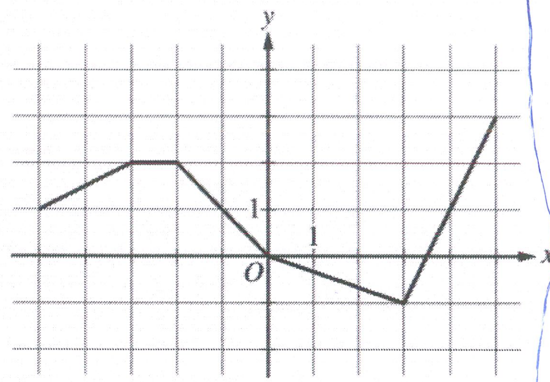
Key

AP Review FRQ Topic: Functions and Tables

1) Non-Calculator

x	$g(x)$	$g'(x)$
−5	10	−3
−4	5	−1
−3	2	4
−2	3	1
−1	1	−2
0	0	−3



Graph of h

d) Since $g(x)$ is continuous and differentiable $[-5,-3]$, by MVT since $\frac{g(-3)-g(-5)}{-3-(-5)} = \frac{2-10}{2} = -4$, then there is at least one value where $g'(c)=-4$

Let f be the function defined by $f(x) = \cos(2x) + e^{\sin x}$.

Let g be a differentiable function. The table above gives values of g and its derivative g' at selected values of x.

Let h be the function whose graph, consisting of five line segments, is shown in the figure above.

(a) Find the slope of the line tangent to the graph of f at $x = \pi$.

(b) Let k be the function defined by $k(x) = h(f(x))$. Find $k'(\pi)$.

(c) Let m be the function defined by $m(x) = g(-2x) \cdot h(x)$. Find $m'(2)$.

(d) Is there a number c in the closed interval $[-5, -3]$ such that $g'(c) = -4$? Justify your answer.

* Recall that $\frac{d}{dx}\cos u = -\sin u \cdot u'$

$\frac{d}{dx}e^u = e^u \cdot u'$

a) *find $f'(x)$ first: $f'(x) = -\sin 2x \cdot 2 + e^{\sin x} \cdot \cos x$

$f'(\pi) = -2\sin(2\pi) + e^{\sin\pi} \cdot \cos(\pi) = 0 + e^0(-1) = \boxed{-1}$

b) $k(x) = h[f(x)]$ ← Apply chain rule $\frac{d}{dx}f[g(x)] = f'(g(x)) \cdot g'(x)$

$k'(x) = h'[f(x)] \cdot f'(x)$

$k'(\pi) = h'[f(\pi)] \cdot f'(\pi)$

* $f(\pi) = \cos(2\pi) + e^{\sin\pi} = 1+1 = 2$

$k'(\pi) = h'[2] \cdot f'(\pi)$

$k'(\pi) = -\frac{1}{3} \cdot (-1)$

$k'(\pi) = \boxed{\frac{1}{3}}$

c) $m(x) = \overbrace{g(-2x)}^{f} \cdot \overbrace{h(x)}^{g}$ ← product Rule

$m'(x) = \overbrace{g'(-2x)(-2)}^{f'} \cdot \overbrace{h(x)}^{g} + \overbrace{g(-2x)}^{f} \cdot \overbrace{h'(x)}^{g'}$

$m'(2) = -2g'(-4) \cdot h(2) + g(-4) \cdot h'(2)$

$m'(2) = -2(-1)(\frac{-2}{3}) + (5)(\frac{-1}{3})$

$m'(2) = -\frac{4}{3} - \frac{5}{3} = -\frac{9}{3} = \boxed{-3}$

2) Non-Calculator

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	–6	3	2	8
2	2	–2	–3	0
3	8	7	6	2
6	4	5	3	–1

The functions f and g have continuous second derivatives. The table above gives values of the functions and their derivatives at selected values of x.

(a) Let $k(x) = f(g(x))$. Write an equation for the line tangent to the graph of k at $x = 3$.

(b) Let $h(x) = \dfrac{g(x)}{f(x)}$. Find $h'(1)$.

(c) Evaluate $\int_1^3 f''(2x)\,dx$.

Apply chain rule: $\frac{d}{dx} f[g(x)] = f'[g(x)] \cdot g'(x)$

a) $k(x) = f(g(x))$

$k'(x) = f'[g(x)] \cdot g'(x)$

$k'(3) = f'[g(3)] \cdot g'(3)$

$k'(3) = f'[6] \cdot g'(3)$

$k'(3) = (5)(2) = 10$

point: $k(3) = f(g(3))$
$= f(6)$
$= 4$

point: $(3,4)$ slope: $m = 10$

$\boxed{y - 4 = 10(x-3)}$

b) $h(x) = \dfrac{g(x)}{f(x)}$ ← quotient rule

$h'(x) = \dfrac{g'(x)\cdot f(x) - g(x)\cdot f'(x)}{[f(x)]^2}$

$h'(1) = \dfrac{g'(1)f(1) - g(1)f'(1)}{[f(1)]^2}$

$h'(1) = \dfrac{(8)(-6) - (2)(3)}{(-6)^2} = \dfrac{-48-6}{36} = \dfrac{-54}{36} = \boxed{\dfrac{-3}{2}}$

c) $\int_1^3 f''(2x)\,dx$

* Apply FFTC: $\int_a^b f'(x)\,dx = f(b) - f(a)$
* Apply u-sub / convert bounds

$u = 2x$

$\dfrac{du}{dx} = 2$

$dx = \dfrac{du}{2}$

$\int f''(u)\dfrac{du}{2}$

$\dfrac{1}{2}\int f''(u)\,du$

$\to \frac{1}{2} f'(u) \to \frac{1}{2} f'(2x)\Big]_1^3 = \frac{1}{2}f'(6) - \frac{1}{2}f'(2)$

$= \frac{1}{2}(5) - \frac{1}{2}(-2) = \boxed{\dfrac{7}{2}}$

OR convert bounds

$x=1,\ u=2x \to u=2$

$x=3,\ u=2x \to u=6$

$\frac{1}{2}f'(u)\Big]_2^6 = \frac{1}{2}f'(6) - \frac{1}{2}f'(2)$

3) Non-Calculator

x	-2	$-2 < x < -1$	-1	$-1 < x < 1$	1	$1 < x < 3$	3
$f(x)$	12	Positive	8	Positive	2	Positive	7
$f'(x)$	-5	Negative	0	Negative	0	Positive	$\frac{1}{2}$
$g(x)$	-1	Negative	0	Positive	3	Positive	1
$g'(x)$	2	Positive	$\frac{3}{2}$	Positive	0	Negative	-2

The twice-differentiable functions f and g are defined for all real numbers x. Values of f, f', g, and g' for various values of x are given in the table above.

(a) Find the x-coordinate of each relative minimum of f on the interval $[-2, 3]$. Justify your answers.

(b) Explain why there must be a value c, for $-1 < c < 1$, such that $f''(c) = 0$.

(c) The function h is defined by $h(x) = \ln(f(x))$. Find $h'(3)$. Show the computations that lead to your answer.

(d) Evaluate $\int_{-2}^{3} f'(g(x))g'(x)\,dx$.

a) $x=1$ is the critical point where $f'(x)$ changes sign from $-$ to $+$. Therefore, $f(x)$ has relative minimum at $x=1$.

b) By MVT, (or Rolle's) since $f'(x)$ is continuous and differentiable on $(-1,1)$ and $\frac{f'(1)-f'(-1)}{1-(-1)} = \frac{0-0}{2} = 0$, there must be a point such that $f''(c)=0$.

c) $h(x) = \ln(f(x))$ ← find $h'(x)$ using chain rule: $\frac{d}{dx} f(g(x)) = f'(g(x)) \cdot g'(x)$

$h'(x) = \frac{f'(x)}{f(x)} \rightarrow h'(3) = \frac{f'(3)}{f(3)} = \frac{1/2}{7} = \frac{1}{2} \cdot \frac{1}{7} = \boxed{\frac{1}{14}}$

d) $\int_{-2}^{3} f'[g(x)] \cdot g'(x)\,dx$

*Apply U-sub:

$u = g(x)$

$\frac{du}{dx} = g'(x)$

$dx = \frac{du}{g'(x)}$

$\int f'(u) \cdot \cancel{g'(x)} \cdot \frac{du}{\cancel{g'(x)}}$

$\int f'(u)\,du \rightarrow f(u)$

$f(g(x))\Big]_{-2}^{3} = f(g(3)) - f(g(-2))$

$= f(1) - f(-1)$

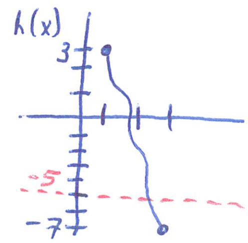
$= 2 - 8$

$= \boxed{-6}$

4) Non-Calculator

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	6	4	2	5
2	9	2	3	1
3	10	−4	4	2
4	−1	3	6	7

The functions f and g are differentiable for all real numbers, and g is strictly increasing. The table above gives values of the functions and their first derivatives at selected values of x. The function h is given by $h(x) = f(g(x)) - 6$.



(a) Explain why there must be a value r for $1 < r < 3$ such that $h(r) = -5$.

(b) Explain why there must be a value c for $1 < c < 3$ such that $h'(c) = -5$.

(c) Let w be the function given by $w(x) = \int_1^{g(x)} f(t)\,dt$. Find the value of $w'(3)$.

(d) If g^{-1} is the inverse function of g, write an equation for the line tangent to the graph of $y = g^{-1}(x)$ at $x = 2$.

a) $h(1) = f(g(1)) - 6 = f(2) - 6 = 9 - 6 = 3$
$h(3) = f(g(3)) - 6 = f(4) - 6 = -1 - 6 = -7$

since $h(3) < -5 < h(1)$ and $h(x)$ is continuous, then by IVT (Intermediate Value Theorem) there exists value such that $h(r) = -5$

b) By MVT, since $\frac{h(3) - h(1)}{3-1} = \frac{-7-3}{3-1} = \frac{-10}{2} = -5$, then there must exist a value c, $1 < c < 3$ such that $h'(c) = -5$.

c) $w(x) = \int_1^{g(x)} f(t)\,dt$. Recall SFTC: $\frac{d}{dx}\int_a^{p(x)} f(t)\,dt = f(p(x)) \cdot p'(x)$

$w'(x) = f(g(x)) \cdot g'(x)$ | $w'(3) = f(g(3)) \cdot g'(3) \rightarrow f(4) \cdot g'(3) \rightarrow (-1) \cdot (2) = \boxed{-2}$

d) Since $g(1) = 2$, then $g^{-1}(2) = 1$
Since $g'(1) = 5$, then $(g^{-1})'(2) = \frac{1}{5}$

point: $(2, 1)$
slope: $m = \frac{1}{5}$

$y - y_1 = m(x - x_1)$
$\boxed{y - 1 = \frac{1}{5}(x - 2)}$