AP Calculus AB Fall Exam Review WS #2

I. Equation of Tangent Lines

1. Find equation of tangent line

a. What is the equation of the tangent to the curve $\sin(\pi x) + 9\cos(\pi y) = x^2 y$ at (3, -1)?

b. Find the equation of the tangent line to the curve f(x) = cos(3x)*sin$^2(2x - \pi)$ at x = $\frac{\pi}{3}$

2. Tangent to curve and parallel/perpendicular to given line

a. Find all points (x,y) on the graph y= x/(x-2) with tangent lines perpendicular to the line y=2x+7. Then find equations of the tangent lines.

b. Find an equation of the tangent line to the graph $y = \sqrt{x-3}$ that is perpendicular to $6x+3y-4=0$.

c. Find an equation of the tangent line to the graph $y = x^3$ that is parallel to $3x - y - 4 = 0$

Problem 14 The tangent to the curve $y = 2x^3 - 3x^2 - 6x$ is parallel to the line $y = 6x - 1$ at the point(s) $x =$ (circle all that apply)

(A) -2 (B) -1 (C) 0 (D) 1 (E) 2 (F) 3

II. Limit Definition of a Derivative

Limit Definition of a Derivative: This is merely an expression for finding derivative or finding derivative at a point.	$f'(c) = \lim_{h\to 0}\frac{f(c+h)-f(c)}{h}$
4a) Find $\lim_{h\to 0}\frac{(x+h)^4 - x^4}{h} =$	a) Find $\lim_{h\to 0}\frac{(2+h)^4 - 2^4}{h}$
b) Find $\lim_{h\to 0}\frac{\sqrt{x+h}-\sqrt{x}}{h}$	c) Find $\lim_{h\to 0}\frac{\sqrt{9+h}-3}{h} =$

III. PVA and Particle Motion

5. A particle moves along a horizontal line. Its position function is given by $x(t) = -t^2 + 6t + 27$ for values $0 \le t \le 4$.

a. Find the displacement of the particle over the given time interval	b. Find the distance the particle traveled over the given time interval	c. At t = 2, is the particle's speed increasing or decreasing?

6. The position of a particle that is moving in a straight line is given by the equation
$s = t^3 - 6t^2 + 9t$

(a) Find the displacement of the particle during the first five seconds.	(b) Find the total distance traveled by the particle during the first 5 seconds.	(c) When is the particle speeding up? When is it slowing down?

7. (Calculator section) An object moving along a line has velocity $v(t) = t\cos(t) - \ln(t+2)$, where $0 \le t \le 10$.

a) For what value(s) of t is the object motionless?	b) How many times does the object reverse direction?

I. Equation of Tangent Lines

1. Find equation of tangent line

a. What is the equation of the tangent to the curve $\sin(\pi x) + 9\cos(\pi y) = x^2 y$ at (3, -1)?

$\cos(\pi x)\cdot\pi + 9\cdot(-\sin(\pi y))\cdot\pi\frac{dy}{dx} = 2xy + x^2\frac{dy}{dx}$

$\pi\cos(3\pi) - 9\pi\sin(-\pi)\frac{dy}{dx} = 2(3)(-1) + 9\frac{dy}{dx}$

$-\pi - 0 = -6 + 9\frac{dy}{dx}$ $\quad \frac{dy}{dx} = \frac{6-\pi}{9}$

$\boxed{y+1 = \frac{6-\pi}{9}(x-3)}$

b. Find the equation of the tangent line to the curve f(x) = cos(3x)*sin$^2(2x - \pi)$ at x = $\frac{\pi}{3}$

*product rule: $f'g + fg'$

$f'(x) = [-\sin(3x)\cdot 3][\sin^2(2x-\pi)] + [\cos(3x)]\cdot 2[\sin(2x-\pi)]\cos(2x-\pi)\cdot 2$

$f'(\pi/3) = (-3\sin\pi)(\sin(\frac{2\pi}{3}-\pi))^2 + 4\cos(\pi)\sin(2\pi/3-\pi)\cos(2\pi/3-\pi)$

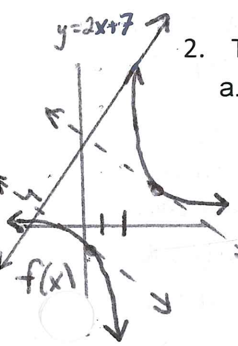
$= (0)(\sin(-\frac{\pi}{3}))^2 + 4\cos\pi\sin(5\pi/3)\cos(5\pi/3) = 4(-1)(-\frac{\sqrt{3}}{2})(\frac{1}{2}) = \sqrt{3}$

$m = \sqrt{3}$ point: $(\pi/3, -3/4)$

$f(\pi/3) = \cos(\pi)[\sin(5\pi/3)]^2$

$= (-1)(-\frac{\sqrt{3}}{2})^2 = -3/4$

$\boxed{y + 3/4 = \sqrt{3}(x - \pi/3)}$



2. Tangent to curve and parallel/perpendicular to given line

a. Find all points (x,y) on the graph y= x/(x-2) with tangent lines perpendicular to the line y=2x+7. Then find equations of tangent lines

Steps:
1) Find $f'(x)$
2) Find slope of given line
3) Find $\perp$ slope of line
4) set $f'(x) = \perp$ slope
5) Solve for x.
6) Find ordered pair on f(x).
7) Plug into linear equation. $y - y_1 = m(x - x_1)$

$f'(x) = \frac{1(x-2)-x(1)}{(x-2)^2} = \frac{-2}{(x-2)^2}$

For $y = 2x+7$, $m = 2$ and perpendicular slope $= -1/2$

$\frac{-2}{(x-2)^2} = \frac{-1}{2}$ | $(x-2)^2 = 4$, $x^2 - 4x + 4 = 4$, $x(x-4) = 0$ | $x = 0, 4$, $f(0) = 0$, $f(4) = \frac{4}{4-2} = \frac{4}{2} = 2$

point: (0,0) slope: $m = -1/2$ $\boxed{y - 0 = -\frac{1}{2}(x-0)}$

point: (4,0) slope: $m = -1/2$ $\boxed{y - 0 = -\frac{1}{2}(x-4)}$

b. Find an equation of the tangent line to the graph $y = \sqrt{x-3}$ that is perpendicular to 6x+3y−4=0. $\rightarrow 3y = -6x - 4 \rightarrow y = -2x - 4/3$

$f'(x) = \frac{1}{2}(x-3)^{-1/2}(1)$

$f'(x) = \frac{1}{2\sqrt{x-3}}$

$y = -2x - 4/3$

$m = -2$ $\quad \perp m = \frac{1}{2}$

$\frac{1}{2\sqrt{x-3}} = \frac{1}{2}$, $2\sqrt{x-3} = 2$, $\sqrt{x-3} = 0$, $x = 3$

$f(3) = \sqrt{3-3} = 0$

point: (3, 0)

slope: $m = 1/2$

$y - y_1 = m(x - x_1)$

$y - 0 = \frac{1}{2}(x-3)$

$\boxed{y = \frac{1}{2}x - \frac{3}{2}}$

c. Find an equation of the tangent line to the graph y = x^3 that is parallel to 3x – y – 4 = 0

$f'(x) = 3x^2$

$3x - y - 4 = 0$

$3x - 4 = y$

$m = 3$

$3x^2 = 3$, $x^2 = 1$, $x = 1, -1$

$f(1) = (1)^3 = 1$

$f(-1) = (-1)^3 = -1$

point: (1,1) $m = 3$ $\boxed{y - 1 = 3(x-1)}$

point (-1,-1) $m = 3$ $\boxed{y + 1 = 3(x+1)}$

Problem #3 The tangent to the curve $y = 2x^3 - 3x^2 - 6x$ is parallel to the line $y = 6x - 1$ at the point(s) $x =$

(A) −2 $\boxed{\text{(B) } -1}$ (C) 0 (D) 1 $\boxed{\text{(E) } 2}$ (F) 3

$y' = 6x^2 - 6x - 6$

For $y = 6x - 1$, $m = 6$

$6x^2 - 6x - 6 = 6$

$6x^2 - 6x - 12 = 0$

$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$\boxed{x = 2, x = -1}$

II. Limit Definition of a Derivative

Limit Definition of a Derivative: This is merely an expression for finding derivative or finding derivative at a point.	$f'(c) = \lim\limits_{h \to 0} \frac{f(c+h) - f(c)}{h}$

4a) Find $\lim\limits_{h \to 0} \frac{(x+h)^4 - x^4}{h} =$

$f'(x) = \lim\limits_{h \to 0} \frac{(x+h)^4 - x^4}{h}$

$f(x) = x^4$

$f'(x) = \boxed{4x^3}$

a) Find $\lim\limits_{h \to 0} \frac{(2+h)^4 - 2^4}{h}$

$f'(2) = \lim\limits_{h \to 0} \frac{(2+h)^4 - 2^4}{h}$

$f(x) = x^4$ | $f'(2) = 4(2)^3 = 4 \cdot 8 = \boxed{32}$

$f'(x) = 4x^3$

b) Find $\lim\limits_{h \to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h}$

$f(x) = \sqrt{x}$

$f'(x) = \frac{1}{2}x^{-1/2} = \boxed{\frac{1}{2\sqrt{x}}}$

c) Find $\lim\limits_{h \to 0} \frac{\sqrt{9+h} - 3}{h} =$

$f(x) = \sqrt{x}$ | $f'(9) = \frac{1}{2\sqrt{9}} = \frac{1}{2(3)} = \boxed{\frac{1}{6}}$

$f'(x) = \frac{1}{2\sqrt{x}}$

III. PVA and Particle Motion

5. A particle moves along a horizontal line. Its position function is given by $x(t) = -t^2 + 6t + 27$ for values $0 \le t \le 4$.

a. Find the displacement of the particle over the given time interval

$x(0) = 0 + 0 + 27 = 27$

$x(4) = -4^2 + 6(4) + 27 = 35$

$x(4) - x(0) = 35 - 27 = \boxed{8}$

b. Find the distance the particle traveled over the given time interval *consider possible direction change

$v(t) = -2t + 6$

$0 = -2t + 6$

$t = 3$

$v(t)$: 0 + 3 − 4

$x(0) = 27$ > 9

$x(3) = 36$ > 1

$x(4) = 35$

distance $= \boxed{10}$

c. At t = 2, is the particle's speed increasing or decreasing?

$a(t) = -2$

$v(2) = -2(2) + 6 = 2$

$a(2) = -2$

Since $v(2)$ and $a(2)$ have opposite signs, speed is decreasing at $t = 2$.

6. The position of a particle that is moving in a straight line is given by the equation

$s = t^3 - 6t^2 + 9t$

$a(t) = 6t - 12$ | $t = 2$

$0 = 6(t - 2)$

$a(t)$: 0 − 2 + 5

(a) Find the displacement of the particle during the first five seconds.

$s(0) = 0 + 0 + 0$

$s(5) = 5^3 - 6(5)^2 + 9(5) = 20$

so $s(5) - s(0) = \boxed{20}$

(b) Find the total distance traveled by the particle during the first 5 seconds.

$v(t) = 3t^2 - 12t + 9$

$0 = 3(t^2 - 4t + 3)$

$0 = 3(t-3)(t-1)$

$t = 1, 3$

$v(t)$: 0 + 1 − 3 + 5

$s(0) = 0$ > 4

$s(1) = 4$ > 4

$s(3) = 0$ > 20

$s(5) = 20$

$\boxed{\text{distance} = 28}$

(c) When is the particle speeding up? When is it slowing down?

$a(t)$	−	−	+	+
$v(t)$	+	−	−	+

0 1 2 3 5

speed up at $1 < t < 2$ and $3 < t < 5$

slow down at $0 < t < 1$ and $2 < t < 3$

7. (Calculator section) An object moving along a line has velocity $v(t) = t\cos(t) - \ln(t+2)$, where $0 \le t \le 10$.

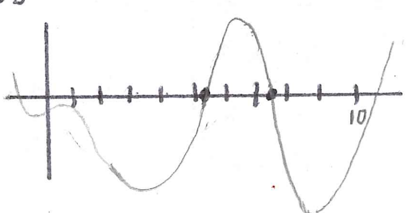
a) For what value(s) of t is the object motionless?

when $v(t) = 0$

$x = 5.107$

$x = 7.55$

v(t)



b) How many times does the object reverse direction? twice, at $t = 5.107, 7.550$

$v(t)$: 0 − 5.107 + 7.55 − 10