I. Similar Triangle Related Rates

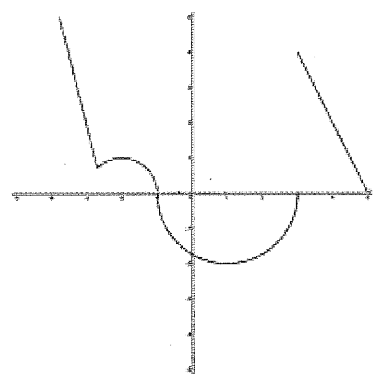
1. A light is on the top of a 12 ft tall pole and a 5ft tall person is walking away from the pole at a rate of 2 ft/sec.

(a) At what rate is the shadow length increasing when the person is 25 ft from the pole?

(b) At what rate is the tip of the shadow moving when the person is 25 ft from the pole?

2. A tank of water in the shape of a cone is leaking water at a constant rate of $2\text{ft}^3/\text{hour}$. The base radius of the tank is 5 ft and the height of the tank is 14 ft.

At what rate is the depth of the water in the tank changing when the depth of the water is 6 ft?



II. Miscellaneous Problems:

3.

If a function is given by $f(x) = \frac{x+3}{x^2-1}$, what is the instantaneous rate of change of the function at $x = 3$?

A. $\frac{7}{16}$

B. $-\frac{7}{16}$

C. $\frac{11}{16}$

D. $-\frac{11}{16}$

E. $\frac{1}{6}$

4.

The function f shown in the graph above has horizontal tangents at $(-2,1)$ and $(1,-2)$ and vertical tangents at $(-1,0)$ and $(3,0)$. For how many values of x in the interval $(-5,5)$ is the function not differentiable?

A. 0

B. 1

C. 2

D. 3

E. 4

5.

x	1	3	5
$f(x)$	4	k	3

Given that f is a continuous function on the interval $[1,5]$ and that f takes values shown in the table. The function f will have two zeros in the interval $[1,5]$ if $k =$

A. -1
B. 0
C. 1
D. 2
E. 3

6.

If $f''(x) = x(x-2)(x+1)^2$, then the graph of f has points of inflection when $x =$

A. -2 and 1
B. 2 and -1
C. 2 and 0
D. -2 and 0
E. 0, 2 and -1

7.

If $f(x) = \begin{cases} x^3 e^x & \text{for } 0 \le x < 1 \\ \dfrac{e^x}{x^3} & \text{for } 1 < x \le 3 \end{cases}$, then $\lim_{x \to 1} f(x)$ is

A. 0
B. 1
C. e
D. e^3
E. nonexistent

8.

If $3x^2 - 4xy = 1$, then when $x = 1$, $\dfrac{dy}{dx} =$

A. $\frac{3}{2}$
B. 1
C. $\frac{1}{2}$
D. 0
E. $-\frac{1}{2}$

9.

The function f is given by $f(x) = 2x^4 - 3x^2 + 1$. On which of the following intervals is f decreasing?

A. $\left(\frac{\sqrt{3}}{2}, \infty\right)$
B. $\left(-\frac{\sqrt{3}}{2}, \frac{\sqrt{3}}{2}\right)$
C. $\left(-\infty, -\frac{\sqrt{3}}{2}\right)$
D. $\left(0, \frac{\sqrt{3}}{2}\right)$
E. $\left(-\infty, -\frac{\sqrt{3}}{2}\right)$ and on $\left(0, \frac{\sqrt{3}}{2}\right)$

10.

If $f(x)$ is continuous on the closed interval $[a,b]$ and differentiable on the open interval (a,b), which of the following must be true?

I. If $f(a) = f(b)$, then for some value c between a and b, $f'(c) = 0$

II. If k is any number between $f(a)$ and $f(b)$, there is a value $c \in (a,b)$ such that $f(c) = k$.

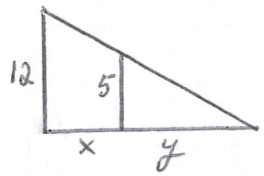
III. There is a value $c \in (a,b)$ such that $f'(c) = \dfrac{f(b) - f(a)}{b - a}$.

A. I only
B. II only
C. III only
D. I and III
E. I, II, and III

AP Calculus AB Fall Exam Review WS #4

I. Similar Triangle Related Rates

1. A light is on the top of a 12 ft tall pole and a 5ft tall person is walking away from the pole at a rate of 2 ft/sec.



$\frac{5}{12} = \frac{y}{x+y}$

$5x + 5y = 12y$

$5x = 7y$

$5\left(\frac{dx}{dt}\right) = 7\left(\frac{dy}{dt}\right)$

$\frac{dx}{dt} = 2 ft/s$

$5(2) = 7\left(\frac{dy}{dt}\right)$

$\frac{10}{7} = \frac{dy}{dt}$

$\frac{dy}{dt} = \frac{10}{7}$ ft/s

(a) At what rate is the shadow length increasing when the person is 25 ft from the pole?

Roc for shadowlength $= \frac{dy}{dt} = \boxed{\frac{10}{7} ft/s}$

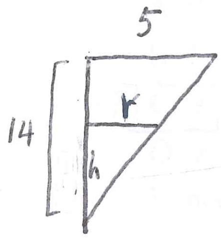
(b) At what rate is the tip of the shadow moving when the person is 25 ft from the pole?

ROC of tip of shadow $= \frac{dx}{dt} + \frac{dy}{dt}$.

$= 2 + \frac{10}{7} = \boxed{\frac{24}{7} ft/s}$

2. A tank of water in the shape of a cone is leaking water at a constant rate of 2ft³/hour . The base radius of the tank is 5 ft and the height of the tank is 14 ft. $V = \frac{\pi}{3}r^2h$

At what rate is the depth of the water in the tank changing when the depth of the water is 6 ft?



$\frac{dv}{dt} = -2 ft^3/hr$

Find $\frac{dh}{dt} =$ ____

$h = 6 ft.$

$\frac{r}{5} = \frac{h}{14}$

$14r = 5h$

$r = \frac{5}{14}h$

*Rewrite Volume equation in terms of h.

$V = \frac{\pi}{3}\left(\frac{5}{14}h\right)^2 h$

$V = \frac{\pi}{3}\left(\frac{25}{196}h^2\right)h$

$V = \frac{25\pi}{588}h^3$

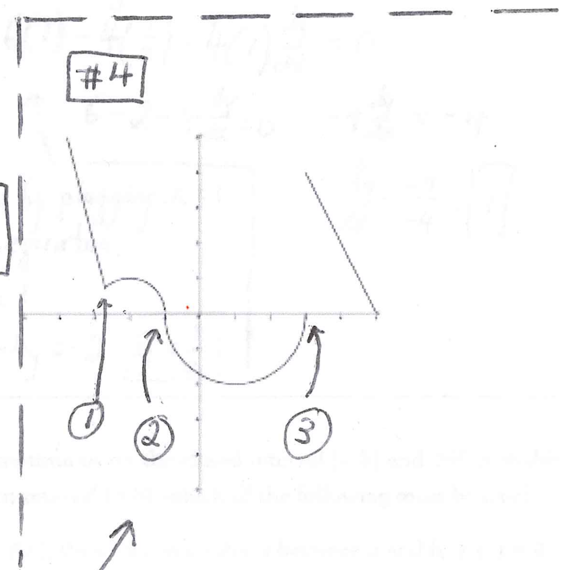
$\frac{dV}{dt} = \frac{25\pi}{588} \cdot 3h^2\left(\frac{dh}{dt}\right)$

$-2 = \frac{25\pi}{588} \cdot 3(6)^2\left(\frac{dh}{dt}\right)$

$-2 = \frac{225\pi}{49}\frac{dh}{dt}$

$-2 \cdot \frac{49}{225\pi} = \frac{dh}{dt}$

$\boxed{\frac{dh}{dt} = \frac{-98}{225\pi} ft/hr.}$



II. Miscellaneous Problems:

3.

If a function is given by $f(x) = \frac{x+3}{x^2-1}$, what is the instantaneous rate of change of the function at $x = 3$?

A. $\frac{7}{16}$

B. $-\frac{7}{16}$

C. $\frac{11}{16}$

D. $-\frac{11}{16}$

E. $\frac{1}{6}$

$f'(x) = \frac{(1)(x^2-1) - (x+3)(2x)}{(x^2-1)^2}$

$f'(x) = \frac{x^2 - 1 - 2x^2 - 6x}{(x^2-1)^2} = \frac{-x^2-6x-1}{(x^2-1)^2}$

$f'(3) = \frac{-3^2 - 6(3) - 1}{(3^2-1)^2} = \frac{-28}{64}$

$= \boxed{\frac{-7}{16}}$

4.

The function f shown in the graph above has horizontal tangents at $(-2,1)$ and $(1,-2)$ and vertical tangents at $(-1,0)$ and $(3,0)$. For how many values of x in the interval $(-5,5)$ is the function not differentiable?

A. 0

B. 1

C. 2

D. 3

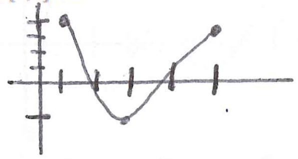
E. 4

5.

x	1	3	5
f(x)	4	k	3

Given that f is a continuous function on the interval $[1,5]$ and that f takes values shown in the table. The function f will have two zeros in the interval $[1,5]$ if $k =$ (x-ints)

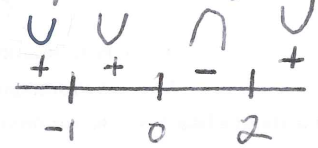
$\boxed{\text{A. } -1}$
B. 0
C. 1
D. 2
E. 3



6.

If $f''(x) = x(x-2)(x+1)^2$, then the graph of f has points of inflection when $x =$ critical points: $x = 0, 2, -1$

A. -2 and 1
B. 2 and -1
$\boxed{\text{C. 2 and 0}}$
D. -2 and 0
E. 0, 2 and -1



7.

If $f(x) = \begin{cases} x^3 e^x & \text{for } 0 \le x < 1 \\ \dfrac{e^x}{x^3} & \text{for } 1 < x \le 3 \end{cases}$, then $\lim_{x \to 1} f(x)$ is

A. 0
B. 1
$\boxed{\text{C. } e}$
D. e^3
E. nonexistent

$\lim_{x \to 1^-} x^3 e^x = 1^3 e^1 = e$

$\lim_{x \to 1^+} \frac{e^x}{x^3} = \frac{e^1}{1} = e$ ✓

8.

✓ product rule

If $3x^2 - 4xy = 1$, then when $x = 1$, $\frac{dy}{dx} =$

A. $\frac{3}{2}$
$\boxed{\text{B. } 1}$
C. $\frac{1}{2}$
D. 0
E. $-\frac{1}{2}$

$6x - (4)(y) + (-4x)\left(\frac{dy}{dx}\right) = 0$

$6x - 4y - 4x\left(\frac{dy}{dx}\right) = 0$

$6(1) - 4\left(\frac{1}{2}\right) - 4(1)\frac{dy}{dx} = 0$

$6 - 2 - 4\frac{dy}{dx} = 0 \qquad -4\frac{dy}{dx} = -4$

$\frac{dy}{dx} = \frac{-4}{-4} = \boxed{1}$

*Find y-value by plugging $x = 1$ into original equation.
$3(1)^2 - 4(1)y = 1$
$3 - 4y = 1 \qquad -4y = -2 \qquad \boxed{y = \frac{1}{2}}$

9.

The function f is given by $f(x) = 2x^4 - 3x^2 + 1$. On which of the following intervals is f decreasing?

A. $\left(\frac{\sqrt{3}}{2}, \infty\right)$
B. $\left(-\frac{\sqrt{3}}{2}, \frac{\sqrt{3}}{2}\right)$
C. $\left(-\infty, -\frac{\sqrt{3}}{2}\right)$
D. $\left(0, \frac{\sqrt{3}}{2}\right)$
$\boxed{\text{E.}}$ $\left(-\infty, -\frac{\sqrt{3}}{2}\right)$ and on $\left(0, \frac{\sqrt{3}}{2}\right)$

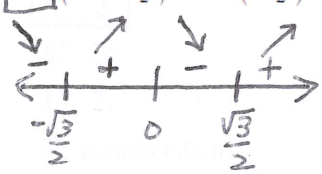
$f'(x) = 8x^3 - 6x$

$0 = 2x(4x^2 - 3)$

$2x = 0 \qquad 4x^2 - 3 = 0$

$x^2 = \frac{3}{4}$

$x = \pm\sqrt{\frac{3}{4}} = \pm\frac{\sqrt{3}}{2}$



$f(x)$ is decreasing $\left(-\infty, -\frac{\sqrt{3}}{2}\right) \cup \left(0, \frac{\sqrt{3}}{2}\right)$
b/c $f'(x) < 0$

10.

If $f(x)$ is continuous on the closed interval $[a,b]$ and differentiable on the open interval (a,b), which of the following must be true?

Rolle's I. If $f(a) = f(b)$, then for some value c between a and b, $f'(c) = 0$

IVT II. If k is any number between $f(a)$ and $f(b)$, there is a value $c \in (a,b)$ such that $f(c) = k$.

MVT III. There is a value $c \in (a,b)$ such that $f'(c) = \frac{f(b) - f(a)}{b - a}$.

A. I only
B. II only
C. III only
D. I and III
$\boxed{\text{E. I, II, and III}}$