1a) What is a one-to-one function? | b) What does monotonic mean? | c.What is a local maximum?

2.

The radius of a sphere is increasing at a constant of 2 cm/sec. At the instant when the volume of the sphere is increasing at 32π cm^3/sec, the surface area of the sphere is

3. A particle moves along the x-axis so that its velocity v at time t, for $0 \leq t \leq 5$, is given by
$$v(t) = \ln(t^2 - 3t + 3).$$
The particle is at the point $x = 8$ at time $t = 0$.

(a) Find the acceleration of the particle at time $t = 4$.

(b) Find all the times in the open interval $0 < t < 5$ at which the particle changes direction. During which time intervals, for $0 < t < 5$, does the particle travel to the left?

4.

A spherical balloon is inflated so that its volume is increasing at the rate of 20 cubic feet per minute. How fast is the surface area of the balloon increasing when the radius is 4 feet?

5.

6.

At what value(s) of x do the graphs of $y = x^2$ and $y = -\sqrt{x}$ have perpendicular tangent lines?

7.

A spherical balloon is being inflated. At the instant when the rate of increase of the volume of the sphere is four times the rate of increase of the radius, the radius of the sphere is

8. A particle moves in a vertical line so that its coordinate at time t is

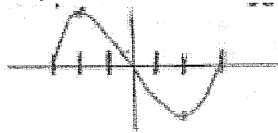$$y(t) = t^3 - 12t + 3, \; t \geq 0 .$$

(a) Find the velocity and acceleration functions.	(b) When is the particle moving upward, and when is it moving downward?
(c) Find the displacement of the particle during the first 3s.	(d) Find the distance that the particle travels in the time interval $0 \leq t \leq 3$.
(e) Given the position function below: 	(f) Graph the velocity function
(g) Graph the acceleration function	h) When is the particle speeding up? When is it slowing down?

1

The $\lim_{h \to 0} \dfrac{\ln(2x+h) - \ln(2x)}{h}$ is

(A) $\dfrac{1}{x}$

(B) $\dfrac{1}{2x}$

(C) $\ln x$

(D) $\ln(2x)$

(E) Undefined

2

What is $\lim_{h \to 0} \dfrac{\tan\left(\dfrac{\pi}{4}+h\right) - \tan\left(\dfrac{\pi}{4}\right)}{h}$?

(A) 0

(B) $\dfrac{\sqrt{2}}{2}$

(C) 1

(D) $\dfrac{\sqrt{3}}{2}$

(E) 2

3

If $f(3) = 8$, then the slope of the tangent line to $(f^{-1})'(8)$ must be

(A) $\dfrac{1}{f'(3)}$

(B) $\dfrac{1}{f'(8)}$

(C) $f'(3)$

(D) 3

(E) 8

4

Let f be the function given by $f(x) = \dfrac{x}{x-3}$. For what value(s) of x is the slope of the line tangent to the graph of f at $(x, f(x))$ equal to $-\dfrac{3}{4}$?

(A) $x = -\dfrac{3}{7}$

(B) $x = \dfrac{9}{7}$

(C) $x = 1$ or $x = 5$

(D) $x = \pm 7$

(E) No solution

5

Find the inflection point(s) for the function $f(x) = 2x(x+4)^3$.

(A) $(0, 0)$

(B) $(-4, 0)$

(C) $(0, 0), (-4, 0)$

(D) $(-4, 0), (0, 0), (4, 0)$

(E) $(-4, 8)$

6

At what value(s) of x do the graphs of $y = e^x$ and $y = x^2 + 5x$ have parallel tangent lines?

7

If the line tangent to $y = f(x)$ at point $(-3, 8)$ passes through the point $(-2, 5)$ then,

(A) $f'(-2) = 3$

(B) $f'(-2) = -3$

(C) $f'(-3) = \dfrac{3}{5}$

(D) $f'(-3) = -3$

(E) $f(-2) = 5$

8

Let $f(x) = x^3 - x$ such that f is continuous on a closed interval $[-1, 1]$. Find the critical number(s), c, that satisfies the mean value theorem for the given function and interval.

(A) $\pm\sqrt{\dfrac{1}{3}}$

(B) $\pm\sqrt{3}$

(C) 0

(D) $\sqrt[3]{\dfrac{1}{3}}$

(E) $\dfrac{1}{3}$

9.

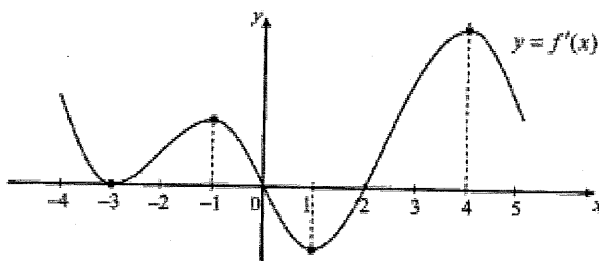The local linear approximation for $f(x) = \sqrt{x^2 + 16}$ near $x = -3$ is

(A) $5 - \dfrac{3}{5}(x-3)$ (B) $5 + \dfrac{3}{5}(x-3)$ (C) $5 - \dfrac{3}{5}(x+3)$

(D) $3 - \dfrac{5}{3}(x-3)$ (E) $3 + \dfrac{3}{5}(x+3)$

10.

At time $t \geq 0$, the position of a particle moving along the x-axis is given by $x(t) = \dfrac{t^3}{3} + 2t + 2$. For what value of t in the interval $[0, 3]$ will the instantaneo[us] velocity of the particle equal the average velocity of the particle from time $t = $ to time $t = 3$?

(A) 1   (B) $\sqrt{3}$ (C) $\sqrt{7}$ (D) 3   (E) 5

11.

$y = f'(x)$

The figure above shows the graph of f', the derivative of the function f, for $-4 \leq x \leq 5$. The graph of f' has horizontal tangent lines at $x = -3, -1, 1,$ and 4.

(a) Find all values of x, for $-4 < x < 5$, for which f is decreasing. Justify your answer.

(b) Find all values of x, for $-4 < x < 5$, at which f attains a relative maximum. Justify your answer.

(c) Find all values of x, for $-4 < x < 5$, for which the graph of f is concave up.

12   A particle moves along the y-axis so that its position at any time $t \geq 0$ is given by $y(t) = t^2 - 4\ln(t+1) - 1$.

(a) Find the velocity $v(t)$ at any time $t \geq 0$.

b. Find all values of t for which the speed of the particle is increasing. Justify your answer.

c. Find the total distance traveled by the particle from time $t = 0$ to time $t = 2$.

If f is a differentiable function and $f(0)=-1$ and $f(4)=3$, then which of the following must be true?

   I.   There exists a c in $[0, 4]$ where $f(c)=0$.

   II. There exists a c in $[0, 4]$ where $f'(c)=0$.

   III. There exists a c in $[0, 4]$ where $f'(c)=1$.

(A) I only   (B) II only   (C) I and II only

(D) I and III only   (E) I, II, and III

If $f'(x)=-5(x-3)^2(x-2)$, which of the following features does the graph of $f(x)$ have?

(A) a local minimum at $x = 2$ and a local maximum at $x = 3$
(B) a local maximum at $x = 2$ and a local minimum at $x = 3$
(C) a point of inflection at $x = 2$ and a local minimum at $x = 3$
(D) a local minimum at $x = 2$ and a point of inflection at $x = 3$
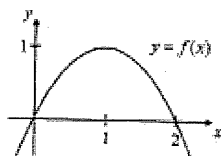(E) a local maximum at $x = 2$ and a point of inflection at $x = 3$
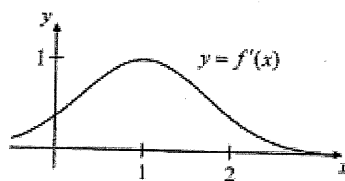
The graph of $f'(x)$ is shown above. Which of the following could be the graph of $f(x)$?
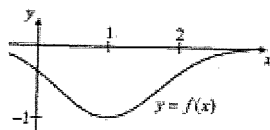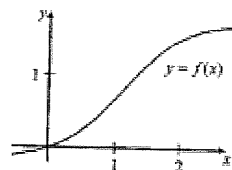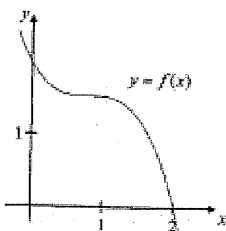
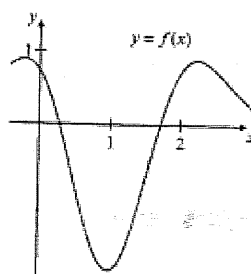(A)   (B)   (C)

(D)   (E)

The radius of a sphere increases at a constant rate of 2 cm/min. At the time when the volume of the sphere is 40 cm³, what is the rate of increase of the volume in cm³/min? $\left(V = \frac{4}{3}\pi r^3 \right)$

(A) 2.122   (B) 9.549   (C) 56.562   (D) 113.124   (E) 293.954

$f(x)$ is a differentiable function with $f(1)=-3$ and $f(5)=4$. Which of the following must be true?

(A) $f(0)=k$ for some k in $(1,5)$

(B) $f(x)$ is increasing on $(1,5)$

(C) $f'(x)=\frac{7}{4}$ for all x in $(1,5)$

(D) $f'(k)=0$ for some k in $(1,5)$

(E) $f'(k)=\frac{7}{4}$ for some k in $(1,5)$
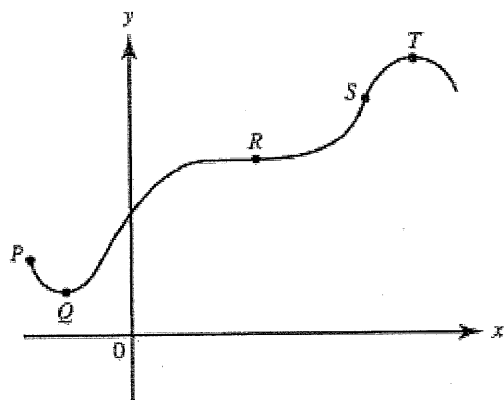
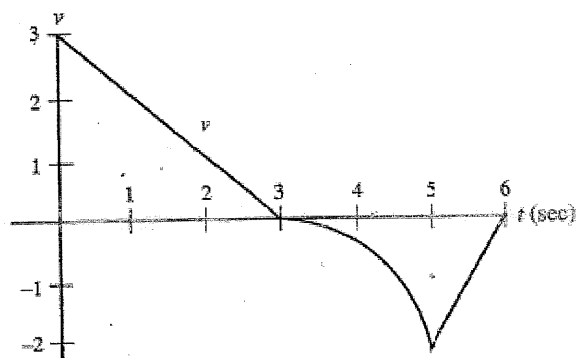At which labeled point do both $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ equal zero?

(A) P (B) Q (C) R (D) S (E) T

At which labeled point is $\frac{dy}{dx}$ positive and $\frac{d^2y}{dx^2}$ equal to zero?

(A) P (B) Q (C) R (D) S (E) T

At which labeled point is $\frac{dy}{dx}$ equal to zero and $\frac{d^2y}{dx^2}$ negative?

(A) P (B) Q (C) R (D) S (E) T

For how many values of t in the interval $0 < t < 6$ is the acceleration undefined?

(A) none   (B) one   (C) two   (D) three   (E) four

During what time interval (in sec) is the speed increasing?

(A) $0 < t < 3$ (B) $3 < t < 5$ (C) $3 < t < 6$
(D) $5 < t < 6$ (E) never

What is the average acceleration (in units/sec²) during the first 5 seconds?

(A) $-\frac{5}{2}$ (B) -1 (C) $-\frac{1}{5}$ (D) $\frac{1}{5}$ (E) $\frac{1}{2}$