Exam Details:

Virtual students take Exam on Teams
F2F students take Exam in classroom

90 minutes: (8:30-10am)

$\boxed{16\ MC}$ Released 8:25am, Due 10:05

* penalty for late submissions

* Multiple Choice Answer Form on last page (complete it at the end)
* Show Work (where suitable) is required for full credit
* Open Notes, calculator
* Curve is applied (TBD)
* will not impact grade negatively
* Virtual students take on Teams

3rd, 4th

90 minutes (10:30-12pm)

Released 10:25am, Due 12:05pm

* penalty for late submission

Reminders: 1) Take the Fall Exam Survey (Remind message link)
2) Fall Exam Detail:
16 MC, 90 mins, open notes, show work for credit
3) Will not go in gradebook if you do poorly
4) Penalty for late submission
5) Exam is curved (TBD)

AP Calculus AB Fall Exam Topics 2020

1) Limits (ch. 1)

2) L'Hopital's Rule (Ch. 2)

3) Continuity and Continuity Conditions (ch. 1.4)

4) Finding Derivatives, Tangent Line Equations (Ch. 2)

*Product Rule, Quotient Rule, Chain Rule, Implicit Differentiation

5) Finding Average Rate of Change (Ch. 2) → slope $\frac{\text{change in position}}{\text{change in time}}$ → Avg. velocity

Fri (12/11) 6) Particle Motion (Position – Velocity – Acceleration) - Ch. 2

*Velocity sign line, intervals moving left/right, increasing/decreasing speed/velocity

Packet 3 7) Continuous/Differentiable Piecewise Functions (Ch. 2) share same slope → set equations equal → set derivatives equal

8) Related Rates (2.6) (Volume, Surface Area, cones, similar triangles)

Mon (12/14) 9) Linear Approximation (Ch. 3) i) find tangent line equation ii) plug in decimal in for x.

Fri (12/11) 10) Theorems: ~~IVT~~, EVT, Rolle's, MVT (Ch. 3)

11) Interpreting Derivative Graphs : f '(x) and f ''(x) (Ch 3)

(Fri 12/11) 12) First Derivative Test, First derivative sign line, 2nd derivative sign line (Ch. 3)

* Relative extrema, intervals increase/decrease

↗ | ↘ | ↗ f'(x) sign line

∪ | ∩ | ∪ f''(x) sign line

Mon (12/14) 13) New Derivative Rules (Ch. 2, 5)

*Derivative Rules involving Trig, Arc Trig, Logs, Exponentials

Mon (12/14) 14) Expanding Log terms before finding derivative (Ch. 5) (product, quotient, power property)

Mon (12/14) 15) Log Differentiation (Ch. 5) $y = (x+3)^{4x}$ → $\ln y = \ln(x+3)^{4x}$

16) Finding Derivative of Inverse at a point (Ch. 5)

$f(a) = b$	$f^{-1}(b) = a$
$f'(a) = n$	$(f^{-1})'(b) = \frac{1}{n}$

55) $y = \ln\left(\dfrac{\sqrt{x^2+1}}{x(2x^3-1)^2}\right)$

*Expand first before taking derivative

$$y = \ln\sqrt{x^2+1} - \ln x - \ln(2x^3-1)^2$$

$$y = \ln(x^2+1)^{1/2} - \ln x - \ln(2x^3-1)^2$$

$$y = \frac{1}{2}\ln(x^2+1) - \ln x - 2\ln(2x^3-1)$$

Take derivative $\frac{d}{dx}\ln u = \frac{u'}{u}$

$$y' = \frac{1}{\cancel{2}}\cdot\frac{\cancel{2}x}{x^2+1} - \frac{1}{x} - 2\cdot\frac{6x^2}{2x^3-1}$$

D

$$\boxed{y' = \frac{x}{x^2+1} - \frac{1}{x} - \frac{12x^2}{2x^3-1}}$$

$6\cdot\frac{1}{5} = \frac{6}{5} = 1.2$

$6\cdot 5/4 \qquad \frac{3\cdot 5}{2} = \frac{15}{2}$

Log differentiation

56) $y = \dfrac{\overbrace{x^3}^{f}}{\underbrace{3^x}_{g}}$

*quotient rule

* $\dfrac{d}{dx} a^u = \ln a \cdot a^u \cdot u'$

$$y' = \frac{\overbrace{3x^2}^{f'} \cdot \overbrace{3^x}^{g} - \overbrace{x^3}^{f} \cdot \overbrace{\ln 3 \cdot 3^x \cdot 1}^{g'}}{\underbrace{(3^x)^2}_{g^2}}$$

$$y' = \frac{\cancel{3^x} x^2 [3 - x \ln 3]}{(3^x)^{\cancel{2}}} = \boxed{\frac{x^2[3 - x\ln 3]}{3^x}} \quad \boxed{D}$$

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58) $y = \log_3(\underbrace{x^3 - 8x}_{u})$

* $\dfrac{d}{dx} \log_a u = \dfrac{1}{\ln a} \cdot \dfrac{u'}{u}$

$\boxed{B}$ $\boxed{y' = \dfrac{1}{\ln 3} \cdot \dfrac{3x^2 - 8}{x^3 - 8x}}$

Log differentiation: example

$$y = (x^2 + 3\cos x)^{4x}$$

* Log differentiation

$$\ln y = \ln(x^2 + 3\cos x)^{4x}$$

← expand the log term (power property)

$$\ln y = \underbrace{4x}_{f}\,\underbrace{\ln(x^2 + 3\cos x)}_{g}$$

← product Rule

Implicit

$\frac{d}{dx}\ln u = \frac{u'}{u}$

$\frac{d}{dx}\cos u = -\sin u \cdot u'$

$$\frac{1}{y}\left(\frac{dy}{dx}\right) = \overbrace{4}^{f'} \cdot \overbrace{\ln(x^2 + 3\cos x)}^{g} + \overbrace{4x}^{f} \cdot \overbrace{\frac{2x - 3\sin x}{x^2 + 3\cos x}}^{g'}$$

$$\frac{dy}{dx} = y\left[4\ln(x^2 + 3\cos x) + \frac{4x(2x - 3\sin x)}{x^2 + 3\cos x}\right]$$

Replace y variable

$$\boxed{\frac{dy}{dx} = (x^2 + 3\cos x)^{4x}\left[4\ln(x^2 + 3\cos x) + \frac{4x(2x - 3\sin x)}{x^2 + 3\cos x}\right]}$$

Packet #1

60) Derivative of Inverse at a point

$f(x) = \frac{1}{27}(x^5 + 2x^3)$ find $g'(-11)$ if $g(-11) = -3$

$f(-3) = -11$	$g(-11) = 3$
$f'(-3) = 17$	$\boxed{g'(-11) = \frac{1}{17}}$

$f(x) = \frac{1}{27}x^5 + \frac{2}{27}x^3$

$f'(x) = \frac{1}{27} \cdot 5x^4 + \frac{2}{27} \cdot 3x^2$

$f'(-3) = \frac{5}{27}(-3)^4 + \frac{6}{27}(-3)^2 = 17$

Linear Approximation:

Given $y = 2x^3 + 1$ at $(1, 3)$ Approximate $y(1.2)$

(a) find tangent line equation

(b) plug in the decimal to find value

$y' = 6x^2$

$y'(1) = 6(1)^2 = 6$

point: $(1,3)$

slope: $m = 6$

$y - 3 = 6(x - 1)$

$\boxed{y = 6(x - 1) + 3}$

$y(1.2) = 6(1.2 - 1) + 3$

$= 1.2 + 3$

$\boxed{y(1.2) = 4.2}$

54) $f(x) = \begin{cases} x^2 + kx - 3 & \text{for } x \le 1 \\ 3x + b & \text{for } x > 1 \end{cases}$

Packet #3

Continuous and differentiable. Find values of constants k and b

share same y-value at $x=1$ (set equations equal)

share same slope at $x=1$ (set derivatives equal)

at $x=1$

$x^2 + kx - 3 = 3x + b$

$1^2 + k(1) - 3 = 3 + b$

$k - 2 = 3 + b$

$k = 5 + b$

$(1) = 5 + b$

$-4 = b$

at $x=1$

$2x + k = 3$

$2(1) + k = 3$

$k = 1$

$f(-3) = -11$	$g(-11) = -3$
$f'(-3) = 17$	$\boxed{g'(-11) = \frac{1}{17}}$

$$f(x) = \frac{1}{27}x^5 + \frac{2}{27}x^3$$

$$f'(x) = \frac{1}{27}\cdot 5x^4 + \frac{2}{27}\cdot 3x^2$$

$$f'(x) = \frac{5}{27}x^4 + \frac{6}{27}x^2$$

$$f'(-3) = \frac{5}{27}(-3)^4 + \frac{6}{27}(-3)^2 = 17$$