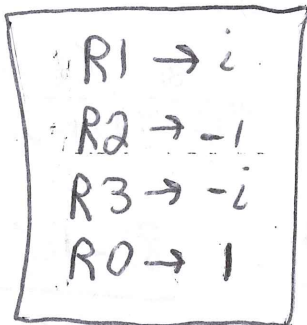


$$\sqrt{-1} = i$$

Essential Question: How do we multiply and divide complex numbers?

Consider about multiplication. What is $(\sqrt{-1})^2$? Let's try to find a pattern to the powers of i :

$$\begin{aligned} i &= i \\ i^2 &= -1 \\ i^3 &= i^2 \cdot i = -i \\ i^4 &= i^2 \cdot i^2 = (-1)(-1) = 1 \\ i^5 &= i \\ i^6 &= -1 \\ i^7 &= -i \\ i^8 &= 1 \end{aligned}$$



$$\begin{aligned} i \cdot i &= \boxed{-1} \\ i^2 &= \boxed{-1} \end{aligned}$$

Notice that every 4 values, the pattern repeats. To figure out where we are in the pattern for bigger exponents, we can divide the exponent by 4 and use the remainder.

Model: For i^{22} consider $22 \div 4$. It has a remainder of 2. So, $i^{22} = i^2 = -1$

Examples: $4 \overline{)12} \begin{matrix} 3R0 \\ \underline{12} \\ 0 \end{matrix}$

$$4 \overline{)22} \begin{matrix} 5R2 \\ \underline{20} \\ +2 \end{matrix}$$

1. $i^{12} = \boxed{1}$

2. $i^{26} = \boxed{-1}$

$$4 \overline{)26} \begin{matrix} 6R2 \\ \underline{-24} \\ +2 \end{matrix}$$

3. $i^{18} = \boxed{-1}$

$$4 \overline{)18} \begin{matrix} 4R2 \\ \underline{-16} \\ +2 \end{matrix}$$

4. $i^{11} = \boxed{-i}$

$$4 \overline{)11} \begin{matrix} 2R3 \\ \underline{8} \\ +3 \end{matrix}$$

When we multiply complex numbers, we multiply similarly to variable expressions. We can distribute and then combine like terms.

Examples:

5. $(8+5i)(2-3i)$

6. $(-6+2i)(5-3i)$

$$16 - 24i + 10i - 15i^2$$

$$16 - 14i - 15(-1)$$

$$16 - 14i + 15$$

$$\boxed{31 - 14i}$$

7. $4(4i+12)(3-2i)$

8. $(11-3i)(11+3i)$

$$(16i+48)(3-2i)$$

$$48i - 32i^2 + 144 - 96i$$

$$-48i - 32(-1) + 144 = \boxed{176 - 48i}$$

$$-30 + 18i + 10i - 6i^2$$

$$-30 + 28i - 6(-1)$$

$$-30 + 28i + 6$$

$$\boxed{-24 + 28i}$$

$$121 + 33i - 33i - 9i^2$$

$$121 - 9(-1)$$

$$121 + 9$$

$$\boxed{130}$$

Notice in our last example what happened to i . This is because the expressions are known as conjugates. We will use this property to divide (or rationalize) complex numbers.

REMEMBER: Whatever we multiply with the denominator, we need to also multiply with the numerator!!

Model: $\frac{3}{9-i} \cdot \frac{(9+i)}{(9+i)} = \frac{27+3i}{9^2+1^2} = \frac{27}{82} + \frac{3}{82}i$

Examples: Write the following in standard form.

9. $\frac{(3-i)(2-7i)}{(2+7i)(2-7i)}$

$$\frac{6-21i-2i+7i^2}{4-14i+14i-49i^2} = \frac{6-23i+7(-1)}{4-49(-1)}$$

$$\frac{6-23i-7}{4+49} = \frac{-1-23i}{53}$$

$$\boxed{\frac{-1}{53} - \frac{23}{53}i} \quad a+bi$$

10. $\frac{8i(1-2i)}{(1+2i)(1-2i)} = \frac{8i-16i^2}{1-2i+2i-4i^2}$

$$\frac{8i-16(-1)}{1-4(-1)} = \frac{8i+16}{1+4} = \frac{16+8i}{5}$$

$$\boxed{\frac{16}{5} + \frac{8}{5}i}$$

11. $\frac{5+2i}{3-i} \cdot \frac{(3+i)}{(3+i)}$

12. $\frac{(2-3i)}{2i} \cdot \frac{i}{i}$

CCGPS Analytic Geometry

Homework: Multiplying and Dividing Complex Numbers

1. $(12 + 7i)(2 - 4i)$

2. $(3 + 8i)(11 - 5i)$

3. $(4i - 6i)(9i + 10)$

4. $6(1 - i)(1 + i)$

5. $\frac{(3+2i)(1+3i)}{(1-3i)(1+3i)}$

6. $\frac{(6-4i)(4-5i)}{(4+5i)(4-5i)}$

7. $\frac{2-3i}{i} \cdot \frac{i}{i}$

8. $\frac{(4+5i)(2+i)}{(2-i)(2+i)}$

9. $\frac{(6+7i)(3-2i)}{(3+2i)(3-2i)}$

