

**I. Curve Sketching**

1. Given the function  $f(x) = x^4 - 4x^2$ . Find the below information:

Relative Minimum: \_\_\_\_\_ Relative Maximum: \_\_\_\_\_ POI: \_\_\_\_\_

2. The total number of local maximum and minimum points of the function whose derivative, for all  $x$ , is given by  $f'(x) = x(x-3)^2(x+1)^4$  is

- a) 0      b) 1      c) 2      d) 3      e) none

**Problem 16** The function  $x^3 - 2x^2 + x - 4$  is decreasing on the interval

- (A)  $(0, \frac{1}{3})$       (B)  $(1, 3)$       (C)  $(-1, \frac{1}{3})$       (D)  $(\frac{1}{3}, 1)$       (E)  $(-1, -\frac{1}{3})$       (F)  $(0, 1)$

**II. Interpreting Derivative Graphs**

4. Given  $f'(x)$  graph on the right, locate  $x$ -values where the following occur on interval  $0 < x < 3.5$

Rel. min: \_\_\_\_\_

Rel. max: \_\_\_\_\_

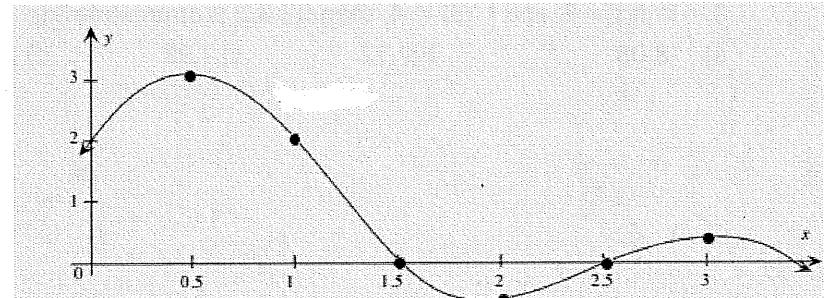
Interval increasing: \_\_\_\_\_

Interval decreasing: \_\_\_\_\_

Interval concave up: \_\_\_\_\_

Interval concave down: \_\_\_\_\_

POI: \_\_\_\_\_



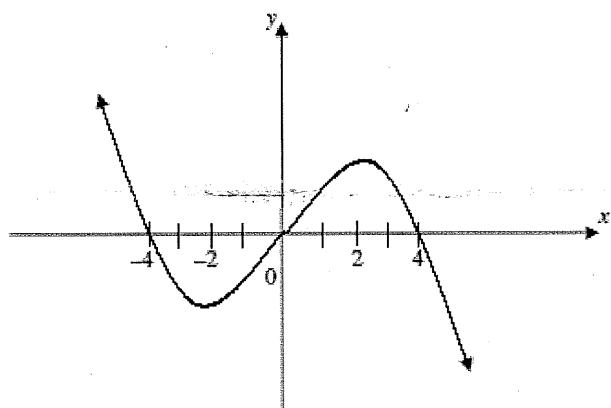
5. Identify POI, concave up/down from  $f''(x)$  graph

- Given  $f''(x)$  graph on the right, locate  $x$ -values where the following occur on interval  $-6 < x < 6$

Interval concave up: \_\_\_\_\_

Interval concave down: \_\_\_\_\_

POI: \_\_\_\_\_



### III. Derivative of Absolute Value function :

rule:  $\frac{d}{dx} |u| = \frac{u}{|u|} * u'$ ,  $u \neq 0$

6. Find  $\frac{dy}{dx}$  for  $y = |x^2 - 4|$

a) Find  $y'(1)$

b) Find  $y'(-1)$

7. Find  $\frac{dy}{dx}$  for  $y = 3|3 - 2x^3|$

a) Find  $y'(2)$

b) Find  $y'(-2)$

### IV. Linear approximation

8. If  $f(6) = 30$  and  $f'(x) = \frac{x^2}{x+3}$ , estimate  $f(6.02)$  using the line tangent to  $f$  at  $x = 6$

- a) 29.92      b) 30.02      c) 30.08      d) 34.00      e) none of these

9. If  $f(3) = 8$ , and  $f'(3) = -4$ , then  $f(3.02)$  is approximately

- a) -8.08      b) 7.92      c) 7.98      d) 8.02      e) 8.08

### V. Derivative of an inverse at a point

10. Let  $f(x) = \frac{3x-1}{2x+1}$ , and let  $g(x)$  be the inverse function of  $f(x)$ . Compute  $g'(1)$  given that  $g(1) = 2$ .

11. Let  $f(x) = \sqrt{x-4}$ . Find  $(f^{-1})'(1)$  given that  $(f^{-1})(1) = 5$ .

12. If  $f(x) = \cos x + 3x$  and  $(f^{-1})\left(\frac{3\pi}{2}\right) = \frac{\pi}{2}$ , find  $(f^{-1})'\left(\frac{3\pi}{2}\right)$

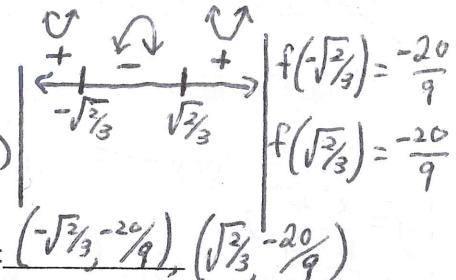
17) If  $g(f(x)) = x$ ,  $g(7) = 2$  and  $g'(7) = 10$ , then  $f'(2)$  is

- a)  $-\frac{1}{10}$   
 b)  $\frac{1}{10}$   
 c)  $\frac{1}{7}$   
 d)  $-\frac{1}{7}$   
 e)  $\frac{7}{10}$

I. Curve Sketching

1. Given the function  $f(x) = x^4 - 4x^2$ . Find the below information:

$$\begin{array}{c|c|c|c} f'(x) = 4x^3 - 8x & f(-\sqrt{2}) = -4 & f''(x) = 12x^2 - 8 & \\ \downarrow \quad \uparrow \quad \downarrow \quad \uparrow & \leftarrow \quad \rightarrow \quad \leftarrow \quad \rightarrow & & \\ O = 4x(x^2 - 2) & f(0) = 0 & O = 4(3x^2 - 2) & \\ X = 0, \sqrt{2}, -\sqrt{2} & f(\sqrt{2}) = -4 & x = \pm \sqrt{\frac{2}{3}} & \\ \text{Relative Minimum: } (-\sqrt{2}, -4), (\sqrt{2}, -4) & \text{Relative Maximum: } (0, 0) & \text{POI: } \left(-\sqrt{\frac{2}{3}}, -\frac{20}{9}\right), \left(\sqrt{\frac{2}{3}}, -\frac{20}{9}\right) & \end{array}$$



2. The total number of local maximum and minimum points of the function whose derivative, for all  $x$ , is given by  $f'(x) = x(x-3)^2(x+1)^4$  is

$$\begin{array}{lllll} O = x(x-3)^2(x+1)^4 & \text{a) 0} & \boxed{\text{b) 1}} & \text{c) 2} & \text{d) 3} \\ X = 0, -1, 3 & \begin{array}{c} \text{sign chart: } \\ \leftarrow \quad \downarrow \quad \uparrow \quad \uparrow \end{array} & \begin{array}{c} \text{sign chart: } \\ \leftarrow \quad \downarrow \quad \uparrow \quad \uparrow \end{array} & & \text{e) none} \end{array}$$

**Problem 3** The function  $x^3 - 2x^2 + x - 4$  is decreasing on the interval

$$\begin{array}{cccccc} \text{(A) } (0, \frac{1}{3}) & \text{(B) } (1, 3) & \text{(C) } (-1, \frac{1}{3}) & \boxed{\text{(D) } (\frac{1}{3}, 1)} & \text{(E) } (-1, -\frac{1}{3}) & \text{(F) } (0, 1) \\ f'(x) = 3x^2 - 4x + 1 & \boxed{x=1, \frac{1}{3}} & \begin{array}{c} \text{sign chart: } \\ \leftarrow \quad \downarrow \quad \uparrow \quad \uparrow \end{array} & \begin{array}{c} \text{graph: } \\ \text{decreasing on } (0, 1) \end{array} & & \begin{array}{c} \text{sign chart: } \\ \leftarrow \quad \uparrow \quad \downarrow \quad \uparrow \end{array} \\ O = (3x-1)(x-1) & & & & & \begin{array}{c} \text{sign chart: } \\ \leftarrow \quad \uparrow \quad \downarrow \quad \uparrow \end{array} \end{array}$$

II. Interpreting Derivative Graphs

4. Given  $f'(x)$  graph on the right, locate  $x$ -values where the following occur on interval  $0 < x < 3.5$

Rel. min:  $x=2.5$

Rel. max:  $x=1.5$

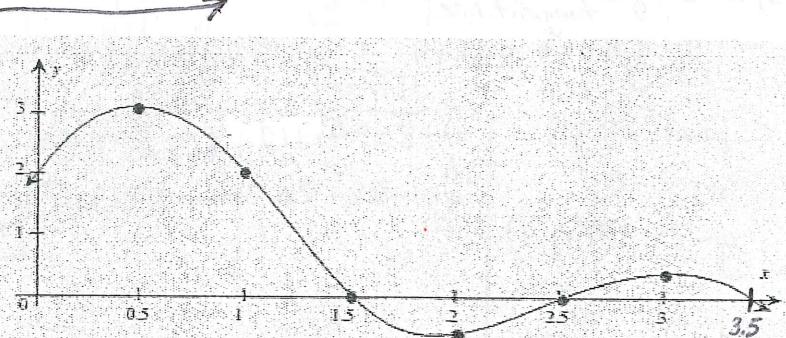
Interval increasing:  $(0, 1.5) \cup (2.5, 3.5)$

Interval decreasing:  $(1.5, 2.5)$

Interval concave up:  $(0, 0.5) \cup (2, 3)$

Interval concave down:  $(0.5, 2) \cup (3, 3.5)$

POI:  $x=0.5, 2, 3$



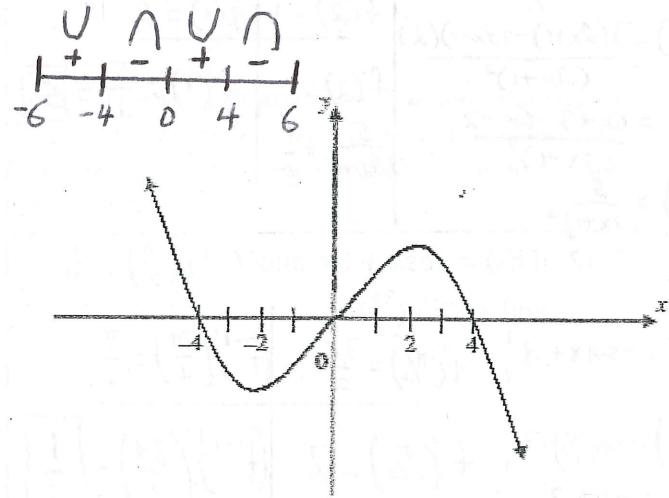
5. Identify POI, concave up/down from  $f''(x)$  graph

Given  $f''(x)$  graph on the right, locate  $x$ -values where the following occur on interval  $-6 < x < 6$

Interval concave up:  $(-6, -4) \cup (0, 4)$

Interval concave down:  $(-4, 0) \cup (4, 6)$

POI:  $x=-4, 0, 4$



### III. Derivative of Absolute Value function :

rule:  $\frac{d}{dx} |u| = \frac{u}{|u|} * u'$ ,  $u \neq 0$

6. Find  $\frac{dy}{dx}$  for  $y = |x^2 - 4|$

$$y' = \frac{x^2 - 4}{|x^2 - 4|} \cdot 2x$$

a) Find  $y'(1)$   $y'(1) = \frac{1-4}{|1-4|} \cdot 2 = \frac{-3}{3} \cdot 2 = \boxed{-2}$

b) Find  $y'(-1)$

$$y'(-1) = \frac{1-4}{|1-4|} \cdot -2 = \frac{-3}{3} \cdot -2 = \boxed{2}$$

7. Find  $\frac{dy}{dx}$  for  $y = 3|3 - 2x^3|$

$$y' = 3 \cdot \frac{3-2x^3}{|3-2x^3|} \cdot -6x^2 = \frac{-18x^2(3-2x^3)}{|3-2x^3|}$$

a) Find  $y'(2)$

$$y'(2) = \frac{-72(-13)}{|-13|} = -72(-1) = \boxed{72}$$

b) Find  $y'(-2)$

$$y'(-2) = \frac{-72(19)}{|19|} = \boxed{-72}$$

### IV. Linear approximation

8. If  $f(6) = 30$  and  $f'(x) = \frac{x^2}{x+3}$ , estimate  $f(6.02)$  using the line tangent to  $f$  at  $x = 6$

a) 29.92

b) 30.02

c) 30.08

d) 34.00

e) none of these

Steps:

- 1) Find derivative
- 2) Evaluate derivative at a point to find slope
- 3) Write equation of tangent line

4) Use tangent line equation to approximate decimal value

$$\begin{aligned} f'(6) &= \frac{6^2}{6+3} = \frac{36}{9} = 4 \\ f(6) &= 30 \\ y - y_1 &= m(x - x_1) \\ y - 30 &= 4(x - 6) \end{aligned}$$

$$\begin{aligned} y - 30 &= 4x - 24 \\ y &= 4x + 6 \end{aligned}$$

$$\begin{aligned} f(6.02) &\approx 4(6.02) + 6 \\ &= 24.08 + 6 = \boxed{30.08} \end{aligned}$$

9. If  $f(3) = 8$ , and  $f'(3) = -4$ , then  $f(3.02)$  is approximately

a) -8.08

b) 7.92

c) 7.98

d) 8.02

e) 8.08

point:  $(3, 8)$

$m = -4$

$$y - 8 = -4(x - 3)$$

$$\begin{aligned} y - 8 &= -4x + 12 \\ y &= -4x + 20 \end{aligned}$$

$$f(3.02) \approx -4(3.02) + 20 = -12.08 + 20$$

$$= \boxed{7.92}$$

### V. Derivative of an inverse at a point

10. Let  $f(x) = \frac{3x-1}{2x+1}$ , and let  $g(x)$  be the inverse function of  $f(x)$ . Compute  $g'(1)$  given that  $g(1) = 2$ .

$$\begin{aligned} f'(x) &= \frac{3(2x+1) - (3x-1)(2)}{(2x+1)^2} & f(2) &= 1 & g(1) &= 2 \\ &= \frac{6x+3 - 6x+2}{(2x+1)^2} & f'(2) &= & g'(1) &= \frac{5}{1} = \boxed{5} \\ &= \frac{5}{(2x+1)^2} & \frac{5}{(2(2)+1)^2} &= \frac{1}{5} \end{aligned}$$

11. Let  $f(x) = \sqrt{x-4}$ . Find  $(f^{-1})'(1)$  given that  $(f^{-1})(1) = 5$ .

$$\begin{aligned} f'(x) &= \frac{1}{2}(x-4)^{-\frac{1}{2}} & f(5) &= 1 & (f^{-1})(1) &= 5 \\ &= \frac{1}{2\sqrt{x-4}} & f'(5) &= \frac{1}{2} & (f^{-1})'(1) &= \frac{2}{1} = \boxed{2} \\ & & f'(5) &= \frac{1}{2\sqrt{5-4}} = \frac{1}{2} & & \end{aligned}$$

12. If  $f(x) = \cos x + 3x$  and  $(f^{-1})\left(\frac{3\pi}{2}\right) = \frac{\pi}{2}$ ,

find  $(f^{-1})'\left(\frac{3\pi}{2}\right)$

$$f'(x) = -\sin x + 3$$

$$f\left(\frac{3\pi}{2}\right) = \frac{3\pi}{2}$$

$$(f^{-1})'\left(\frac{3\pi}{2}\right) = \frac{\pi}{2}$$

17) If  $g(f(x)) = x$ ,  $g(7) = 2$  and  $g'(7) = 10$ , then  $f'(2)$  is

a)  $-\frac{1}{10}$

b)  $\frac{1}{10}$

c)  $\frac{1}{7}$

d)  $-\frac{1}{7}$

e)  $\frac{7}{10}$

$$f(2) = 7 \quad g(7) = 2$$

$$f'(2) = \boxed{\frac{1}{10}} \quad g'(7) = 10$$