Key

Unit 2 Differentiation AP Review MC WS

Find the derivative, $\frac{dy}{dx}$ *of the function:*

1) $y=\sqrt{3-2x}$

(A) $\frac{1}{2\sqrt{3-2x}}$ $\boxed{\text{(B) } -\frac{1}{\sqrt{3-2x}}}$ (C) $-\frac{(3-2x)^{3/2}}{3}$

(D) $-\frac{1}{3-2x}$ (E) $\frac{2}{3}(3-2x)^{3/2}$

$y=(3-2x)^{1/2}$

*chain rule
out: $(\ )^{1/2}$
in: $3-2x$

$y'=\frac{1}{2}(3-2x)^{-1/2}(-2)$

$\boxed{y'=\frac{-1}{\sqrt{3-2x}}}$

2) $y=\frac{2}{(5x+1)^3}$

(A) $-\frac{30}{(5x+1)^2}$ $\boxed{\text{(B) } -30(5x+1)^{-4}}$ (C) $\frac{-6}{(5x+1)^4}$

(D) $-\frac{10}{3}(5x+1)^{-4/3}$ (E) $\frac{30}{(5x+1)^4}$

$y=2(5x+1)^{-3}$

*chain rule:
out: $2(\ )^{-3}$
in: $5x+1$

$y'=-6(5x+1)^{-4}(5)$

$\boxed{y'=\frac{-30}{(5x+1)^4}}$

3) $y=2\sqrt{x}-\frac{1}{2\sqrt{x}}$

(A) $x+\frac{1}{x\sqrt{x}}$ (B) $x^{-1/2}+x^{-3/2}$ (C) $\frac{4x-1}{4x\sqrt{x}}$

$\boxed{\text{(D)}}$ $\frac{1}{\sqrt{x}}+\frac{1}{4x\sqrt{x}}$ (E) $\frac{4}{\sqrt{x}}+\frac{1}{x\sqrt{x}}$

$y=2x^{1/2}-\frac{1}{2}x^{-1/2}$

*power rule

$y'=2\cdot\frac{1}{2}x^{-1/2}+\frac{1}{4}(x)^{-3/2}$

$\boxed{y'=\frac{1}{\sqrt{x}}+\frac{1}{4x^{3/2}}}$

4) $y=\ln\left(\frac{e^x}{e^x-1}\right)$

(A) $x-\frac{e^x}{e^x-1}$ (B) $\frac{1}{e^x-1}$ $\boxed{\text{(C)}}$ $-\frac{1}{e^x-1}$

(D) 0 (E) $\frac{e^x-2}{e^x-1}$

*quotient property
$\ln\left(\frac{a}{b}\right)=\ln a-\ln b$

*expand 1st:

$y=\ln(e^x)-\ln(e^x-1)$

$\frac{d}{dx}\ln u=\frac{u'}{u}$

$y=x-\ln(e^x-1)$

$y'=1-\frac{e^x}{e^x-1}$

$y'=\frac{e^x-1}{e^x-1}-\frac{e^x}{e^x-1}=\frac{\cancel{e^x-e^x}-1}{e^x-1}$

$\boxed{y'=\frac{-1}{e^x-1}}$

Find the derivative $\frac{dy}{dx}$

$\frac{d}{dx}\tan^{-1}(u) = \frac{u'}{1+u^2}$

5) $y = \tan^{-1}\frac{x}{2}$

(A) $\frac{4}{4+x^2}$ (B) $\frac{1}{2\sqrt{4-x^2}}$ (C) $\frac{2}{\sqrt{4-x^2}}$

(D) $\frac{1}{2+x^2}$ $\boxed{\text{(E)}}$ $\frac{2}{x^2+4}$

$y = \tan^{-1}\left(\frac{1}{2}x\right)$

$y' = \frac{\frac{1}{2}}{1+\left(\frac{1}{2}x\right)^2} = \frac{\frac{1}{2}}{1+\frac{x^2}{4}} \cdot \frac{4}{4}$

$\boxed{y' = \frac{2}{4+x^2}}$

6) $x + \cos(x+y) = 0$

$\boxed{\text{(A)}}$ $\csc(x+y) - 1$ (B) $\csc(x+y)$ (C) $\frac{x}{\sin(x+y)}$

(D) $\frac{1}{\sqrt{1-x^2}}$ (E) $\frac{1-\sin x}{\sin y}$

* Implicit differentiation

$1 - \sin(x+y)\cdot\left(1+\frac{dy}{dx}\right) = 0$

$1 - \sin(x+y) - \frac{dy}{dx}(\sin(x+y)) = 0$

$1 - \sin(x+y) = \frac{dy}{dx}(\sin(x+y))$

$\frac{1-\sin(x+y)}{\sin(x+y)} = \frac{dy}{dx}$

$\frac{1}{\sin(x+y)} - \frac{\sin(x+y)}{\sin(x+y)}$

$= \boxed{\csc(x+y) - 1}$

7) $3x^2 - \overbrace{2x}^{f}\overbrace{y}^{g} + 5y^2 = 1$ *Implicit, product rule

(A) $\frac{3x+y}{x-5y}$ $\boxed{\text{(B)}}$ $\frac{y-3x}{5y-x}$ (C) $3x+5y$

(D) $\frac{3x+4y}{x}$ (E) none of these

$6x - \overbrace{2}^{f'}\cdot\overbrace{y}^{g} + \overbrace{-2x}^{f}\overbrace{\left(\frac{dy}{dx}\right)}^{g'} + 10y\left(\frac{dy}{dx}\right) = 0$

$6x - 2y = 2x\left(\frac{dy}{dx}\right) - 10y\left(\frac{dy}{dx}\right)$

$6x - 2y = \frac{dy}{dx}(2x - 10y)$

$\frac{6x-2y}{2x-10y} = \frac{dy}{dx}$

$\boxed{\frac{dy}{dx} = \frac{3x-y}{x-5y} \text{ or } \frac{y-3x}{5y-x}}$

8) If a point moves on the curve $x^2 + y^2 = 25$, then, at $(0, 5)$, $\frac{d^2y}{dx^2}$ is

(A) 0 (B) $\frac{1}{5}$ (C) -5 $\boxed{\text{(D)}}$ $-\frac{1}{5}$ (E) nonexistent

$2x + 2y\left(\frac{dy}{dx}\right) = 0$

$2y\left(\frac{dy}{dx}\right) = -2x$

$\frac{dy}{dx} = \frac{-2x}{2y}$

$\frac{dy}{dx} = \frac{-x}{y}$

quotient Rule

$\frac{d^2y}{dx^2} = \frac{(-1)(y) - (-x)\left(\frac{dy}{dx}\right)}{y^2}$

$\frac{d^2y}{dx^2} = \frac{-y + x\left(\frac{-x}{y}\right)}{y^2}$

$\frac{d^2y}{dx^2} = \frac{-y - \frac{x^2}{y}}{y^2}$

$\left.\frac{d^2y}{dx^2}\right|_{(0,5)} = \frac{-5 - \frac{0^2}{5}}{5^2} = \frac{-5}{25} = \boxed{\frac{-1}{5}}$

9) $\lim_{h\to 0} \frac{\sqrt[3]{8+h} - 2}{h}$ is

(A) 0 $\boxed{\text{(B)}}$ $\frac{1}{12}$ (C) 1 (D) 192 (E) ∞

* L'Hopital's Rule

$\lim_{h\to 0} \frac{\sqrt[3]{8+h} - 2}{h} = \frac{\sqrt[3]{8} - 2}{0} = \frac{0}{0} \xrightarrow{L'H} \lim_{h\to 0} \frac{(8+h)^{1/3} - 2}{h} \rightarrow \lim_{h\to 0} \frac{\frac{1}{3}(8+h)^{-2/3}(1)}{1} = \frac{1}{3}(8)^{-2/3} = \frac{1}{3\cdot 4} = \boxed{\frac{1}{12}}$

85. Suppose $y = f(x) = 2x^3 - 3x$. If $h(x)$ is the inverse function of f, then $h'(-1) =$

(A) -1 (B) $\frac{1}{5}$ $\boxed{\text{(C)}}$ $\frac{1}{3}$ (D) 1 (E) 3

$f(1) = -1$	$h(-1) = 1$
$f'(\ ) = \_\_$	$h'(-1) = \_\_$

1) set $f(x) = -1$
$2x^3 - 3x = -1$
$2x^3 - 3x + 1 = 0$
$\boxed{x = 1}$ ← guess and check

2) find $f'(x) \to f'(x) = 6x^2 - 3$
3) $f'(1) = 6(1)^2 - 3 = 3$
4) flip slope to find $h'(-1) = \boxed{\frac{1}{3}}$

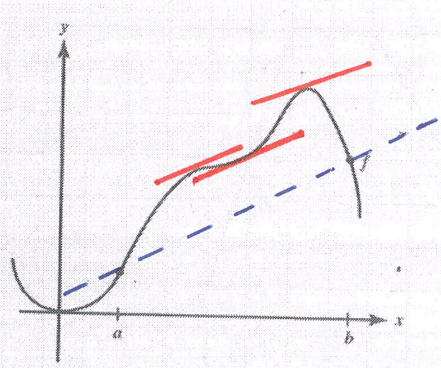
86. Suppose $f(1) = 2$, $f'(1) = 3$, and $f'(2) = 4$. Then $(f^{-1})'(2)$

(A) equals $-\frac{1}{3}$ (B) equals $-\frac{1}{4}$ (C) equals $\frac{1}{4}$

$\boxed{\text{(D)}}$ equals $\frac{1}{3}$ (E) cannot be determined

$f(1) = 2$	$f^{-1}(2) = 1$
$f'(1) = 3$	$(f^{-1})'(2) = \boxed{\frac{1}{3}}$

103. At how many points on the interval $[a,b]$ does the function graphed satisfy the Mean Value Theorem?



(A) none (B) 1 (C) 2 $\boxed{\text{(D)}\ 3}$ (E) 4

*MVT states that for a smooth curve, the slope between endpoints can be found at least once on the interval between endpoints: