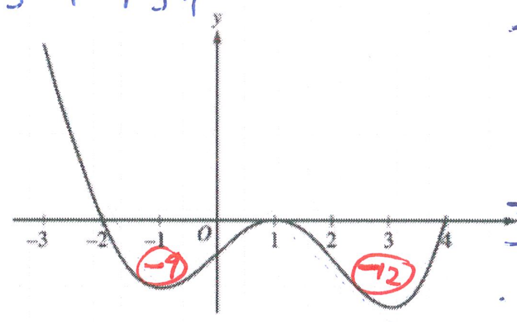
Derivative Graph FRQs **AP FRQ Review WS**

1) (Non-Calculator)

$f''(x)$: $\cap$ $\cup$ $\cap$ $\cup$ on $-3, -1, 1, 3, 4$

$f'(x)$: $+$ $\nearrow$, $-$ $\searrow$, $-$ $\searrow$ on $-3, -2, 1, 4$

The figure above shows the graph of f', the derivative of a twice-differentiable function f, on the interval $[-3, 4]$. The graph of f' has horizontal tangents at $x = -1$, $x = 1$, and $x = 3$. The areas of the regions bounded by the x-axis and the graph of f' on the intervals $[-2, 1]$ and $[1, 4]$ are 9 and 12, respectively.



Graph of f'

pos. slope $\nearrow$

$\to m = 0$

neg slope $\searrow$

(a) Find all x-coordinates at which f has a relative maximum. Give a reason for your answer.

(b) On what open intervals contained in $-3 < x < 4$ is the graph of f both concave down and decreasing? Give a reason for your answer.

(c) Find the x-coordinates of all points of inflection for the graph of f. Give a reason for your answer.

(d) Given that $f(1) = 3$, write an expression for $f(x)$ that involves an integral. Find $f(4)$ and $f(-2)$.

a) Relative max at $x = -2$ since f' changes from $+$ to $-$.

b) $f(x)$ is concave down and decreasing $-2 < x < 1$ and $1 < x < 3$ b/c f' is negative and decreasing

$f'(x)$: $\nearrow$ $\searrow$ $\searrow$ on $-3, -2, 1, 4$

$f''(x)$: $\cap$ $\cup$ $\cap$ $\cup$ on $-3, -1, 1, 3, 4$

c) POI at $x = -1, 1, 3$ b/c f' changes from inc/dec or dec/inc.

d) $* \; f(b) = f(a) + \int_a^b f'(x)\,dx$

final pos. = initial pos. + displacement

$f(4) = f(1) + \int_1^4 f'(x)\,dx$

$f(4) = 3 + (-12) = -9$

$\boxed{f(4) = -9}$

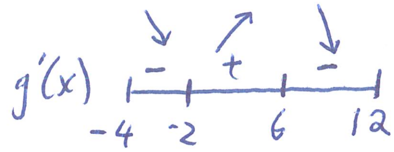
$f(-2) = f(1) + \int_1^{-2} f'(x)\,dx$

$f(-2) = f(1) - \int_{-2}^{1} f'(x)\,dx$

$= 3 - (-9)$

$\boxed{f(-2) = 12}$

2) (Non-Calculator)



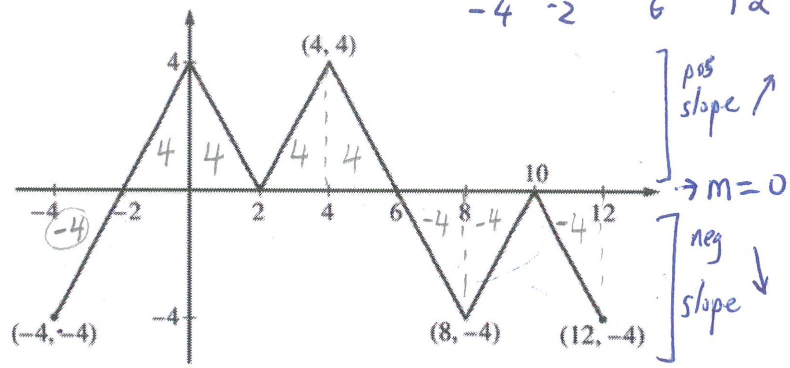
The figure above shows the graph of the piecewise-linear function f. For $-4 \le x \le 12$, the function g is defined by $g(x) = \int_2^x f(t)\,dt$.

(a) Does g have a relative minimum, a relative maximum, or neither at $x = 10$? Justify your answer.

(b) Does the graph of g have a point of inflection at $x = 4$? Justify your answer.

(c) Find the absolute minimum value and the absolute maximum value of g on the interval $-4 \le x \le 12$. Justify your answers.

(d) For $-4 \le x \le 12$, find all intervals for which $g(x) \le 0$.



Graph of f $= g'(x)$

a) The function g has neither min or max at $x=10$ since $g'(x)$ does not change signs at $x=10$.

b) POI at $x=4$ since $g'(x)$ changes from increasing to decreasing

c) Abs max/min *apply EVT (find critical points $(x=-2,6)$, test endpoints $(x=-4,12)$)

$g(-4) = \int_2^{-4} f(t)dt = -\int_{-4}^{2} f(t)dt = -(-4+8) = -4$

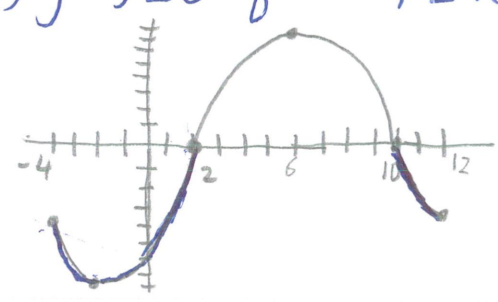
$g(-2) = \int_2^{-2} f(t)dt = -\int_{-2}^{2} f(t)dt = -(8) = -8$

$g(6) = \int_2^{6} f(t)dt = 8$

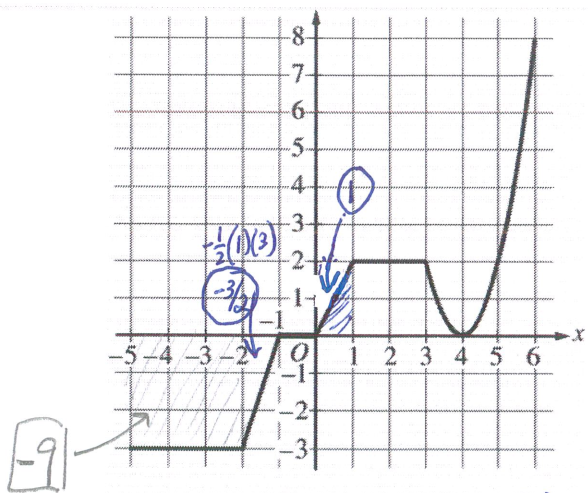
$g(12) = \int_2^{12} f(t)dt = -4$

Abs max value is 8 at $x=6$
Abs min value is -8 at $x=-2$

d) $g(x) \le 0$ for $-4 \le x \le 2$ and $10 \le x \le 12$



3) Non-Calculator



Graph of g $= f'(x)$

3. The graph of the continuous function g, the derivative of the function f, is shown above. The function g is piecewise linear for $-5 \le x < 3$, and $g(x) = 2(x-4)^2$ for $3 \le x \le 6$.

(a) If $f(1) = 3$, what is the value of $f(-5)$?

(b) Evaluate $\int_1^6 g(x)\,dx$.

(c) For $-5 < x < 6$, on what open intervals, if any, is the graph of f both increasing and concave up? Give a reason for your answer.

(d) Find the x-coordinate of each point of inflection of the graph of f. Give a reason for your answer.

a) $* \; f(b) = f(a) + \int_a^b f'(x)dx$ | $f(-5) = f(1) + \int_1^{-5} g(x)dx$

final = initial + displacement | $f(-5) = f(1) - \int_{-5}^{1} g(x)dx$

$f(-5) = 3 - (-9 - \frac{3}{2} + 1) = 3 - (-\frac{19}{2}) = \boxed{\frac{25}{2}}$

b) $\int_1^6 g(x)dx = \int_1^3 g(x)dx + \int_3^6 g(x)dx \rightarrow \int_3^6 2(x-4)^2 dx$

$u = x-4 \quad dx = du$

$\frac{du}{dx} = 1 \quad 2\int u^2 du$

$2(\frac{u^3}{3})$

$\frac{2}{3}(x-4)^3 \Big]_3^6 = \frac{2}{3}(2)^3 - \frac{2}{3}(-1)^3$

$\frac{16}{3} + \frac{2}{3} = 6$

$= 4 + 6 = \boxed{10}$

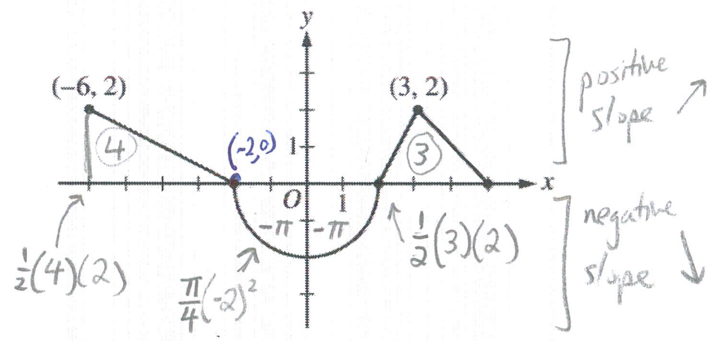
c) finding Increasing and concave up of $f(x)$ from a $f'(x)$ graph means interval where $f'(x)$ is above x-axis and positive slope:

Intervals are $0 < x < 1$ and $4 < x < 6$

d) POI of $f(x)$ from $f'(x)$ graph are peaks/valleys.

POI at $x = 4$ b/c $f'(x)$ changes from decreasing to increasing.

4) Non-Calculator



Graph of f'

The function f is differentiable on the closed interval $[-6, 5]$ and satisfies $f(-2) = 7$. The graph of f', the derivative of f, consists of a semicircle and three line segments, as shown in the figure above.

(a) Find the values of $f(-6)$ and $f(5)$.

(b) On what intervals is f increasing? Justify your answer.

(c) Find the absolute minimum value of f on the closed interval $[-6, 5]$. Justify your answer.

(d) For each of $f''(-5)$ and $f''(3)$, find the value or explain why it does not exist.

a) * $f(b) = f(a) + \int_a^b f'(x)dx$

final = given + displacement

$f(-6) = f(-2) + \int_{-2}^{-6} f'(x)dx$

$f(-6) = f(-2) - \int_{-6}^{-2} f'(x)dx$

$f(-6) = 7 - 4 = \boxed{3}$

$f(5) = f(-2) + \int_{-2}^{5} f'(x)dx$

$f(5) = 7 + (-\pi - \pi + 3)$

$f(5) = \boxed{10 - 2\pi}$

b) f is increasing when $f'(x)$ is above axis: f increasing on interval $(-6, -2)$ and $(2, 5)$ because $f'(x) > 0$

c) Abs max/min apply EVT, test critical points and endpts. (critical pts: $x = -2, x = 2$)

$f(-2) = 7$

$f(2) = f(-2) + \int_{-2}^{2} f'(x)dx = 7 - 2\pi \approx 1$

$f(-6) = 3$

$f(5) = 10 - 2\pi \approx 4$

$\boxed{\text{Abs min value is } 7 - 2\pi \text{ at } x = 2}$

d) $f''(-5) = \frac{0-2}{-2-(-6)} = \frac{-2}{4} = -\frac{1}{2}$

$f''(3)$ does not exist since slope doesn't exist:

$\lim_{x \to 3^-} f'(x) = 2 \neq \lim_{x \to 3^+} f'(x) = -1$