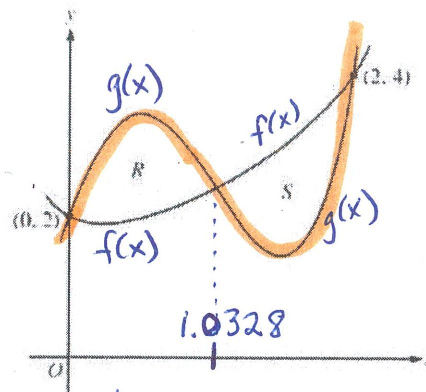
Key

Area/Volume FRQ Practice Problems WS

1) **(Calculator Active)**

Let f and g be the functions defined by $f(x) = 1 + x + e^{x^2-2x}$ and $g(x) = x^4 - 6.5x^2 + 6x + 2$. Let R and S be the two regions enclosed by the graphs of f and g shown in the figure above.



(a) Find the sum of the areas of regions R and S.

(b) Region S is the base of a solid whose cross sections perpendicular to the x-axis are squares. Find the volume of the solid.

(c) Let h be the vertical distance between the graphs of f and g in region S. Find the rate at which h changes with respect to x when $x = 1.8$.

$h'(x)$

(5)

* Use $2^{nd} \to$ Trace $\to$ Intersect feature on calculator to find $x \approx 1.0328$

a) Area is Top-bottom split into 2 integrals:

$$\text{Area} = \int_0^{1.0328} x^4 - 6.5x^2 + 6x + 2 - (1 + x + e^{x^2-2x})\,dx + \int_{1.0328}^{2} 1 + x + e^{x^2-2x} - (x^4 - 6.5x^2 + 6x + 2)\,dx$$

$$\text{Area} = 0.99743 + 1.00692 = \boxed{2.004}$$

b) cross-section volume: Top-bottom

$$\text{base} = 1 + x + e^{x^2-2x} - (x^4 - 6.5x^2 + 6x + 2)$$

$$\text{Area(square)} = (\text{base})^2$$

$$\text{Area} = \left[1 + x + e^{x^2-2x} - x^4 + 6.5x^2 - 6x - 2\right]^2$$

$$\text{Volume} = \int_{1.0328}^{2} \left[1 + x + e^{x^2-2x} - x^4 + 6.5x^2 - 6x - 2\right]^2 dx = \boxed{1.283}$$

c) $h(x)$ = change between f and g

$h(x) = f(x) - g(x)$

$h'(x) = f'(x) - g'(x)$

$h'(1.8) = f'(1.8) - g'(1.8)$

* use math $\to$ 8 $\to$ nderiv

$\boxed{h'(1.8) = -3.812}$

(Calculator-active)

Question 2

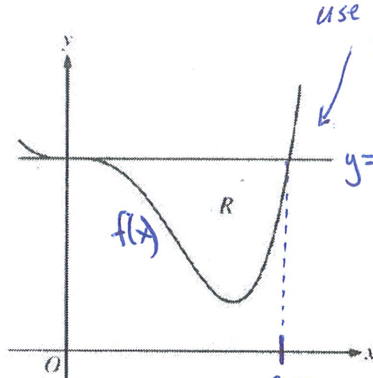
Let R be the region enclosed by the graph of $f(x) = x^4 - 2.3x^3 + 4$ and the horizontal line $y = 4$, as shown in the figure above.

(a) Find the volume of the solid generated when R is rotated about the horizontal line $y = -2$.

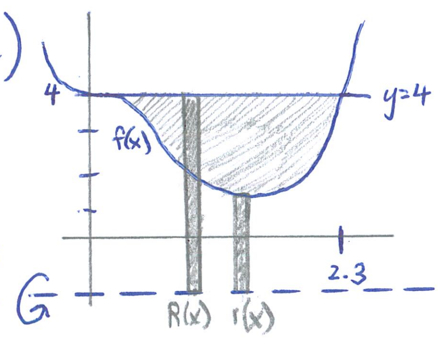
(b) Region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is an isosceles right triangle with a leg in R. Find the volume of the solid.

(c) The vertical line $x = k$ divides R into two regions with equal areas. Write, but do not solve, an equation involving integral expressions whose solution gives the value k.

use 2nd → Trace → Intersect to find x=2.3



a)



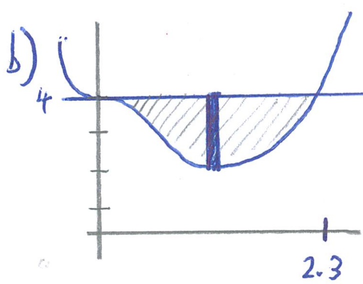
*washer method

$R(x) = 4-(-2) = 6$

$r(x) = x^4 - 2.3x^3 + 4 - (-2) = x^4 - 2.3x^3 + 6$

$$V = \pi \int_0^{2.3} 6^2 - (x^4 - 2.3x^3 + 6)^2 \, dx = \boxed{98.868}$$

b)



*vertical base: top-bottom

$\text{base} = 4 - (x^4 - 2.3x^3 + 4) = \cancel{4} - x^4 + 2.3x^3 - \cancel{4}$

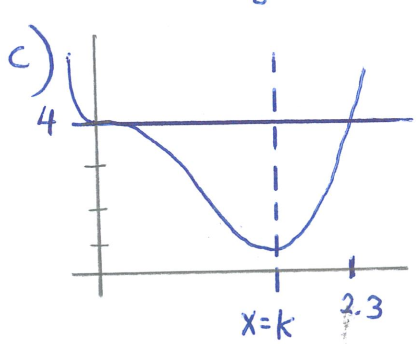
$\text{base} = -x^4 + 2.3x^3$

$\text{Area(isosceles triangle leg on base)} = \frac{1}{2}(\text{base})^2$

$\text{Area} = \frac{1}{2}(-x^4 + 2.3x^3)^2$

$$\text{Volume} = \int_0^{2.3} \frac{1}{2}(-x^4 + 2.3x^3)^2 \, dx = \boxed{3.574}$$

c)

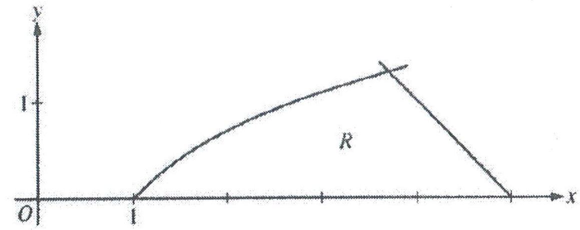


*2 regions with equal areas. Set up to find K.

$$\boxed{\int_0^k 4 - (x^4 - 2.3x^3 + 4) \, dx = \int_k^{2.3} 4 - (x^4 - 2.3x^3 + 4) \, dx}$$

3) (Calculator – Active)

Let R be the region in the first quadrant bounded by the x-axis and the graphs of $y = \ln x$ and $y = 5 - x$, as shown in the figure above.



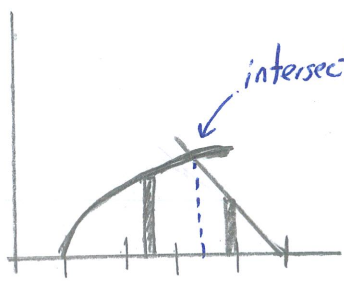
(a) Find the area of R.

(b) Region R is the base of a solid. For the solid, each cross section perpendicular to the x-axis is a square. Write, but do not evaluate, an expression involving one or more integrals that gives the volume of the solid.

(c) The horizontal line $y = k$ divides R into two regions of equal area. Write, but do not solve, an equation involving one or more integrals whose solution gives the value of k.

a)

intersection (3.693, 1.3065)



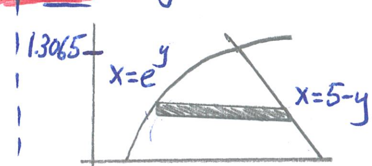
If Top-bottom, split into 2 integrals:

$$\text{Area} = \int_1^{3.693} \ln x - 0\,dx + \int_{3.693}^{5} 5 - x - 0\,dx$$

$$\boxed{\text{Area} = 2.986}$$

OR Right-Left

$y = \ln x \quad e^y = e^{\ln x} \quad \downarrow \quad x = e^y$ | $y = 5 - x \quad x = 5 - y$

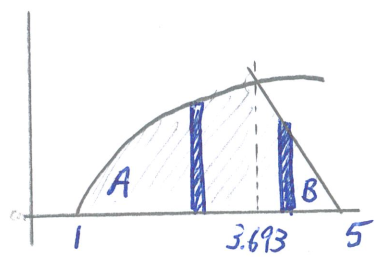


Right-Left

$$\text{Area} = \int_0^{1.3065} 5 - y - (e^y)\,dy$$

$$\boxed{\text{Area} = 2.986}$$

b)



* volume-cross section
* vertical base (top-bottom)

base (region A) $= \ln x - 0 = \ln x$

$\downarrow$

Area $= (\ln x)^2$

base (region B) $= 5 - x - 0$

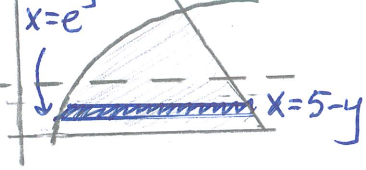
$\downarrow$

Area $= (5-x)^2$

Area $= (\text{base})^2$ (square)

$$\boxed{\text{Volume} = \int_1^{3.693} (\ln x)^2 dx + \int_{3.693}^{5} (5-x)^2 dx}$$

c)

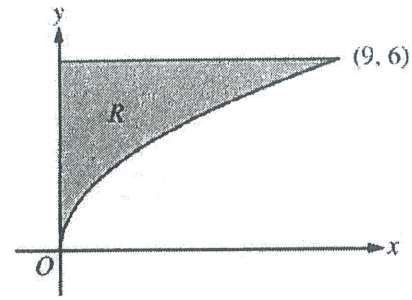


*Area is 2.986 (from part a)

$$\boxed{\int_0^k 5 - y - e^y\,dy = \frac{1}{2}(2.986)}$$

*bottom half region is half of overall Area.

4) Non-Calculator



Let R be the region in the first quadrant bounded by the graph of $y = 2\sqrt{x}$, the horizontal line $y = 6$, and the y-axis, as shown in the figure above.

(a) Find the area of R.

(b) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = 7$.

(c) Region R is the base of a solid. For each y, where $0 \le y \le 6$, the cross section of the solid taken perpendicular to the y-axis is a rectangle whose height is 3 times the length of its base in region R. Write, but do not evaluate, an integral expression that gives the volume of the solid.

(d) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when region R is rotated about the vertical line x = -1.

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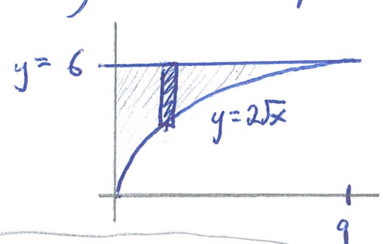
a) Area: Top-Bottom

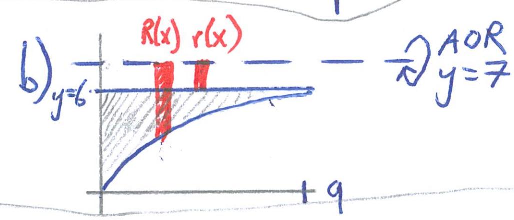


$$\text{Area} = \int_0^9 6-2\sqrt{x}\,dx \to \int 6-2x^{1/2}\,dx$$

$$\to 6x - 2\left(\frac{x^{3/2}}{3/2}\right)\Big]_0^9 = 6(9)-2\left(\frac{2}{3}\right)(9)^{3/2}$$

$$= 54-\frac{4}{3}(27) = \boxed{18}$$

b)



Washer Method

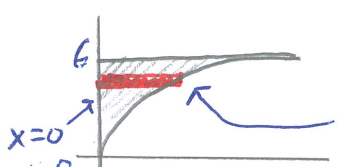
$R(x) = 7-2\sqrt{x}$

$r(x) = 7-6 = 1$

$$\boxed{V=\pi\int_0^9 (7-2\sqrt{x})^2-(1)^2\,dx}$$

c) Volume cross-section

$$\boxed{V = 3\int_0^6 \left(\frac{y^2}{4}\right)^2 dy}$$



$y=2\sqrt{x}$

$\downarrow$

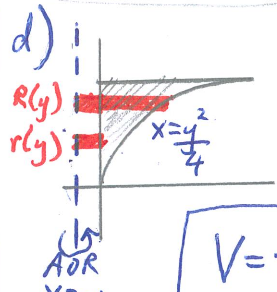
$x=\left(\frac{y}{2}\right)^2$

*horizontal base (Right-Left)

base $= \frac{y^2}{4}-0$ | height $= 3\left(\frac{y^2}{4}\right)$

Area = base x height | Area $=\left(\frac{y^2}{4}\right)\cdot 3\left(\frac{y^2}{4}\right)$

d)



*washer Method

*Right-Left

$R(y) = \frac{y^2}{4}-(-1) = \frac{y^2}{4}+1$

$r(y) = 0-(-1) = 1$

$$\boxed{V=\pi\int_0^6 \left[\frac{y^2}{4}+1\right]^2-[1]^2\,dy}$$