

I. Equation of Tangent Lines

1. Find equation of tangent line

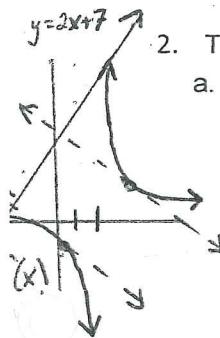
a. What is the equation of the tangent to the curve $\sin(\pi x) + 9 \cos(\pi y) = x^2 y$ at $(3, -1)$?

$$\begin{aligned} \cos(\pi x) \cdot \pi + 9 \cdot (-\sin(\pi y)) \cdot \pi \frac{dy}{dx} &= 2xy + x^2 \frac{dy}{dx} \\ \pi \cos(3\pi) - 9\pi \sin(-\pi) \frac{dy}{dx} &= 2(3)(-1) + 9 \frac{dy}{dx} \\ -\pi - 0 &= -6 + 9 \frac{dy}{dx} \quad \frac{dy}{dx} = \frac{6-\pi}{9} \end{aligned}$$

b. Find the equation of the tangent line to the curve $f(x) = \cos(3x) * \sin^2(2x - \pi)$ at $x = \frac{\pi}{3}$.*product rule: $f'g + fg'$

$$\begin{aligned} f(x) &= [-\sin(3x) \cdot 3] [\sin^2(2x - \pi)] + [\cos(3x)] \cdot 2 [\sin(2x - \pi)] \cos(2x - \pi) \cdot 2 \\ f'(\frac{\pi}{3}) &= (-3 \sin \pi) (\sin(\frac{2\pi}{3} - \pi))^2 + 4 \cos(\pi) \sin(\frac{5\pi}{3} - \pi) \cos(\frac{5\pi}{3} - \pi) \\ &= (0) (\sin(-\frac{\pi}{3}))^2 + 4 \cos \pi \sin(\frac{5\pi}{3}) \cos(\frac{5\pi}{3}) = 4(-1)(-\frac{\sqrt{3}}{2})(\frac{1}{2}) = \sqrt{3} \end{aligned}$$

2. Tangent to curve and parallel/perpendicular to given line

a. Find all points (x, y) on the graph $y = x/(x-2)$ with tangent lines perpendicular to the line $y = 2x+7$. Then find equations of tangent lines.

Steps:

- 1) Find $f'(x)$
- 2) Find slope of given line
- 3) Find \perp slope of line
- 4) set $f'(x) = \perp$ slope

5) Solve for x .6) Find ordered pair on $f(x)$.

7) Plug into linear equation.

$$y - y_1 = m(x - x_1)$$

$$f'(x) = \frac{1(x-2) - x(1)}{(x-2)^2} = \frac{-2}{(x-2)^2}$$

For $y = 2x+7$, $m = 2$ and perpendicular slope $= -\frac{1}{2}$

$$\begin{aligned} -2 &= -\frac{1}{2} \\ \frac{-2}{(x-2)^2} &= \frac{-1}{2} \\ (x-2)^2 &= 4 \\ x^2 - 4x + 4 &= 4 \\ x(x-4) &= 0 \\ f(0) &= 0 \\ f(4) &= \frac{4}{4-2} = \frac{4}{2} = 2 \end{aligned}$$

point: $(0, 0)$ slope: $m = -\frac{1}{2}$ point: $(4, 2)$ slope: $m = -\frac{1}{2}$ b. Find an equation of the tangent line to the graph $y = \sqrt{x-3}$ that is perpendicular to

$$6x+3y-4=0 \rightarrow 3y = -6x+4 \rightarrow y = -2x + \frac{4}{3}$$

$$y-2 = -\frac{1}{2}(x-4)$$

$$\begin{aligned} f'(x) &= \frac{1}{2}(x-3)^{-\frac{1}{2}}(1) \\ f'(x) &= \frac{1}{2\sqrt{x-3}} \\ \frac{1}{2\sqrt{x-3}} &= -2 \\ 2\sqrt{x-3} &= -1 \\ \sqrt{x-3} &= 1 \\ x-3 &= 1 \\ x &= 4 \end{aligned}$$

$$\begin{aligned} \frac{1}{2\sqrt{x-3}} &= \frac{1}{2} \\ f(3) &= \sqrt{3-3} = 0 \\ f(4) &= \sqrt{4-3} = 1 \quad (4, 1) \\ 2\sqrt{x-3} &= 2 \\ \sqrt{x-3} &= 1 \\ x-3 &= 1 \\ x &= 4 \end{aligned}$$

$$y - y_1 = m(x - x_1)$$

$$y - 0 = \frac{1}{2}(x - 3)$$

$$y - 1 = \frac{1}{2}(x - 4)$$

$$y = \frac{1}{2}x - \frac{3}{2}$$

c. Find an equation of the tangent line to the graph $y = x^3$ that is parallel to $3x - y - 4 = 0$

$$\begin{array}{ll|ll|ll} f'(x) = 3x^2 & m = 3 & f(1) = (1)^3 = 1 & \text{point: } (1, 1) & \text{point: } (-1, -1) \\ 3x - y - 4 = 0 & 3x^2 = 3 & f(-1) = (-1)^3 = -1 & m = 3 & m = 3 \\ 3x - 4 = y & x^2 = 1 & & y - 1 = 3(x - 1) & y + 1 = 3(x + 1) \\ x = 1, -1 & & & & & \end{array}$$

Problem #3 The tangent to the curve $y = 2x^3 - 3x^2 - 6x$ is parallel to the line $y = 6x - 1$ at the point(s) $x =$

- (A) -2 (B) -1 (C) 0 (D) 1 (E) 2 (F) 3

$$y' = 6x^2 - 6x - 6$$

$$6x^2 - 6x - 6 = 6$$

$$6x^2 - 6x - 12 = 0$$

$$x^2 - x - 2 = 0$$

$$(x-2)(x+1) = 0$$

$$x = 2, x = -1$$

For $y = 6x - 1$,

$$m = 6$$

II. Limit Definition of a Derivative

Limit Definition of a Derivative:

This is merely an expression for finding derivative or finding derivative at a point.

$$f(x) = \lim_{h \rightarrow 0} \frac{(x+h)^4 - x^4}{h}$$

4a) Find $\lim_{h \rightarrow 0} \frac{(x+h)^4 - x^4}{h}$

$$f(x) = x^4$$

$$f'(x) = \boxed{4x^3}$$

$$f'(c) = \lim_{h \rightarrow 0} \frac{f(c+h) - f(c)}{h}$$

$$a) \text{ Find } \lim_{h \rightarrow 0} \frac{(2+h)^4 - 2^4}{h}$$

$$f(x) = x^4 \quad | \quad f'(2) = 4(2)^3 = 4 \cdot 8 = \boxed{32}$$

$$f'(x) = 4x^3$$

$$b) \text{ Find } \lim_{h \rightarrow 0} \frac{\sqrt{x+h} - \sqrt{x}}{h}$$

$$f(x) = \sqrt{x}$$

$$f'(x) = \frac{1}{2}x^{-\frac{1}{2}} = \boxed{\frac{1}{2\sqrt{x}}}$$

$$c) \text{ Find } \lim_{h \rightarrow 0} \frac{\sqrt{9+h} - 3}{h}$$

$$f(x) = \sqrt{x}$$

$$f'(x) = \frac{1}{2\sqrt{x}} \quad | \quad f'(9) = \frac{1}{2\sqrt{9}} = \frac{1}{2(3)} = \boxed{\frac{1}{6}}$$

III. PVA and Particle Motion

5. A particle moves along a horizontal line. Its position function is given by

$$x(t) = -t^2 + 6t + 27 \text{ for values } 0 \leq t \leq 4.$$

- a. Find the displacement of the particle over the given time interval

$$x(0) = 0 + 0 + 27 = 27$$

$$x(4) = -4^2 + 6(4) + 27 = 35$$

$$x(4) - x(0) = 35 - 27 = \boxed{8}$$

- b. Find the distance the particle traveled over the given time interval ~~*consider possible direction change~~

$$\begin{array}{l|l} v(t) = -2t + 6 & x(0) = 27 \\ 0 = -2t + 6 & x(3) = 36 \\ t = 3 & x(4) = 35 \end{array} \quad \begin{array}{l|l} & x(0) = 27 \\ & x(3) = 36 \\ & x(4) = 35 \end{array} \quad \begin{array}{c} + \\ - \\ + \end{array} \quad \text{distance} = \boxed{10}$$

- c. At $t = 2$, is the particle's speed increasing or decreasing?

$$a(t) = -2$$

$$v(2) = -2(2) + 6 = 2$$

$$a(2) = -2$$

Since $v(2)$ and $a(2)$ have opposite signs, speed is decreasing at $t = 2$.

6. The position of a particle that is moving in a straight line is given by the equation

$$s = t^3 - 6t^2 + 9t$$

- (a) Find the displacement of the particle during the first five seconds.

$$s(0) = 0 + 0 + 0$$

$$s(5) = 5^3 - 6(5)^2 + 9(5) = 20$$

$$s(5) - s(0) = \boxed{20}$$

- (b) Find the total distance traveled by the particle during the first 5 seconds.

$$v(t) = 3t^2 - 12t + 9$$

$$0 = 3(t^2 - 4t + 3)$$

$$0 = 3(t-3)(t-1)$$

$$t=1, 3$$

$$\begin{array}{c} + \\ - \\ + \end{array} \quad v(t)$$

$$\begin{array}{l|l} s(0) = 0 & 4 \\ s(1) = 4 & 4 \\ s(3) = 0 & 20 \\ s(5) = 20 & \end{array} \quad \text{distance} = \boxed{28}$$

- (c) When is the particle speeding up? When is it slowing down?

$$a(t) \begin{array}{c} - \\ - \\ + \\ + \end{array}$$

$$v(t) \begin{array}{c} + \\ - \\ - \\ + \end{array}$$

$$\begin{array}{c} + \\ - \\ + \\ + \end{array} \quad \begin{array}{c} 0 \\ 1 \\ 2 \\ 3 \\ 5 \end{array}$$

speed up at $1 < t < 2$ and $3 < t < 5$
slow down at $0 < t < 1$ and $2 < t < 3$

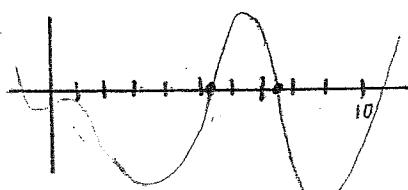
7. (Calculator section) An object moving along a line has velocity $v(t) = t \cos(t) - \ln(t+2)$, where $0 \leq t \leq 10$.

- a) For what value(s) of t is the object motionless?

$$x = 5.107 \quad \text{when } v(t) = 0$$

$$x = 7.55$$

$$V(t)$$



- b) How many times does the object reverse direction? twice, at $t = 5.107, 7.55$

$$v(t) \begin{array}{c} - \\ + \\ - \end{array} \quad \begin{array}{c} 0 \\ 5.107 \\ 7.55 \\ 10 \end{array}$$