Calculus AB FRQ day **Curve Sketching**

1.

1994 AB 1

Let f be the function given by $f(x) = 3x^4 + x^3 - 21x^2$.

a. Write an equation of the line tangent to the graph of f at the point (2, -28)

b. Find the absolute minimum value of f. Show the analysis that leads to your conclusion.

c. Find the x-coordinate of each point of inflection on the graph of f. Show the analysis that leads to your conclusion.

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2.

1981 AB 3 BC 1

Let f be the function defined by $f(x) = 12x^{(2/3)} - 4x$.

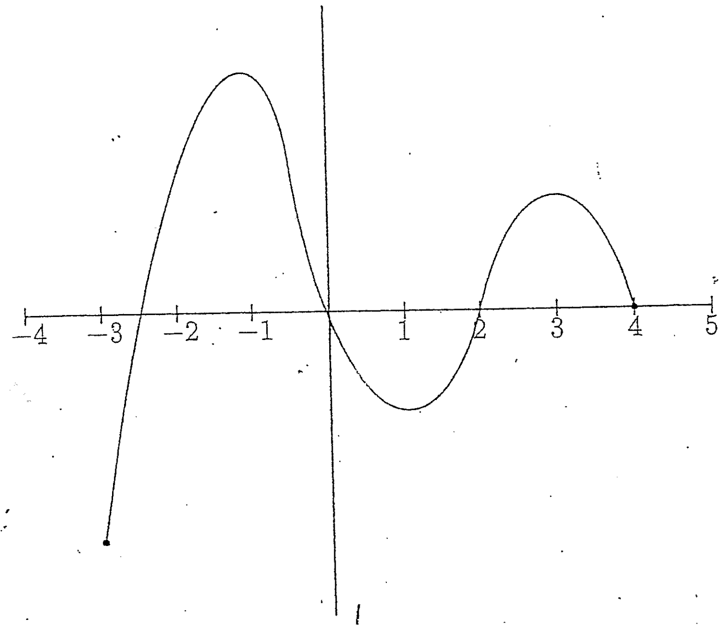
a. Find the intervals on which f is increasing.
b. Find the x- and y-coordinates of all relative maximum points.
c. Find the x- and y-coordinates of all relative minimum points.
d. Find the intervals on which f is concave downward.
e. Using the information found in parts a, b, c, and d, sketch the graph of f on the axes provided.

733. The figure below shows the graph of $g'(x)$, the derivative of a function g, with domain $[-3, 4]$.

a) Determine the values of x for which g has a relative minimum and a relative maximum. Justify your answer.

b) Determine the values of x for which g is concave down and concave up. Justify your answer.

c) Based on the information given and the fact that $g(-3) = 3$ and $g(4) = 6$, sketch a possible graph of g.



Calculus AB FRQ day **Curve Sketching** Solution Key

1.

1994 AB 1

Let f be the function given by $f(x) = 3x^4 + x^3 - 21x^2$.

a. Write an equation of the line tangent to the graph of f at the point (2, -28)

b. Find the absolute minimum value of f. Show the analysis that leads to your conclusion.

c. Find the x-coordinate of each point of inflection on the graph of f. Show the analysis that leads to your conclusion.

a) $f'(x) = 12x^3 + 3x^2 - 42x$

$f'(2) = 12(2)^3 + 3(2)^2 - 42(2) = 24$

point: $(2, -28)$

slope: $m = 24$

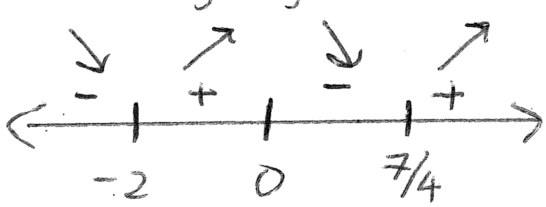
$\boxed{y + 28 = 24(x-2)}$

b) $f'(x) = 12x^3 + 3x^2 - 42x$

$0 = 3x(4x^2 + x - 14)$

$0 = 3x(4x-7)(x+2)$

$x = 0, 7/4, -2$



Abs. min must be either at $x = -2$ or $x = 7/4$ since $f'(x) < 0$ for all $x < -2$ and $f'(x) > 0$ for all $x > 7/4$

$f(-2) = -44$

$f(7/4) = -30.816$

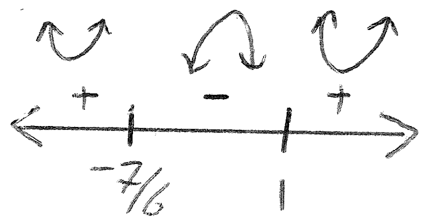
$\boxed{\text{Abs. min is } \underline{-44} \text{ at } x = -2}$

c) $f''(x) = 36x^2 + 6x - 42$

$= 6(6x^2 + x - 7)$

$0 = 6(6x+7)(x-1)$

$x = -7/6, x = 1$



$\boxed{\text{POI at } x = -7/6,\ x = 1 \text{ b/c } f''(x) \text{ change signs}}$

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Let f be the function defined by f(x) = $12x^{(2/3)} - 4x$.

a. Find the intervals on which f is increasing.
b. Find the x- and y-coordinates of all relative maximum points.
c. Find the x- and y-coordinates of all relative minimum points.
d. Find the intervals on which f is concave downward.
e. Using the information found in parts a, b, c, and d, sketch the graph of f on the axes provided.

a) $f'(x) = 12 \cdot \frac{2}{3} x^{-1/3} - 4$

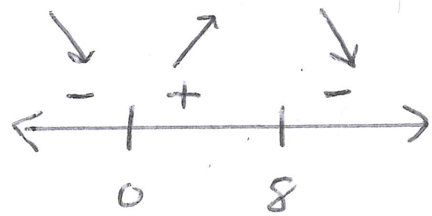
$f'(x) = \frac{8}{x^{1/3}} - 4 = \frac{8 - 4x^{1/3}}{x^{1/3}}$

$8 - 4x^{1/3} = 0$ | $x^{1/3} = 0$

$4x^{1/3} = 8$ | $\boxed{x = 0}$

$x^{1/3} = 2$

$\boxed{x = 8}$



a) f(x) increasing (0, 8] b/c $f'(x) > 0$

b) Rel. max at $(8, \underline{16})$ b/c $f'(x)$ changes from + to −

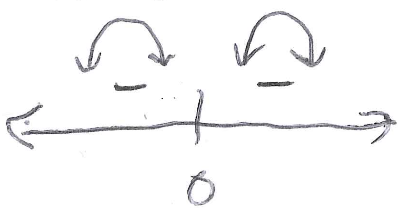
c) Rel. min at $(0, \underline{0})$ b/c $f'(x)$ changes from − to +

$f'(x) = 8x^{-1/3} - 4$

d) $f''(x) = 8 \cdot -\frac{1}{3} x^{-4/3} + 0$

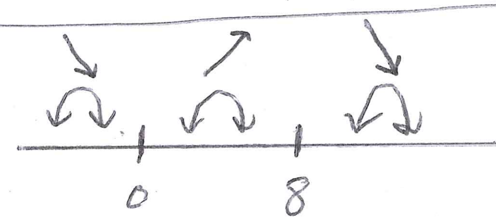
$0 = \frac{-8}{3x^{4/3}}$

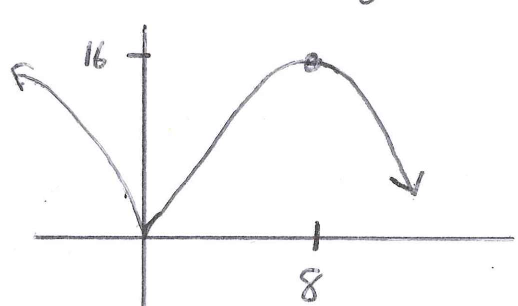
$x = 0$



f(x) concave down $(-\infty, 0) \cup (0, \infty)$ b/c $f''(x) < 0$

e)





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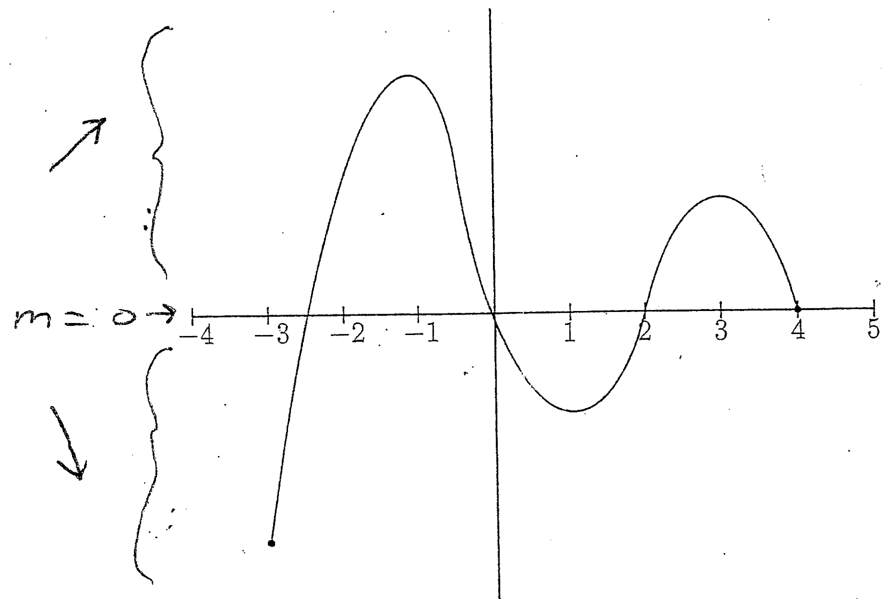
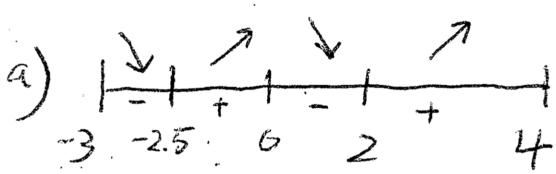
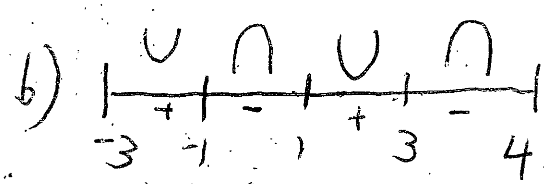


a) 

Rel. min at $x=-2.5$, $x=2$ b/c $f'(x)$ changes sign from − to +

Rel. max at $x=0$ b/c $f'(x)$ changes sign from + to −

b) 

$f(x)$ is concave up $(-3,-1) \cup (1,3)$ b/c $f''(x) > 0$

$f(x)$ is concave down $(-1,1) \cup (3,4)$ b/c $f''(x) < 0$

c)