 \end{cases}$$

III. Intermediate Value Theorem: If a function is continuous on $[a, b]$, then it passes through every value between $f(a)$ and $f(b)$.

4. **Multiple Choice:** Suppose that f is a continuous function defined for all real numbers x and $f(-5) = 3$ and $f(-1) = -2$. If $f(x) = 0$ for one and only one value of x , then which of the following could be x ?

(A) -7 (B) -2 (C) 0 (D) 1 (E) 2

IV. Extreme Value Theorem : If f is continuous over a closed interval, then f has a maximum and minimum value over that interval. (Test critical points, test endpoints)

5. Locate the value(s) where the function attains an absolute maximum and the value(s) where the function attains an absolute minimum, if they exist, on the given interval.

$$f(x) = x^3 - 3x + 1 \text{ on } [-3, 3]$$

- A) absolute min value: -3 absolute max value: 3
B) absolute min value: -17 absolute max value: 19
C) absolute min value: -1 absolute max value: 1
D) absolute min value: 0 absolute max value: 19
E) absolute min value: -19 absolute max value: 17

V. Mean Value Theorem and Rolle's Theorem

6. The function $f(x) = x^{2/3}$ on $[-8, 8]$ does not satisfy the conditions of the mean-value theorem because

- (A) $f(0)$ is not defined (B) $f(x)$ is not continuous on $[-8, 8]$ (C) $f'(-1)$ does not exist
(D) $f(x)$ is not defined for $x < 0$ (E) $f'(0)$ does not exist

7. If $f(a) = f(b) = 0$ and $f(x)$ is continuous on $[a, b]$, then

- (A) $f(x)$ must be identically zero. (B) $f'(x)$ may be different from zero for all x on $[a, b]$.
(C) there exists at least one number c , $a < c < b$, such that $f'(c) = 0$.
(D) $f'(x)$ must exist for every x on (a, b) . (E) none of the preceding is true.

8. (Calculator) For what value of c on $0 < x < 1$ is the tangent to the graph of $f(x) = e^x - x^2$ parallel to the secant line on the interval $[0, 1]$?

- a) -0.248 b) 0.351 c) 0.500 d) 0.693 e) 0.718

I. Limits:

1. Using the graph $f(x)$, find the below values.

a. $\lim_{x \rightarrow -\infty} f(x) = 0$

b. $\lim_{x \rightarrow -2} f(x) = \text{DNE}$

c. $\lim_{x \rightarrow -2^-} f(x) = -\infty$

d. $\lim_{x \rightarrow -2^+} f(x) = +\infty$

e. $\lim_{x \rightarrow 0} f(x) = \text{DNE}$

f. $\lim_{x \rightarrow 0^+} f(x) = 4$

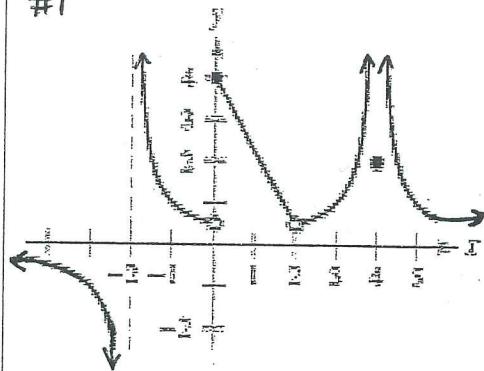
g. $\lim_{x \rightarrow 0^-} f(x) = 0.5$

h. $\lim_{x \rightarrow 2} f(x) = 0.5$

i. $\lim_{x \rightarrow 4} f(x) = +\infty$

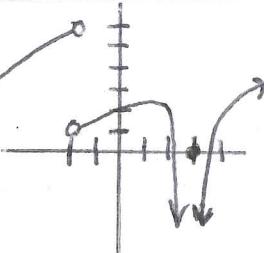
j. $\lim_{x \rightarrow \infty} f(x) = 0.5$

#1



1b) Draw graph with given characteristics:

- a) $f(-2)$ undefined d) $\lim_{x \rightarrow 3} f(x) = -\infty$
- b) $\lim_{x \rightarrow 2^-} f(x) = 6$ e) $f(3) = 0$
- c) $\lim_{x \rightarrow -2^+} f(x) = 1$



2.

- a. For what value(s) of a is it true that $\lim_{x \rightarrow a} f(x)$ exists and $f(a)$ exists, but $\lim_{x \rightarrow a} f(x) \neq f(a)$
- A) -1 only B) 1 only C) 2 only
D) -1 or 1 only E) -1 or 2 only

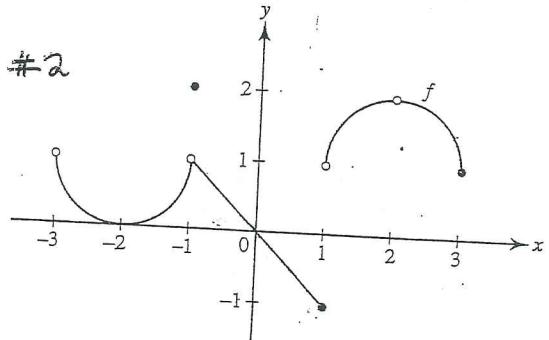
For $-3 < x < 3$

- b. $\lim_{x \rightarrow a} f(x)$ does not exist for $a =$ _____

- A) -1 only B) 1 only C) 2 only
D) 1 and 2 only E) -1, 1, and 2

c. Which statements about limits at $x = 1$ are true?

- I. $\lim_{x \rightarrow 1^-} f(x)$ exists
II. $\lim_{x \rightarrow 1^+} f(x)$ exists
III. $\lim_{x \rightarrow 1} f(x)$ exists
A) none B) I only C) II only
D) I and II only E) all 3

II. Continuity Conditions:

1) $f(c)$ is defined

2) $\lim_{x \rightarrow c} f(x)$ exists \rightarrow meaning that $\lim_{x \rightarrow c^-} f(x) = \lim_{x \rightarrow c^+} f(x)$

3) $\lim_{x \rightarrow c} f(x) = f(c)$

3. Step through continuity conditions to determine if function is continuous or discontinuous. If discontinuous, state whether removable or nonremovable discontinuity. Use continuity condition to justify continuity or discontinuity

a. $f(x) = \begin{cases} \frac{1}{2}x + 1, & x \leq 2 \\ 3 - x, & x > 2 \end{cases}$

1) $f(2) = \frac{1}{2}(2) + 1 = 2$

2) $\lim_{x \rightarrow 2^-} \frac{1}{2}x + 1 = \frac{1}{2}(2) + 1 = 2$ Since

$\lim_{x \rightarrow 2^+} 3 - x = 3 - 2 = 1$ $\lim_{x \rightarrow 2^-} f(x) \neq \lim_{x \rightarrow 2^+} f(x)$

$\lim_{x \rightarrow 2} f(x)$ does not exist, therefore nonremovable discontinuity

b. $f(x) = \begin{cases} \tan \frac{\pi x}{4}, & -1 < x < 1 \\ x, & x \leq -1, x \geq 1 \end{cases}$

1) $f(-1) = \tan \frac{-\pi}{4} = -1$

2) $\lim_{x \rightarrow -1^-} x = -1$

$\lim_{x \rightarrow -1^+} \tan \frac{\pi x}{4} = -1$

3) $f(-1) = \lim_{x \rightarrow -1} f(x)$

1) $f(1) = \tan \frac{\pi}{4} = 1$

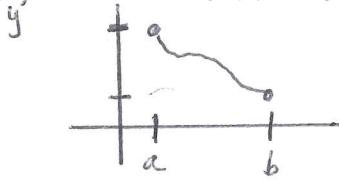
2) $\lim_{x \rightarrow 1^-} \tan \frac{\pi x}{4} = 1$

$\lim_{x \rightarrow 1^+} x = 1$

3) $f(1) = \lim_{x \rightarrow 1} f(x)$

So $f(x)$ is continuous at $x = 1$ and $x = -1$, so f is continuous for all x .

III. Intermediate Value Theorem: If a function is continuous on $[a, b]$, then it passes through every value between $f(a)$ and $f(b)$.



4. **Multiple Choice:** Suppose that f is a continuous function defined for all real numbers x and $f(-5) = 3$ and $f(-1) = -2$. If $f(x) = 0$ for one and only one value of x , then which of the following could be x ?

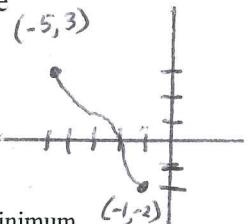
(A) -7

(B)

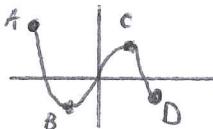
(C) 0

(D) 1

(E) 2



IV. Extreme Value Theorem : If f is continuous over a closed interval, then f has a maximum and minimum value over that interval. (Test critical points, test endpoints)



5. Locate the value(s) where the function attains an absolute maximum and the value(s) where the function attains an absolute minimum, if they exist, on the given interval.

$$f(x) = x^3 - 3x + 1 \text{ on } [-3, 3]$$

$$f'(x) = 3x^2 - 3 \quad 3x^2 - 3 = 0 \quad x = 1, -1$$

$$f(-3) = -17 \quad f(1) = -1$$

$$f(3) = 19 \quad f(-1) = 3$$

- A) absolute min value: -3 absolute max value: 3
 B) absolute min value: -17 absolute max value: 19
 C) absolute min value: -1 absolute max value: 1
 D) absolute min value: 0 absolute max value: 19
 E) absolute min value: -19 absolute max value: 17

V. Mean Value Theorem and Rolle's Theorem

$$f'(x) = \frac{2}{3}x^{-\frac{1}{3}} = \frac{2}{3x^{\frac{1}{3}}}$$

@ $f(x)$ continuous on $[-8, 8]$

① $f(x)$ not differentiable on $(-8, 8)$

6. The function $f(x) = x^{2/3}$ on $[-8, 8]$ does not satisfy the conditions of the mean-value theorem because

- (A) $f(0)$ is not defined (B) $f(x)$ is not continuous on $[-8, 8]$ (C) $f'(-1)$ does not exist

- (D) $f(x)$ is not defined for $x < 0$

- (E) $f'(0)$ does not exist

The condition that " $f(x)$ is differentiable on (a, b) " is not specified, so Rolle's theorem does not apply.

7. If $f(a) = f(b) = 0$ and $f(x)$ is continuous on $[a, b]$, then

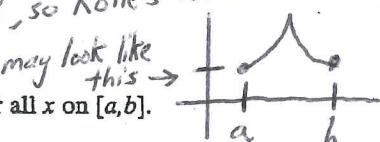
- (A) $f(x)$ must be identically zero.

- (B) $f'(x)$ may be different from zero for all x on $[a, b]$.

- (C) there exists at least one number c , $a < c < b$, such that $f'(c) = 0$.

- (D) $f'(x)$ must exist for every x on (a, b) .

- (E) none of the preceding is true.



8. (Calculator) For what value of c on $0 < x < 1$ is the tangent to the graph of $f(x) = e^x - x^2$ parallel to the secant line on the interval $[0, 1]$?

$$\frac{f(b)-f(a)}{b-a}$$

a) -0.248

b) 0.351

c) 0.500

d) 0.693

e) 0.718

$$e^x - 2x = e - 2$$

use calculator to find intersection b/t 2 graphs

$$\text{OR... } e^x - 2x - e + 2 = 0$$

Graph and look for x-int between interval.

Find c -value using Mean Value Theorem set $f'(c) = \frac{f(b)-f(a)}{b-a}$ and solve for c

$$f'(x) = e^x - 2x$$

$$f(0) = e^0 - 0 = 1$$

$$f(1) = e^1 - 1 = e - 1$$

$$\frac{f(1)-f(0)}{1-0} = \frac{e-1-1}{1} = e-2$$