CCGPS Analytic Geometry – Dec. 8, 2014 (Mon)
Notes: Complex Numbers Day 2 –
Homework: Multiplying and Dividing Complex Numbers Worksheet $\sqrt{-1} = i$

Essential Question: How do we multiply and divide complex numbers?

Consider about multiplication. What is $(\sqrt{-1})^2$? Let's try to find a pattern to the powers of i:

$i \cdot i = \boxed{-1}$

$\boxed{i^2 = -1}$

$i = i$
$i^2 = -1$
$i^3 = i^2 \cdot i = -i$
$i^4 = i^2 \cdot i^2 \quad (-1)(-1) = 1$
$i^5 = i$
$i^6 = -1$
$i^7 = -i$
$i^8 = 1$

$$\boxed{\begin{array}{l} R1 \to i \\ R2 \to -1 \\ R3 \to -i \\ R0 \to 1 \end{array}}$$

Notice that every 4 values, the pattern repeats. To figure out where we are in the pattern for bigger exponents, we can divide the exponent by 4 and use the remainder.

Model: For i^{22} consider $22 \div 4$. It has a remainder of 2. So, $i^{22} = i^2 = -1$

$4\overline{)22}$ = 5 R2; -20; $+2$

Examples: $4\overline{)12}$ = 3 R0

1. $i^{12} = \boxed{1}$

2. i^{26} $4\overline{)26}$ = 6 R2; -24; 2 $= \boxed{-1}$

3. i^{18} $4\overline{)18}$ = 4 R$\boxed{2}$; -16; $+2$ $= \boxed{-1}$

4. i^{11} $4\overline{)11}$ = 2 $\boxed{R3}$; 8 $= \boxed{-i}$

When we multiply complex numbers, we multiply similarly to variable expressions. We can distribute and then combine like terms. $\boxed{i^2 = -1}$

Examples:

5. $(8 + 5i)(2 - 3i)$

$16 - 24i + 10i - 15i^2$
$16 - 14i - 15(-1)$
$16 - 14i + 15$
$\boxed{31 - 14i}$

6. $(-6 + 2i)(5 - 3i)$

$-30 + 18i + 10i - 6i^2$
$-30 + 28i - 6(-1)$
$-30 + 28i + 6$
$\boxed{-24 + 28i}$

7. $4(4i + 12)(3 - 2i)$

$(16i + 48)(3 - 2i)$
$\cancel{48i} - 32i^2 + 144 - \cancel{96i}$
$-48i - 32(-1) + 144 = \boxed{176 - 48i}$

8. $(11 - 3i)(11 + 3i)$

$121 + \cancel{33i} - \cancel{33i} - 9i^2$
$121 - 9(-1)$
$121 + 9$
$\boxed{130}$

Notice in our last example what happened to *i*. This is because the expressions are known as <u>conjugates</u>. We will use this property to divide (or rationalize) complex numbers. REMEMBER: Whatever we multiply with the denominator, we need to also multiply with the numerator!!

Model: $\frac{3}{9-i} * \frac{(9+i)}{(9+i)} = \frac{27+3i}{9^2+1^2} = \frac{27}{82} + \frac{3}{82}i$

Examples: Write the following in standard form.

9. $\frac{(3-i)}{(2+7i)} \cdot \frac{(2-7i)}{(2-7i)}$

$\frac{6-21i-2i+7i^2}{4-\cancel{14i}+\cancel{14i}-49i^2} = \frac{6-23i+7(-1)}{4-49(-1)}$

$\frac{6-23i-7}{4+49} = \frac{-1-23i}{53}$

$\boxed{\frac{-1}{53} - \frac{23}{53}i}$ $a+bi$

10. $\frac{8i}{(1+2i)} \cdot \frac{(1-2i)}{(1-2i)} = \frac{8i-16i^2}{1-\cancel{2i}+\cancel{2i}-4i^2}$

$\frac{8i-16(-1)}{1-4(-1)} = \frac{8i+16}{1+4} = \frac{16+8i}{5}$

$\boxed{\frac{16}{5} + \frac{8}{5}i}$

11. $\frac{5+2i}{(3-i)} \cdot \frac{(3+i)}{(3+i)}$

12. $\frac{(2-3i)}{(2i)} \cdot \frac{i}{i}$

CCGPS Analytic Geometry

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1. $(12 + 7i)(2 - 4i)$

2. $(3 + 8i)(11 - 5i)$

3. $(4i - 6i)(9i + 10)$

4. $6(1 - i)(1 + i)$

5. $\dfrac{(3+2i)}{(1-3i)} \cdot \dfrac{(1+3i)}{(1+3i)}$

6. $\dfrac{(6-4i)}{(4+5i)} \cdot \dfrac{(4-5i)}{(4-5i)}$

7. $\dfrac{2-3i}{i} \cdot \dfrac{i}{i}$

8. $\dfrac{(4+5i)}{(2-i)} \cdot \dfrac{(2+i)}{(2+i)}$

9. $\dfrac{(6+7i)}{(3+2i)} \cdot \dfrac{(3-2i)}{(3-2i)}$