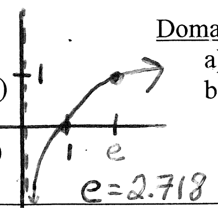
Calculus AB Fall Semester Summary / Formula Sheet

Even Functions: $f(-x) = f(x)$ (Symmetry about y-axis)

- $y = \cos(x)$ is an even function

Odd Function: $f(-x) = -f(x)$ (Symmetry about origin)

- $y = \sin(x)$ is an odd function



Domain: Defines where x-values exist in the function

a) Consider denominator (set denominator = 0)

b) Consider domain of numerator

i) For $y = \ln(x)$ domain $= \{x > 0\}$

ii) For $y = \sqrt{x}$ domain $= \{x \geq 0\}$

iii) For $y = e^x$ domain = All Real Numbers

Vertical Asymptote: Set denominator = 0

Horizontal Asymptote:

(Same as finding Limits at Infinity,

Evaluating Limits Approaching $\pm\infty$

1. If N < D $\lim_{x\to\infty} f(x) = \lim_{x\to-\infty} f(x) = 0$
2. If N=D $\lim_{x\to\infty} f(x) = \lim_{x\to-\infty} f(x) =$ ratio of coeff
3. If N>D then $\lim_{x\to\infty} f(x) = \pm\infty$ Plug in a large positive or large negative value to help you determine the sign at infinity)

*Remember that function can still exist where there is a horizontal asymptote. Horizontal asymptote describes end behavior

$$\lim_{x\to-\infty} \frac{a}{\sqrt{b}} = \lim_{x\to-\infty} \frac{a/x}{-\sqrt{b/x}}$$

Trig Limit Definitions:

$$\lim_{x\to 0} \frac{\sin ax}{bx} = \frac{a}{b} \qquad \lim_{x\to 0} \frac{1-\cos(x)}{(x)} = 0$$

Sum/Difference of Cubes:

$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

Evaluating Limits Approaching Real #

1. Plug in x-value.(Ignore one-sided limits) If real number results, this is your limit (answer) $\left(\frac{\textbf{zero}}{\textbf{nonzero}} = 0\right)$ **2.** If indeterminate form $\left(\frac{0}{0}\right)$ results, then factor, multiply by conjugate, or use trig limit rules to cancel out. Then re-evaluate. **3.** If $\left(\frac{\text{nonzero}}{zero}\right)$, then limit = DNE. Evaluate further ONLY IF this is one-sided limit problem.

a. Choose between $+\infty$ and $-\infty$ **b.** Plug in the appropriate decimal value to determine $\pm\infty$

Extreme Value Theorem (EVT) If f is continuous on $[a, b]$, then it has both a minimum and a maximum on that interval. **Steps**: 1) Find critical points (set $f'(x) = 0$). 2) Plug critical points and endpoints into f(x) 3) Compare y-values to determine abs max and abs min values.

Removable discontinuity is where variable from denominator cancels out with numerator (hole exists in graph)

NonRemovable discontinuity is where variable does not cancel out in denominator (Vertical asymptote exits in graph)

Squeeze Theorem:

If $h(x) \leq f(x) \leq g(x)$ for all x in an open interval containing c, and if

$\lim_{x\to c} h(x) = L = \lim_{x\to c} g(x)$, then

$\lim_{x\to c} f(x) = L$

Intermediate Value Theorem (IVT):

If f is continuous on a closed interval [a, b] and k is any number between f(a) and f(b), then there is at least one number c in [a, b] such that f(c) = k.

In other words, if a function is continuous on an interval, then it has to hit all of the y- values in between the endpoints somewhere in the interval.

Continuity Conditions:

For a function, f, to be continuous at c,

1. $f(c)$ is defined (the point exists)
2. $\lim_{x\to c} f(x)$ exists

*This means $\lim_{x\to c^-} f(x) = \lim_{x\to c^+} f(x)$

3. $\lim_{x\to c} f(x) = f(c)$

This means the point and the limits are equal to each other

Finding equation of tangent line: Steps:

1. Find slope by finding $f'(x)$, plugging in given x-value
2. If not given, find y-value using original function f(x)
3. Plug into point-slope: $y - y_1 = m(x - x_1)$

Piecewise functions:

*If piecewise function is continuous at a point, then set the two parts equal to each other.

*If piecewise function is differentiable at a point, then set the derivatives of each part equal to each other.

Comparative Growth Rates $\lim_{x\to\infty} \frac{f(x)}{g(x)}$

logs < radical < polynomial < exponential

Note: If a line is tangent to a curve at a point , **2 important properties hold true**:

a) The Curve and the tangent line share the same ordered pair at that point (they meet at the point) (set 2 functions equal to each other)

b) The Curve and the tangent line share the same slope at that point as well. (set their derivatives equal to each other)

*To find where function is not continuous set the denominator of $f(x) = 0$

* To find where function is not differentiable, set the denominator of $f'(x) = 0$

Limit Definition of Derivative:

1) General Definition

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

2) Alternative Definition

$$f'(c) = \lim_{x\to c} \frac{f(x) - f(c)}{x - c}$$

Derivative Rules:

Power Rule:

If $f(x) = u^n$, then $f'(x) = nu^{n-1} * u'$

Product Rule:

$$\frac{d}{dx}[f(x)g(x)] = f'g + f g'$$

Quotient Rule:

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{f'g - f g'}{g^2}$$

Chain Rule: (multiply outside function's derivative by inside function's derivative)

$$\frac{d}{dx}[f(g(x))] = f'(g(x)) \cdot g'(x)$$

Avg. ROC vs. Instantaneous ROC

(ROC = Rate of change)

Avg. ROC (avg velocity) $= \dfrac{f(b) - f(a)}{b - a}$

Inst. ROC (Inst. Velocity) $= f'(c)$

*Remember: Average Rate of change has - nothing to do with the derivative. (Just find slope between the endpoints!)

Increasing velocity means acceleration is pos.

Decreasing velocity means acceleration is neg.

Velocity is increasing if $a(t) > 0$

Velocity is decreasing if $a(t) < 0$

* Velocity $s'(t)$ is the derivative of position function

* Acceleration $s''(t)$ is the the 2nd derivative of position function. (acceleration is the derivative of velocity, the rate of change of velocity)

*Speed = |velocity| (speed is always positive)

*Speed is increasing when velocity and acceleration. have the same sign.

*Speed is decreasing when velocity and acceleration have opposite signs.

Displacement how far you are from where you started

Distance: total amount you have traveled

Linear Particle Motion – PVA

1. To find when particle at rest, set $v(t) = 0$ and solve for t.
2. Create sign line, and test intervals with v(t). Pos(+) means particle is moving right. Neg(–) means particle is moving left.

Linear approximation:

a) Find equation of tangent line
*$y - y_1 = m(x - x_1)$

b) Plug decimal value into tangent line equation to approximate function value: $y(x) \approx mx + b$

Implicit Differentiation:
When finding derivative, use this method if y is not defined explicitly in terms of x (In other words: there are multiple y's in the equation or y is mixed together with x's)
Steps: 1. Find derivative of each term 2.For derivatives involving y, be sure to attach $\frac{dy}{dx}$ 3. Solve for $\frac{dy}{dx}$

Curve Sketching:
*First Derivative Test: used for finding increasing/decreasing intervals and relative extrema:
1. Find critical points (set f '(x) = 0) 2. Make Slope Sign Line
3.Test each interval to determine + slope (inc.) or – slope (dec)
4. Rel. max if f '(x) changes from + to –
5. Rel. min if f '(x) changes from – to +
*Critical points can come from numerator or denominator
Finding Concavity/POI: 1. Find critical points [set f ''(x) = 0]
2.. Make 2nd derivative concavity sign line 3. Test interval.
+ means concave up, – means concave down. 4. POI exists if graph is continuous and change in concavity at critical point

L'Hopital's Rule: If plugging in x-value to find limit results in indeterminate form $\left(\frac{0}{0} \text{ or } \pm\frac{\infty}{\infty}\right)$

1) Apply L'Hopital's Rule (take derivative of top and derivative of bottom separately)

If $\lim\limits_{x\to c}\dfrac{f(x)}{g(x)} = \dfrac{0}{0} \text{ or } \pm\dfrac{\infty}{\infty}$, then $\lim\limits_{x\to c}\dfrac{f(x)}{g(x)} = \lim\limits_{x\to c}\dfrac{f'(x)}{g'(x)}$

2) then try to find limit again (plug in x-value)

Sketching Derivative graphs: (Free Response #6)

Related Rates: a) Pythag Theoreom problems b) Problems involving geometric shapes c) Trig angle problems
d) Similar Triangle Problems(see below)
Steps: 1)Write what's given 2)Write what you're finding 3)Write equation relating variables in problem 4)Differentiate with respect to t 5)Substitute and solve.
Ex: Similar triangle problem:

$\frac{dy}{dt}$ = R.O.C. of length of shadow; $\frac{dy}{dt} + \frac{dx}{dt}$ = R.O.C. of tip of shadow

Optimization steps: 1. Write equation for variable you want to optimize 2. Substitute to get equation in terms of one variable on the right side 3. Set derivative = 0 and solve.

Log Derivative Rules:

$$\frac{d}{dx}\ln|u| = \frac{1}{u} * u'$$

$$\frac{d}{dx}\log_b u = \frac{1}{\ln b} * \frac{1}{u} * u'$$

Exponential Derivatives Rules

$$\frac{d}{dx}e^u = e^u * u'$$

$$\frac{d}{dx}b^u = \ln b * b^u * u'$$

Trig Identities: $\sin^2\theta + \cos^2\theta = 1$
Double Angle: $\sin(2\theta) = 2\sin\theta\cos\theta$
$\cos(2\theta) = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$

Trig Derivatives:

$$\frac{d}{dx}\sin u = \cos u * u' \qquad \frac{d}{dx}\cos u = -\sin u * u'$$

$$\frac{d}{dx}\tan u = \sec^2 u * u' \qquad \frac{d}{dx}\cot u = -\csc^2 u * u'$$

$$\frac{d}{dx}\sec u = \sec u \tan u * u' \qquad \frac{d}{dx}\csc u = -\csc u \cot u * u'$$

Finding Absolute Extrema(EVT): (Closed Interval)
1.Find first derivative 2. Find critical numbers
3.Test critical numbers and endpoints by plugging them into the original function 4. Determine absolute max and min.
2nd Derivative Test: Used for finding relative extrema (finds max/min, **not** POI) 1.Set f '(x) = 0 and find critical numbers. 2. Find f ''(x). 3. Plug critical points (from step 1) into f ''(x) and evaluate 4. **If positive**, then rel. min occurs (b/c f '(x) = 0 and f ''(x) > 0 and therefore concave up.)
5. If **negative**, then rel. min occurs (b/c f '(x) = 0 and f ''(x) < 0 and therefore concave down

"**Morgan's Method**" for evaluating and interpreting Derivative Graphs

	f(x)	f '(x)	f ''(x)
X – x-ints	X		
M – max & mins	M	X	
P – POI	P	M	X
		P	M
			P

Rolle's Theorem: 3 Conditions
1. Continuous on [a,b] 2. Differentiable on (a,b)
3. f(a) = f(b) (endpoints have the same y-values)
Set derivative = 0 and solve for x
*Make sure c-value(x) resides **between** endpoints (a, b)
*Test if discontinuous in [a,b], set denom of f(x) = 0
*Test if not differentiable in (a, b), set denom of f '(x) = 0

Mean Value Theorem (MVT) 2 conditions:
1. Continuous on [a,b] 2. Differentiable on (a,b)

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$ (slope of secant line, slope b/t endpoint)

Set f '(x) = Avg slope(slope of secant line) and solve for x.
*Make sure x value resides between endpoints (a, b)

Log/Exponent Properties:
ln(1) = 0
ln(e) = 1
$\ln(a^n) = n*\ln(a)$
ln(ab) = ln(a) + ln(b)
$\ln\left(\frac{a}{b}\right) = \ln a - \ln b$
e≈ 2.718

$a^{\log_a x} = x$
$\log_a a^x = x$

Change of Base:
$$\log_a x = \frac{\ln x}{\ln a}$$

Exponent Properties:
$e^a e^b = e^{a+b}$ $(e^a)^b = e^{ab}$ $e^0 = 1$
$\ln e^x = x$ $\ln x = \log_e x$ $\log x = \log_{10} x$ $e^{\ln x} = x$

Log Differentiation steps: (Be sure no logs in the problem initially!) 1) Take **ln** of both sides. 2) Expand right side.
3)Find derivative 4) Solve for $\frac{dy}{dx}$ *Use Log Diff if $y = \text{variable}^{\text{variable}}$

Evaluate derivative of inverse at a point: (find $(f^{-1})'(a)$

$f(b) = a$	$(f^{-1})(a) = b$
$f'(b) = m$	$(f^{-1})'(a) = \frac{1}{m}$

Inverse Trig Derivatives:

$$\frac{d}{dx}\arcsin u = \frac{u'}{\sqrt{1-u^2}} \qquad \frac{d}{dx}\arccos u = -\frac{u'}{\sqrt{1-u^2}}$$

$$\frac{d}{dx}\arctan u = \frac{u'}{1+u^2} \qquad \frac{d}{dx}\operatorname{arc}\cot u = -\frac{u'}{1+u^2}$$

$$\frac{d}{dx}\operatorname{arc}\sec u = \frac{u'}{|u|\sqrt{u^2-1}} \qquad \frac{d}{dx}\operatorname{arc}\csc u = -\frac{u'}{|u|\sqrt{u^2-1}}$$