AP Calculus AB First Semester Final Exam Review Packet

1. The domain of $g(x) = \frac{\sqrt{x-2}}{x^2-x}$ is

a. $(-\infty, 0) \cup (0, 1) \cup (1, \infty)$ b. $(-\infty, 0) \cup (0, 1) \cup (1, 2]$ c. $(-\infty, 2]$ d. $[2, \infty)$ e. $(2, \infty)$

2. Which of the following functions is not odd?

a. $f(x) = \sin x$ b. $f(x) = \sin 2x$ c. $f(x) = x^3 + 1$ d. $f(x) = \frac{x}{x^2+1}$ e. $f(x) = \sqrt[3]{2x}$

3. Which of the following is a reflection of the graph of $y = f(x)$ over the x-axis?

a. $y = -f(x)$ b. $y = f(-x)$ c. $y = |f(x)|$ d. $y = f(|x|)$ e. $y = -f(-x)$

4. $\lim_{x\to 3} \frac{x-3}{x^2-2x-3} =$

a. 0 b. 1 c. $\frac{1}{4}$ d. ∞ e. none of these

5. $\lim_{x\to 0} \frac{x}{x} =$

a. 1 b. 0 c. $\frac{0}{0}$ d. -1 e. nonexistent

6. $\lim_{x\to 2} \frac{x^3-8}{x^2-4} =$

a. 4 b. 0 c. 1 d. 3 e. ∞

7. $\lim_{x\to \infty} \frac{4-x^2}{4x^2-x-2} =$

a. -2 b. $-\frac{1}{4}$ c. 1 d. 2 e. nonexistent

8. $\lim_{x\to\infty} \frac{50x^3+27}{20x^2+10x+9} =$

a. ∞ b. $\frac{1}{4}$ c. 3 d. 0 e. 1

9. $\lim_{x\to\infty} \frac{3x^2+27}{x^3-27} =$

a. 3 b. ∞ c. 1 d. -1 e. 0

10. $\lim_{x\to 0} \frac{\tan x}{x} =$

a. 0 b. 1 c. π d. ∞ e. nonexistent

11. $\lim_{x\to 0} \frac{|x|}{x} =$

a. 0 b. 1 c. -1 d. nonexistent e. none of these

12. $\lim_{x\to\infty} \sin x =$

a. 0 b. -1 or 1 c. oscillates between -1 and 1 d. ∞ e. nonexistent

13. $\lim_{x\to 0} \frac{\sin 3x}{\sin 4x} =$

a. 0 b. 1 c. $\frac{3}{4}$ d. $\frac{4}{3}$ e. nonexistent

14. $\lim_{x\to 0} \frac{1-\cos x}{x} =$

a. 1 b. 2 c. ∞ d. nonexistent e. none of these

15. Let $f(x) = \begin{cases} x^2 - 1, & x \neq 1 \\ 4, & x = 1 \end{cases}$ **Which of the following statements are true?**

I. $\lim_{x\to 1} f(x)$ exists II. $f(1)$ exists III. f is continuous at $x = 1$

a. I only b. II only c. I and II d. I, II and III e. none of them

16. If $f(x) = \frac{x^2-x}{2x}$ for $\neq 0$, $f(0) = k$ and f is continuous at $x = 0$, then $k =$

a. -1 b. $-\frac{1}{2}$ c. 0 d. $\frac{1}{2}$ e. 1

17. If $f(x) = \frac{3x(x-1)}{x^2-3x+2}$ for $x \neq 1, 2$ and $f(1) = -3$ and $f(2) = 4$, then $f(x)$ is continuous

a. except at $x = 1$ b. except at $x = 2$ c. except at $x = 1$ or 2 d. except at $x = 0$, 1, or 2 e. at all real numbers

18. If $y = \frac{x}{\sqrt{1-x^2}}$, find $\frac{dy}{dx}$

a. $\frac{1-2x^2}{(1-x^2)^{3/2}}$ b. $\frac{1-2x^2}{(1-x^2)^{1/2}}$ c. $\frac{1}{\sqrt{1-x^2}}$ d. $\frac{1}{1-x^2}$ e. none of these

19. If $y = \cos x^2$, find y'

a. $2x \sin x^2$ b. $-\sin x^2$ c. $-2 \sin x \cos x$ d. $-2x \sin x^2$ e. $\sin 2x$

20. If $y = \sin^2 3x + \cos^2 3x$, find $\frac{dy}{dx}$

a. $-6 \sin 6x$ b. 0 c. $12 \sin 3x \cos 3x$ d. $6(\sin 3x + \cos 3x)$ e. 1

21. If $y = \cos^2 x$, find y'

a. $-\sin^2 x$ b. $2 \sin x \cos x$ c. $-\sin 2x$ d. $2 \cos x$ e. $-2 \sin x$

22. If $y = \frac{1}{2\sin 2x}$, find $\frac{dy}{dx}$

a. $-\csc 2x \cot 2x$ b. $\frac{1}{4\cos 2x}$ c. $-4\csc 2x \cot 2x$ d. $\frac{\cos 2x}{2\sqrt{\sin 2x}}$ e. $-\csc^2 2x$

23. If $x + \cos(x+y) = 0$, find $\frac{dy}{dx}$

a. $\csc(x+y) - 1$ b. $\csc(x+y)$ c. $\frac{x}{\sin(x+y)}$ d. $\frac{1}{\sqrt{1-x^2}}$ e. $\frac{1-\sin x}{\sin y}$

24. If $x^3 - xy + y^3 = 1$, find $\frac{dy}{dx}$

a. $\frac{3x^2}{x-3y^2}$ b. $\frac{3x^2-1}{1-3y^2}$ c. $\frac{y-3x^2}{3y^2-x}$ d. $\frac{3x^2+3y^2-y}{x}$ e. $\frac{3x^2+3y^2}{x}$

25. If $f(x) = 16\sqrt{x}$, then $f'''(4) =$

a. $\frac{3}{16}$ b. -4 c. $-\frac{1}{2}$ d. 0 e. 6

26. If $x^2 + y^2 = 25$, then at $(0, 5)$ $\frac{d^2y}{dx^2} =$

a. 0 b. $\frac{1}{5}$ c. -5 d. $-\frac{1}{5}$ e. nonexistent

27. If $f(x) = \frac{1}{x^2+1}$ and $g(x) = \sqrt{x}$, then the derivative of $f(g(x))$ is

a. $\frac{-\sqrt{x}}{(x^2+1)^2}$ b. $-(x+1)^{-2}$ c. $\frac{-2x}{(x^2+1)^2}$ d. $\frac{1}{(x+1)^2}$ e. $\frac{1}{2\sqrt{x}(x+1)}$

28. $\lim_{h\to 0} \frac{(1+h)^6-1}{h} =$

a. 0 b. 1 c. 6 d. ∞ e. nonexistent

29. $\lim_{h\to 0} \frac{\sqrt[3]{8+h}-2}{h} =$

a. 0 b. $\frac{1}{12}$ c. 1 d. 192 e. ∞

30. Suppose $\lim_{x\to 0} \frac{g(x)-g(0)}{x} = 1$. It follows that

a. g is not defined at $x = 0$ **b**. g is not continuous at $x = 0$

c. The limit of $g(x)$ as x approaches 0 equals 1 **d**. $g'(0) = 1$ **e**. $g'(1) = 0$

31. The function $f(x) = x^{2/3}$ on [-8, 8] does not satisfy the conditions of the Mean Value Theorem because

a. $f(0)$ is not defined b. $f(x)$ is not continuous on [-8, 8]

c. $f'(-1)$ does not exist d. $f(x)$ is not defined for $x < 0$ e. $f'(0)$ does not exist

32. If $f(a) = f(b) = 0$ and $f(x)$ is continuous on $[a, b]$, then

a. $f(x)$ must be identically zero b. $f'(x)$ may be different from zero for all x on $[a, b]$

c. there exists at least one number c, $a < c < b$, such that $f'(c) = 0$

d. $f'(x)$ must exist for every x on (a, b) e. none of the preceding is true

33. If c is the number defined by Rolle's Theorem, then for $f(x) = 2x^3 - 6x$ on the interval $[0, \sqrt{3}]$, c is

a. 1 b. -1 c. $\sqrt{2}$ d. 0 e. $\sqrt{3}$

34. The slope of the curve $y^3 - xy^2 = 4$ at the point where $y = 2$ is

a. -2 b. $\frac{1}{4}$ c. $-\frac{1}{2}$ d. $\frac{1}{2}$ e. 2

35. The equation of the line tangent to the curve $y = x \sin x$ at the point $\left(\frac{\pi}{2}, \frac{\pi}{2}\right)$ is

a. $y = x - \pi$ b. $y = \frac{\pi}{2}$ c. $y = \pi - x$ d. $y = x + \frac{\pi}{2}$ e. $y = x$

36. The minimum value of the slope of the curve $y = x^5 + x^3 - 2x$ is

a. 0 b. 2 c. 6 d. -2 e. none of these

37. The equation of the line tangent to the hyperbola $x^2 - y^2 = 12$ at the point (4, 2) is

a. $x - 2y + 6 = 0$ b. $y = 2x$ c. $y = 2x - 6$ d. $y = \frac{x}{2}$ e. $x + 2y = 6$

38. The line tangent to the curve $y^2 - xy + 9 = 0$ is vertical when

a. $y = 0$ b. $y = \pm\sqrt{3}$ c. $y = \frac{1}{2}$ d. $y = \pm 3$ e. none of these

39. The total number of relative maximum and minimum points on the function whose derivative, for all x, is given by $f'(x) = x(x-3)^2(x+1)^4$ is

a. 0 b. 1 c. 2 d. 3 e. none of these

40. On the closed interval $[0, 2\pi]$, find the maximum value of the function $f(x) = 4\sin x - 3\cos x$. (This is a bit of a challenge problem. ☺)

a. 3 b. 4 c. $\frac{24}{5}$ d. 5 e. none of these

41. The line $y = 3x + k$ is tangent to the curve $y = x^3$ when k is equal to

a. 1 or -1 b. 0 c. 3 or -3 d. 4 or -4 e. 2 or -2

42. A balloon is being filled with helium at the rate of $4\,\frac{\text{ft}^3}{\text{min}}$. Find the rate, in square feet per minute, at which the surface area is increasing when the volume is $\frac{32\pi}{3}\,\text{ft}^3$. Note, the volume of a sphere is $\frac{4}{3}\pi r^3$ and the surface area of a sphere is $4\pi r^2$.

a. 4π b. 2 c. 4 d. 1 e. 2π

For # **43 – 46**, the motion of a particle on a straight line is given by $x(t) = t^3 - 6t^2 + 12t - 8$.

43. The distance x is increasing for

a. $t < 2$ b. all t except $t = 2$ c. $1 < t < 3$ d. $t < 1$ or $t > 3$ e. $t > 2$

44. The minimum value of the speed is

a. 1 b. 2 c. 3 d. 0 e. none of these

For questions 43 – 46, the motion of a particle on a straight line is given by $x(t) = t^3 - 6t^2 + 12t - 8$.

45. The acceleration is positive when

a. $t > 2$ b. $t \neq 2$ c. $t < 2$ d. $1 < t < 3$ e. $1 < t < 2$

46. The speed of the particle is decreasing for

a. $t > 2$ b. $t < 3$ c. all t d. $t < 1$ or $t > 2$ e. none of these

47. A circular conical reservoir, vertex down, has depth 20 ft and radius of the top 10 ft. Water is leaking out so that the surface is falling at the rate of ½ ft/hour. The rate, in cubic feet per hour, at which the water is leaving the reservoir when the water is 8 ft deep is

a. 4π b. 8π c. 16π d. $\frac{1}{4\pi}$ e. $\frac{1}{8\pi}$

48. Two cars are traveling along perpendicular roads, car A at 40 mi/hour, car B at 60 mi/hour. At noon, when car A reaches the intersection, car B is 90 miles away and moving toward the intersection. At 1 pm, the distance between the cars is changing, in miles per hour, at a rate of

a. -40 b. 68 c. 4 d. -4 e. 40

49. Which statement below is true about the curve $= \frac{x^2+4}{2+7x-4x^2}$?

a. The line $x = -\frac{1}{4}$ is a vertical asymptote
b. The line $x = 1$ is a vertical asymptote
c. The line $y = \frac{1}{4}$ is a horizontal asymptote
d. The graph has no vertical or horizontal asymptotes
e. The line $y = 2$ is a horizontal asymptote

50. Find all open intervals for which the function $f(x) = \frac{x}{x^2+x-2}$ is decreasing.

a. $(-\infty, \infty)$ b. $(-\infty, 0)$ c. $(-\infty, -2)$ and $(1, \infty)$ d. $(-\infty, -2)$, $(-2,\ 1)$ and $(1, \infty)$ e. none of these

51. Find the values of x that give relative extrema for the function $f(x) = (x+1)^2(x-2)$.

a. Relative maximum at x = -1; relative minimum at x = 1
b. Relative maxima at x = 1, 3; relative minimum at x = -1
c. Relative minimum at x = 2
d. Relative maximum at x = -1; relative minimum at x = 2
e. None of these

52. Find all intervals on which the graph of the function $f(x) = \frac{x-1}{x+3}$ is concave upward.

a. $(-\infty, \infty)$ b. $(-\infty, -3)$ c. $(1, \infty)$ d. $(-3, \infty)$ e. none of these

53. Let $f''(x) = 3x^2 - 4$ and let $f(x)$ have critical numbers at -2, 0 and 2. Use the Second Derivative Test to determine which critical numbers, if any, give a relative maximum.

a. -2 b. 2 c. 0 d. -2 and 2 e. none of these

54. Find the coordinates of all extrema (relative and absolute) on the interval $[0, 2\pi]$ for $y = x - \cos x$.

a. $\left(\frac{3\pi}{2}, \frac{3\pi}{2}\right)$, $(2\pi, 2\pi - 1)$ **b.** $\left(\frac{3\pi}{2}, \frac{3\pi}{2}\right)$, (0, -1) **c.** $(2\pi, 2\pi - 1)$, (0, -1) **d.** (0, -1), $\left(\frac{3\pi}{2}, \frac{3\pi}{2}\right)$, $(2\pi, 2\pi - 1)$ **e.** none of these

55. Find the derivative of $f(x) = \ln \frac{\sqrt{x^2+1}}{x(2x^3-1)^2}$

a. $\frac{x}{x^2+1} - \frac{1}{x} + \frac{12x^2}{2x^3-1}$ **b.** $\frac{x}{x^2+1} - \frac{1}{x} + \frac{6x^2}{2x^3-1}$ **c.** $\frac{1}{(x^2+1)^{1/2}(4x^2)(2x^3-1)}$ **d.** $\frac{x}{x^2+1} - \frac{1}{x} - \frac{12x^2}{2x^3-1}$ **e.** none of these

56. Find y' if $y = \frac{x^3}{3^x}$

a. $\frac{x}{3^{x-2}}$ b. $\frac{3x^2}{3^x(\ln 3)}$ c. $\frac{x^2(9-x^2)}{3^{x+1}}$ d. $\frac{x^2[3-x(\ln 3)]}{3^x}$ e. none of these

57. Find $f'(x)$ for $f(x) = \frac{2}{2x+e^{2x}}$

a. 0 b. $\frac{1}{1+e^{2x}}$ c. $\frac{-4(1+e^{2x})}{(2x+e^{2x})^2}$ d. $\frac{1+xe^{2x-1}}{(2x+e^{2x})^2}$ e. none of these

58. If $y = \log_3(x^3 - 8x)$, find $\frac{dy}{dx}$

a. $\frac{3x^2-8}{x^3-8x}$ b. $\frac{3x^2-8}{(x^3-8x)(\ln 3)}$ c. $(3x^2 - 8)(\ln 3)$ d. $\frac{(3x^2-8)(\ln 3)}{(x^3-8x)}$ e. none of these

59. Find $\frac{dy}{dx}$ for $y = \arctan \frac{x}{2}$

a. $\frac{4}{4+x^2}$ b. $\frac{4}{1+x^2}$ c. $\frac{1}{\sqrt{4-x^2}}$ d. $\frac{1}{2}\sec^2\left(\frac{x}{2}\right)$ e. $\frac{2}{4+x^2}$

60. If $f(x) = \frac{1}{27}(x^5 + 2x^3)$, and $g(x) = f^{-1}(x)$, find $(g)'(-11)$ if $g(-11) = -3$.

a. 17 b. $\frac{1}{17}$ c. $\frac{27}{73931}$ d. $\frac{73931}{27}$ e. none of these