Key

AP FRQ Review WS: Particle Motion

1) Calculator-Active

* make sure calculator in radian mode

A particle moves along the x-axis with velocity given by $v(t) = \dfrac{10\sin\left(0.4t^2\right)}{t^2 - t + 3}$ for time $0 \le t \le 3.5$.

The particle is at position $x = -5$ at time $t = 0$.

(a) Find the acceleration of the particle at time $t = 3$.

(b) Find the position of the particle at time $t = 3$.

(c) Evaluate $\int_0^{3.5} v(t)\,dt$, and evaluate $\int_0^{3.5} |v(t)|\,dt$. Interpret the meaning of each integral in the context of the problem.

(d) A second particle moves along the x-axis with position given by $x_2(t) = t^2 - t$ for $0 \le t \le 3.5$. At what time t are the two particles moving with the same velocity?

a) $a(3) = v'(3) = -2.118$ * use calculator: math 8 → nderiv

b) * final position = initial position + displacement

$x(b) = x(a) + \int_a^b v(t)\,dt$

$x(3) = x(0) + \int_0^3 \frac{10\sin(0.4t^2)}{t^2-t+3}\,dt$

$x(3) = -5 + 3.2397$

$x(3) = -1.760$

c) $\int_0^{3.5} v(t)\,dt = 2.844$

$\int_0^{3.5} |v(t)|\,dt = 3.737$

c) $\int_0^{3.5} v(t)\,dt$ is the displacement of particle over time interval $0 \le t \le 3.5$

$\int_0^{3.5} |v(t)|\,dt$ is total distance traveled by particle in $0 \le t \le 3.5$

d) $x_2(t) = t^2 - t$

$v_2(t) = 2t - 1$

* set $v_1(t) = v_2(t)$

$\frac{10\sin(0.4t^2)}{t^2-t+3} = 2t - 1$

$\frac{10\sin(0.4t^2)}{t^2-t+3} - (2t-1) = 0$

Graph this equation → $x = 1.5705$ and find x-intercept

same velocity at $t = 1.571$

2) Non-Calculator

Two particles move along the x-axis. For $0 \le t \le 8$, the position of particle P at time t is given by $x_P(t) = \ln\left(t^2 - 2t + 10\right)$, while the velocity of particle Q at time t is given by $v_Q(t) = t^2 - 8t + 15$. Particle Q is at position $x = 5$ at time $t = 0$.

(a) For $0 \le t \le 8$, when is particle P moving to the left?

(b) For $0 \le t \le 8$, find all times t during which the two particles travel in the same direction.

(c) Find the acceleration of particle Q at time $t = 2$. Is the speed of particle Q increasing, decreasing, or neither at time $t = 2$? Explain your reasoning.

(d) Find the position of particle Q the first time it changes direction.

a) * Find $v(t)$ equation, find critical points, create sign line, test intervals

$v(t) = \frac{2t-2}{t^2-2t+10} \rightarrow \frac{2(t-1)}{t^2-2t+10}$

$v_p(t)$: − on $(0, 1)$, + on $(1, 8)$; test values $\frac{1}{2}$, 2

Particle moves left $[0, 1)$ since $v(t) < 0$

b) $v_Q(t) = t^2 - 8t + 15$

$0 = (t-5)(t-3)$

$t = 5, t = 3$

$v_Q(t)$: + on $(0, 3)$, − on $(3, 5)$, + on $(5, 8)$; test values 1, 4, 6

Both particles move same directions in $1 < t < 3$ and $5 < t \le 8$ since $v_p(t)$ and $v_Q(t)$ have same signs.

c) $a_Q(t) = 2t - 8$

$a_Q(2) = 2(2) - 8 = -4 < 0$

$v_Q(2) = 3 > 0$

since at $t = 2$, $a(t)$ and $v(t)$ have opposite signs, speed of particle decreasing at $t = 2$.

d) Particle Q first change direction at $t = 3$.

* final pos. = initial pos. + displacement

$x(b) = x(a) + \int_a^b v(t)\,dt$

$x(3) = x(0) + \int_0^3 t^2 - 8t + 15\,dt$

$x(3) = 5 + \left[\frac{t^3}{3} - \frac{8t^2}{2} + 15t\right]_0^3$

$x(3) = 5 + \left(\frac{3^3}{3} - 4(3)^2 + 15(3)\right) - (0 - 0 + 0)$

$x(3) = 5 + 9 - 36 + 45$

$\boxed{x(3) = 23}$

3) Non - Calculator

For $0 \le t \le 12$, a particle moves along the x-axis. The velocity of the particle at time t is given by $v(t) = \cos\left(\frac{\pi}{6}t\right)$. The particle is at position $x = -2$ at time $t = 0$. $\rightarrow x(0) = -2$

(a) For $0 \le t \le 12$, when is the particle moving to the left?

(b) Write, but do not evaluate, an integral expression that gives the total distance traveled by the particle from time $t = 0$ to time $t = 6$.

(c) Find the acceleration of the particle at time t. Is the speed of the particle increasing, decreasing, or neither at time $t = 4$? Explain your reasoning.

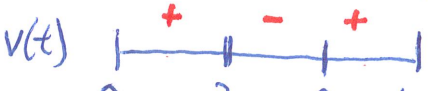
(d) Find the position of the particle at time $t = 4$.

a) *set $v(t) = 0$, find critical points, create sign line, test intervals.

$v(t) = \cos\left(\frac{\pi}{6}t\right)$ | $\frac{\pi}{6}t = \cos^{-1}(0)$ | $\frac{\pi}{6}t = \frac{\pi}{2}$ | $\frac{\pi}{6}t = \frac{3\pi}{2}$

$0 = \cos\left(\frac{\pi}{6}t\right)$ | $\frac{\pi}{6}t = \frac{\pi}{2}, \frac{3\pi}{2}$ | $t = \frac{\pi}{2}\cdot\frac{6}{\pi} = 3$ | $t = \frac{3\pi}{2}\cdot\frac{6}{\pi} = 9$

$t = 3$ | $t = 9$



Particle moves left when $v(t) < 0$, and this occurs in the interval $3 < t < 9$.

b) Total distance: $\int_0^6 |v(t)|\,dt$

c) $a(t) = v'(t) = -\sin\left(\frac{\pi}{6}t\right)\cdot\frac{\pi}{6}$

$a(t) = \frac{-\pi}{6}\sin\left(\frac{\pi}{6}t\right)$

$a(4) = \frac{-\pi}{6}\sin\left(\frac{4\pi}{6}\right) = \frac{-\sqrt{3}\pi}{12} < 0$

$v(4) = \cos\left(\frac{2\pi}{3}\right) = -\frac{1}{2} < 0$

The speed of particle is increasing at time $t=4$ since velocity and acceleration have the same signs.

d) * final position = initial position + displacement

$x(b) = x(a) + \int_a^b v(t)\,dt$

$x(4) = x(0) + \int_0^4 v(t)\,dt$

$x(4) = -2 + \int_0^4 \cos\left(\frac{\pi}{6}t\right)dt$

$u = \frac{\pi}{6}t$ | $\pi\,dt = 6\,du$

$\frac{du}{dt} = \frac{\pi}{6}$ | $dt = \frac{6}{\pi}\,du$

$\int \cos u \cdot \frac{6}{\pi}\,du \rightarrow \frac{6}{\pi}\int \cos u\,du$

$= \frac{6}{\pi}\sin u \rightarrow \frac{6}{\pi}\sin\left(\frac{\pi}{6}t\right)\Big]_0^4$

$\frac{6}{\pi}\sin\left(\frac{\pi}{6}\cdot 4\right) - \frac{6}{\pi}\sin(0) = \frac{6}{\pi}\left(\frac{\sqrt{3}}{2}\right)$

$x(4) = -2 + \frac{6\sqrt{3}}{2\pi} = \boxed{-2 + \frac{3\sqrt{3}}{\pi}}$

4) Calculator

A particle moves along a straight line. For $0 \le t \le 5$, the velocity of the particle is given by $v(t) = -2 + \left(t^2 + 3t\right)^{6/5} - t^3$, and the position of the particle is given by $s(t)$. It is known that $s(0) = 10$.

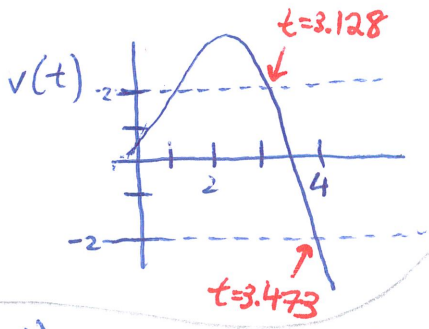
(a) Find all values of t in the interval $2 \le t \le 4$ for which the speed of the particle is 2.

(b) Write an expression involving an integral that gives the position $s(t)$. Use this expression to find the position of the particle at time $t = 5$.

(c) Find all times t in the interval $0 \le t \le 5$ at which the particle changes direction. Justify your answer.

(d) Is the speed of the particle increasing or decreasing at time $t = 4$? Give a reason for your answer.

a) If speed is 2, then $|v(t)| = 2$, which means speed is 2 if $v(t) = 2$ or $v(t) = -2$. (* Graph $v(t) = -2 + (t^2+3t)^{6/5} - t^3$ and see where the graph intersects $y=2$ and $y=-2$)



$t = 3.128$ and $t = 3.473$

b)

* final position = initial position + displacement

$$x(b) = x(a) + \int_a^b v(t)\,dt$$

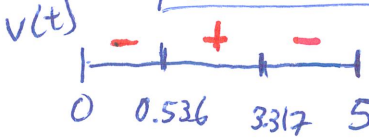
$$s(5) = s(0) + \int_0^5 v(t)\,dt$$

$s(5) = 10 - 19.207$

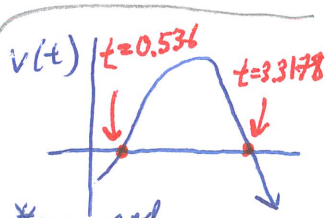
$s(5) = -9.207$

c) * particle change direction when $v(t)$ crosses x-axis's

$t = 0.536$ and $t = 3.3178$ when $v(t) = 0$.



and $v(t)$ change signs at these x-intercepts.



*use 2nd → Trace → Zero

d) $v(4) = -11.4757 < 0$

$a(4) = -22.2957 < 0$

Speed of particle is increasing at $t = 4$ since velocity and acceleration have the same signs.