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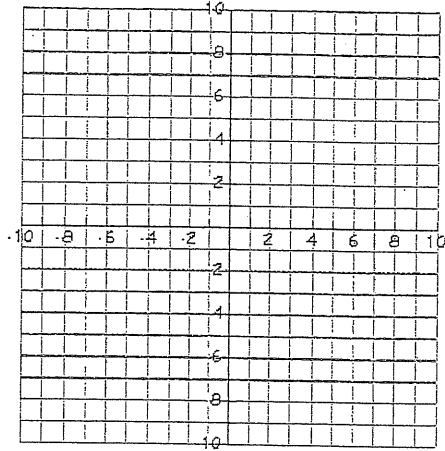
CCGPS Analytic Geometry
Notes: Distance and Midpoint Formulas

Essential Question: How do you find the distance between two points in the coordinate plane?

Warm Up: Use the coordinate plane to the right to plot points A through F. Then find the indicated distances.

A (5, 3) B (-2, 0) C (-2, -4)

D (4, -6) E (0, -3) F (-6, 3)



a. Find AF.

b. Find CB.

:

c. What would make finding DE different from finding AF or CB? What could you do?

The Distance Formula: The distance, d , between any two points (x_1, y_1) and (x_2, y_2)

is $d =$

Examples: Find the distance between the two points.

1. (-2, 7) and (3, -5)

2. (5, 2) and (3, 8)

3. (8, -1) and (-4, 3)

Name: _____ Date: _____ Period: _____

Examples: The distance d between two points is given. Find possible values for b .

4. $d = 15$; $(3, b)$ and $(-6, 2)$

5. $d = \sqrt{37}$; $(-5, 3)$ and $(b, 2)$

The Midpoint Formula: The midpoint, M , of the line segment with endpoints $A(x_1, y_1)$ and $B(x_2, y_2)$ is

Examples: Find the midpoint of the line segment with the given endpoints.

1. $(14, 3)$ and $(6, 9)$

2. $(0, 9)$ and $(-4, -6)$

3. $(-11, -2)$ and $(2, -5)$

Examples: The midpoint and endpoint of a line segment are given. Find the other endpoint.

4. Midpoint $(-3, 5)$ and Endpoint $(-8, 2)$

5. Midpoint $(2.5, -6)$ and Endpoint $(-1, -10)$

Name: _____ Date: _____ Period: _____

CCGPS Analytic Geometry

Homework: Distance and Midpoint Review

Recall:

Midpoint formula: $(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2})$

Distance Formula: $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Find the distance between the two points.

1. (8, 3), (10, 4)

2. (2, 7), (5, 6)

3. (0, 4), (8, 22)

Find the midpoint of the line segment with the given endpoints.

4. (2, 5), (4, 12)

5. (27, 2), (210, 14)

6. (29, 25), (7, 214)

The midpoint and an endpoint of a line segment are given. Find the other endpoint.

7. Midpoint: (24, 6)

8. Midpoint: (23, 3)

Endpoint: (2, 1)

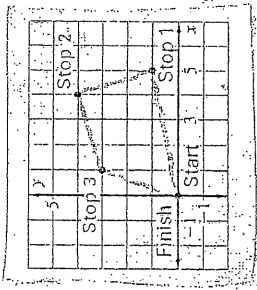
Endpoint: (24, 22)

The distance d between 2 points is given. Find the possible values of b :

9. (b, 4), (2, -1) ; $d = 5$

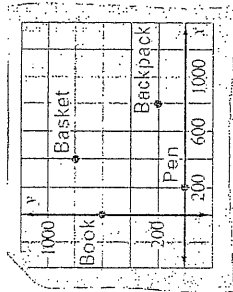
10. (-3, 2), (7, b); $d = 10$

11. A walking trail follows the path shown on the map. The distance between consecutive grid lines is 1 mile. Find the total distance of the trail from start to finish. Round your answer to the nearest mile.

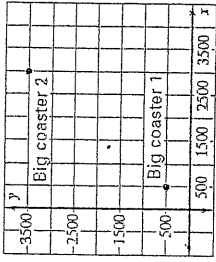


12. Suppose now that on your walk you get to Stop 2 and decide you are too tired to continue. You decide to take the short cut and walk diagonally to the Finish. What is your total distance if you take this path? (use same picture as above)

13. You set up a treasure hunt with the items placed according to the map shown. The distance between consecutive grid lines is 200 feet. What is the distance between each pair of objects? Which two objects are closest together? Which two objects are farthest apart?



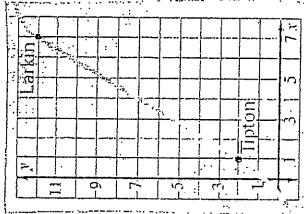
14. An amusement park designer wants to place a Ferris wheel midway between the two largest coasters. The distance between consecutive grid lines is 500 feet.



a. Determine the coordinates of where the Ferris wheel should be.

b. How far will the Ferris wheel be from each of the coasters? Round to the nearest foot.

15. You are biking a straight line distance between the two towns shown on the map. The distance between the grid lines is 1 mile.



a. How far is your bike ride ONE way?

b. How far is your bike ride round trip?

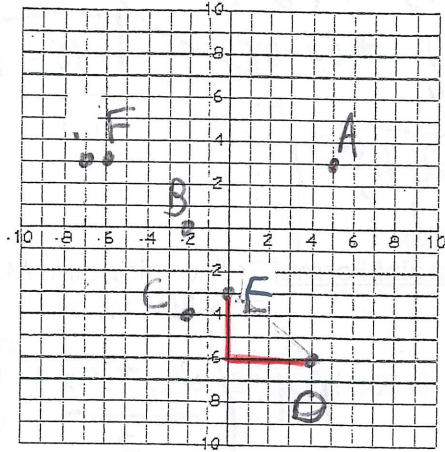
c. You stop for a snack break halfway between the two towns. What are the coordinates of your location?

CCGPS Analytic Geometry
Notes: Distance and Midpoint Formulas

Essential Question: How do you find the distance between two points in the coordinate plane?

Warm Up: Use the coordinate plane to the right to plot points A through F. Then find the indicated distances.

- A (5, 3) B (-2, 0) C (-2, -4)
D (4, -6) E (0, -3) F (-6, 3)

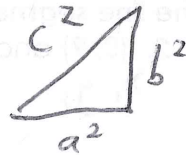


a. Find AF. *11 units*

b. Find CB. *4 units*

c. What would make finding DE different from finding AF or CB? What could you do?

*diagonal points
require distance formula*



The Distance Formula: The distance, d , between any two points (x_1, y_1) and (x_2, y_2)

is $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Examples: Find the distance between the two points.

1. $(-2, 7)$ and $(3, -5)$

$$d^2 = (3 - (-2))^2 + (-5 - 7)^2$$

$$d^2 = 5^2 + (-12)^2$$

$$d^2 = 25 + 144$$

$$\sqrt{d^2} = \sqrt{169}$$

$$d = 13$$

2. $(5, 2)$ and $(3, 8)$

$$d^2 = (3 - 5)^2 + (8 - 2)^2$$

$$d^2 = (-2)^2 + (6)^2$$

$$d^2 = 4 + 36$$

$$\sqrt{d^2} = \sqrt{40}$$

$$d = 2\sqrt{10}$$

3. $(8, -1)$ and $(-4, 3)$

$$d^2 = (-4 - 8)^2 + (3 - (-1))^2$$

$$d^2 = (-12)^2 + 4^2$$

$$d^2 = 144 + 16$$

$$\sqrt{d^2} = \sqrt{160}$$

$$d = 4\sqrt{10}$$

Name: _____ Date: _____ Period: _____

$$15^2 = (-9)^2 + (2-b)^2 \quad d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$

Examples: The distance d between two points is given. Find possible values for b .

4. $d = 15$; $(3, b)$ and $(-6, 2)$

$$15^2 = (-6-3)^2 + (2-b)^2$$

$$= (-9)^2 + (2-b)^2$$

$$225 = 81 + (2-b)^2$$

$$-81 \quad -81$$

$$\pm \sqrt{144} = (2-b)^2$$

$$\pm 12 = 2-b$$

$12 = 2-b$	$-12 = 2-b$
$-2 \quad -2$	$-2 \quad -2$
$10 = -b$	$-14 = -b$
$-1 \quad -1$	$-1 \quad -1$
$-10 = b$	$14 = b$

5. $d = \sqrt{37}$; $(-5, 3)$ and $(b, 2)$

$$(\sqrt{37})^2 = (b-(-5))^2 + (2-3)^2$$

$$37 = (b+5)^2 + 1$$

$$+ \sqrt{36} = \sqrt{(b+5)^2}$$

$$\pm 6 = b+5$$

$6 = b+5$	$-6 = b+5$
$-5 \quad -5$	$-5 \quad -5$
$1 = b$	$-11 = b$

The Midpoint Formula: The midpoint, M , of the line segment with endpoints $A(x_1, y_1)$ and $B(x_2, y_2)$ is

$$M \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

Examples: Find the midpoint of the line segment with the given endpoints.

1. $(14, 3)$ and $(6, 9)$

$$M = \left(\frac{14+6}{2}, \frac{3+9}{2} \right)$$

$$M = \left(\frac{20}{2}, \frac{12}{2} \right)$$

$$M(10, 6)$$

2. $(0, 9)$ and $(-4, -6)$

$$M = \left(\frac{0-4}{2}, \frac{9-6}{2} \right)$$

$$M = \left(-2, \frac{3}{2} \right)$$

3. $(-11, -2)$ and $(2, -5)$

$$M = \left(\frac{-11+2}{2}, \frac{-2-5}{2} \right)$$

$$M = \left(-\frac{9}{2}, -\frac{7}{2} \right)$$

Examples: The midpoint and endpoint of a line segment are given. Find the other endpoint.

4. Midpoint $(-3, 5)$ and Endpoint $(-8, 2)$

E	-8	2
$+5$		
M	-3	5
$+5$		
	$(2, 8)$	

5. Midpoint $(2.5, -6)$ and Endpoint $(-1, -10)$

E	-1	-10
$+3.2$		
M	2.5	-6
$+3.2$		
	$(6, -2)$	

Name: _____ Date: _____ Period: _____

CCGPS Analytic Geometry

Homework: Distance and Midpoint Review

$$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$

Recall:

Midpoint formula: $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

Distance Formula: $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Find the distance between the two points.

1. (8, 3), (10, 4)

$$d^2 = (10 - 8)^2 + (4 - 3)^2$$

$$d^2 = 2^2 + 1^2$$

$$d^2 = 5$$

$$d = \sqrt{5}$$

2. (2, 7), (5, 6)

$$d^2 = (2 - 5)^2 + (7 - 6)^2$$

$$d^2 = 3^2 + 1^2$$

$$d^2 = 10$$

$$d = \sqrt{10}$$

3. (0, 4), (8, 22)

$$d^2 = (8 - 0)^2 + (22 - 4)^2$$

$$d^2 = 64 + 324$$

$$d^2 = 388$$

$$d = 2\sqrt{97}$$

Find the midpoint of the line segment with the given endpoints.

4. (2, 5), (4, 12)

$$\frac{2+4}{2}, \frac{5+12}{2}$$

$$M(3, 8.5)$$

5. (27, 2), (210, 14)

$$\frac{27+210}{2}, \frac{2+14}{2}$$

$$M(118.5, 8)$$

6. (29, 25), (7, 214)

$$\frac{29+7}{2}, \frac{25+214}{2}$$

$$M(18, 119.5)$$

The midpoint and an endpoint of a line segment are given. Find the other endpoint.

7. Midpoint: (24, 6)

Endpoint: (2, 1)

$$\begin{matrix} 22 \\ \left[\begin{matrix} E_1 & 2 & 1 \\ M & 24 & 6 \end{matrix} \right] \end{matrix} \quad \left. \vphantom{\begin{matrix} 22 \\ \left[\begin{matrix} E_1 & 2 & 1 \\ M & 24 & 6 \end{matrix} \right] \end{matrix}} \right\} 5$$

$$\begin{matrix} 22 \\ \left[\begin{matrix} E_2 & 46 & 11 \end{matrix} \right] \end{matrix} \quad \left. \vphantom{\begin{matrix} 22 \\ \left[\begin{matrix} E_2 & 46 & 11 \end{matrix} \right] \end{matrix}} \right\} 5$$

$$(46, 11)$$

8. Midpoint: (23, 3)

Endpoint: (24, 22)

$$\begin{matrix} \left[\begin{matrix} E_2 & 22 & -16 \\ M & 23 & 3 \end{matrix} \right] \end{matrix} \quad \left. \vphantom{\begin{matrix} \left[\begin{matrix} E_2 & 22 & -16 \\ M & 23 & 3 \end{matrix} \right] \end{matrix}} \right\} 19$$

$$\begin{matrix} \left[\begin{matrix} E_1 & 24 & 22 \end{matrix} \right] \end{matrix} \quad \left. \vphantom{\begin{matrix} \left[\begin{matrix} E_1 & 24 & 22 \end{matrix} \right] \end{matrix}} \right\} 19$$

$$(22, -16)$$

The distance d between 2 points is given. Find the possible values of b:

9. (b, 4), (2, -1); d = 5

$$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$

$$25 = (2 - b)^2 + (5)^2$$

$$0 = (2 - b)^2$$

$$0 = 2 - b \quad \boxed{b = 2}$$

10. (-3, 2), (7, b); d = 10

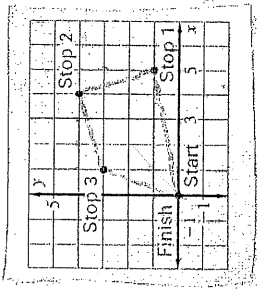
$$10^2 = (-3 - 7)^2 + (2 - b)^2$$

$$100 = 100 + (2 - b)^2$$

$$0 = (2 - b)^2$$

$$\boxed{b = 2}$$

11. A walking trail, as the path shown on the map. The distance between consecutive grid lines is 1 mile. Find the total distance of the trail from start to finish. Round your answer to the nearest mile.

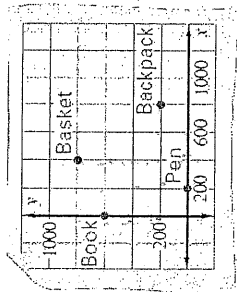


$$\begin{aligned} S \rightarrow 1 & d = 5^2 + 1^2 = \sqrt{26} \\ 1 \rightarrow 2 & d = 3^2 + 1^2 = \sqrt{10} \\ 2 \rightarrow 3 & d = 3^2 + 1^2 = \sqrt{10} \\ 3 \rightarrow F & d = 3^2 + 1^2 = \sqrt{10} \end{aligned}$$

12. Suppose now that on your walk you get to Stop 2 and decide you are too tired to continue. You decide to take the short cut and walk diagonally to the Finish. What is your total distance if you take this path? (use same picture as above)

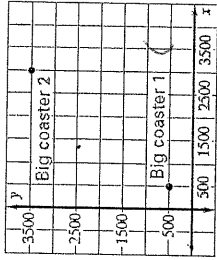
$$\begin{aligned} S \rightarrow 1 & = \sqrt{26} \\ 1 \rightarrow 2 & = \sqrt{10} \\ 2 \rightarrow F & = 4^2 + 4^2 = \sqrt{32} = 4\sqrt{2} \end{aligned}$$

13. You set up a treasure hunt with the items placed according to the map shown. The distance between consecutive grid lines is 200 feet. What is the distance between each pair of objects? Which two objects are closest together? Which two objects are farthest apart?



$$\begin{aligned} \text{Book} \rightarrow \text{Basket} & = 200^2 + 400^2 \\ \text{Pen} \rightarrow \text{Book} & = 600^2 + 200^2 \\ \text{Basket} \rightarrow \text{Pen} & = 600^2 + 200^2 \\ \text{Basket} \rightarrow \text{Pen} & = 800^2 + 200^2 \\ \text{Book} \rightarrow \text{Backpack} & = 400^2 + 800^2 \end{aligned}$$

14. An amusement park designer wants to place a Ferris wheel midway between the two largest coasters. The distance between consecutive grid lines is 500 feet.



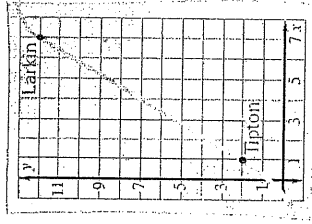
$$\begin{aligned} M & = \left(\frac{3000 + 3500}{2}, \frac{3500 + 500}{2} \right) \\ M & = (1750, 2000) \end{aligned}$$

- a. Determine the coordinates of where the Ferris wheel should be.

- b. How far will the Ferris wheel be from each of the coasters? Round to the nearest foot.

$$d^2 = (1750 - 500)^2 + (2000 - 500)^2$$

15. You are biking a straight line distance between the two towns shown on the map. The distance between the grid lines is 1 mile.



- a. How far is your bike ride ONE way?

$$d^2 = 6^2 + 10^2$$

- b. How far is your bike ride round trip?

- c. You stop for a snack break halfway between the two towns. What are the coordinates of your location?

$$M = \left(\frac{1+7}{2}, \frac{2+12}{2} \right)$$

$$M(4, 7)$$