CCGPS Geometry
Spring Summary Sheet

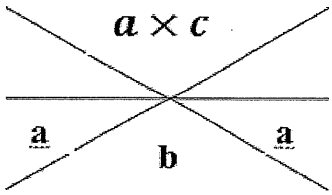
Monomial: number,variable,or product of a number and one or more variables with whole number expoents: (ex. $-6a^3b^2$)

Binomial: expression of the sum or the difference of two terms

Polynomial: is an expression consisting of variables and coefficients, that involves the operations of non-negative integer exponents.

Factoring Steps

1) Factor out GCF (greatest common factor)
2) **Star method** for expression in standard form of $\mathbf{ax^2 + bx + c}$
3) Find a pair of numbers that multiply to be $a \times c$ and adds to be **b**



4) Put factors in the form (x)(x)
5) Reduce the fractions
6) Pull denominator in front of the x's

Steps for Completing the square:

1) Rearrange equation in standard form: $ax^2 + bx + c = 0$

2) divide each term in the equation by **a** if a $\neq 1$ (We need the new **a** value to be 1)

3) Move the constant to the other side of the equation.

4) Add spaces "+___" to the equation: $\mathbf{x^2 + bx +}$ ___ $\mathbf{= c +}$ __

5) Find $\left(\frac{b}{2}\right)^2$ and enter this value into the blank spaces ___ on both sides of the equation

6) Rewrite left side in factored form and add the numbers on the right side

) take the square root ($\sqrt{\ }$) of both sides (don't forget $\pm$)

8) solve for x

* A **quadratic equation** is as an equation of degree 2, **meaning** that the highest exponent of this function is 2.

* The quadratic formula is used to solve an equation of the form $\mathbf{ax^2 + bx + c = 0}$

*This formula can solve any equation that can be solved by factoring and completing the square

Quadratic Formula: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

given $ax^2 + bx + c = 0$

The **Discriminant** is the number (from the expression) inside the square root of the quadratic formula.

Since the quadratic formula is $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, the discriminant is the $\mathbf{b^2 - 4ac}$

The discriminant describes the **nature**, or the type, of solutions (or roots)

If the Discriminant is **positive**, there are 2 real answers (2 real roots or **solutions**)

If the Discriminant is **negative**, there are 2 imaginary answers (2 imaginary roots)

If the Discriminant is **zero**, there is 1 real answer. (2 real answers being the same value) (1 real root)

Unit 5B: Graphing Quadratic Functions

1. **Standard form:** $\mathbf{y = ax^2 + bx + c}$

a. The x-coordinate of the vertex is located at $x = \frac{-b}{2a}$

b. The axis of symmetry (AOS) is the vertical line $x = \frac{-b}{2a}$

c. Make a T-table, put the vertex in the middle of the t-table.

d. Fill in the rest of the values (use calculator)

e. Graph the points with a smooth, U-shaped curve

f. **Domain** : always all Real Numbers $(-\infty, \infty)$

g. **Range**: lowest y-value to the highest y-value

h. Determine **Increasing/Decreasing** Intervals

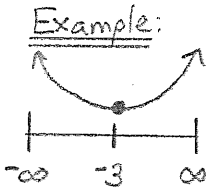
*Determine the x-value of the vertex

*Create number line with endpoints as $-\infty$ *and* $+\infty$

*Sketch the parabola above number line

* determine interval where graph is rising (increasing) and interval where graph is falling (decreasing)

Example:



Increasing: $(-3, \infty)$

Decreasing: $(-\infty, -3)$

i. Determine **Positive/Negative** intervals

*Determines x-values of the x-intercepts

*Create number line with endpoints as $-\infty$ *and* $+\infty$ and x-values of intercepts

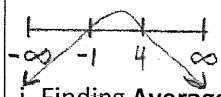
*Sketch the shape of parabola through the x-intercepts

* determine interval where graph is above the number line (positive) and interval where graph is below the number line(negative)

Example:

positive: $(-1, 4)$

negative: $(-\infty, -1) \cup (4, \infty)$



j. Finding **Average Rate of Change**

* Find the 2 ordered pairs using the x-values given in the interval

* Find the slope between the 2 ordered pairs: m $m = \frac{y_2 - y_1}{x_2 - x_1}$

2. **Intercept Form**: $\mathbf{y = a(x - p)(x - q)}$

a) If "a" is positive (> 0) the parabola opens up

If "a" is negative (< 0) the parabola opens down.

b) The x-intercepts are the points x = p and x = q. Set factors equal to 0 and solve to get p and q.

c) The **x-coordinate of the vertex** is half way between the x-intercepts

d) Make a T-table, put the vertex in the middle of the t-table.

e) Fill in the rest of the values (use calculator)

3. Vertex Form: $y = a(x - h)^2 + k$

a) If "a" is positive (> 0) the parabola opens up If "a" is negative (< 0) the parabola opens down.

b) The **vertex** is the point (h, k)

c) Make a T-table, put the vertex in the middle of the t-table.

d) Fill in the rest of the values (use calculator)

Quadratic Function Transformations in Vertex Form $y = a(x - h)^2 + k$

1. If **a** is negative, there is a vertical reflection and the parabola will open downwards.
2. $|a|$ is the vertical stretch factor.
   - If $|a| > 1$, vertical stretch
   - $|a| < 1$, vertical compress
3. h is the horizontal translation (shift)
   " **–h**" means shift **right** h units
   " **+h**" means shift **left** h units
4. k is the vertical translation (shift)
   " **–k**" means shift **down** k units
5. " **+k**" means shift **up** k units

Quadratic Inequalities Steps

1. Determine x-values of the x-intercepts
2. Create number line with endpoints as $-\infty$ *and* $+\infty$ and x-values of intercepts
3. Sketch the shape of parabola through the x-intercepts (if a > 0, parabola opens up. If a < 0, parabola opens down)

* determine interval where graph is above the number line (positive) and interval where graph is below the number line(negative)

a) $f(x) > 0$ is everything ***above*** the x-axis, ***not including*** the intercepts (use parenthesis)
b) $f(x) < 0$ is everything ***below*** the x-axis, ***not including*** the intercepts (use parenthesis)
c) $f(x) \geq 0$ is everything ***above*** the x-axis, ***including*** the intercepts (use brackets)
d) $f(x) \leq 0$ is everything ***below*** the x-axis, ***including*** the intercepts (use brackets)

Projectile Motion for Quadratic Word Problems Projectile Motion Formula

$h(t) = \frac{1}{2}at^2 + v_i t + h_i$

h(t) = final height (at the end of the problem)

a = acceleration due to gravity (-32 ft/s^2)

v_i = initial velocity

h_i = initial height (at the beginning of the problem)

t = time (from initial height to final height)

Unit 6: Modeling Geometry

Distance Formula:

$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$ or

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Midpoint Formula:

$M = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

A **circle** is the set of all points equidistant(same distance) from a fixed point called the **radius**. The standard equation for a circle with center (h,k) and radius r is $(x-h)^2 + (y-k)^2 = r^2$

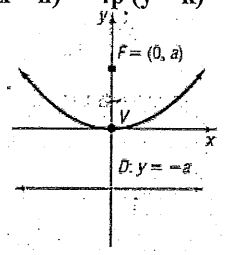
Circle: Converting from general form to standard form steps:

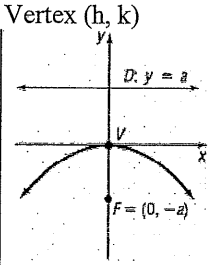
1. Group x's and y's on the left side, then move the constant to the other side of equation.
2. Complete the square with x's, using $\left(\frac{b}{2}\right)^2$ be sure to **balance** the equation.
3. Complete the square with y's, using $\left(\frac{b}{2}\right)^2$ be sure to **balance** the equation.
4. Express each perfect square trinomial as a binomial squared to become standard form $(x-h)^2 + (y-k)^2 = r^2$

Parabola: a conic section where the distance from 1 fixed point (focus) and a line (directrix) is equal.

If graph opens up or down: (Think $y = x^2$)

$(x-h)^2 = 4p\,(y-k)$ Vertex (h, k)





p: distance from vertex to focus (inside parabola)

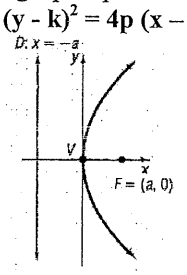
p: distance from vertex to directrix (line outside parabola)

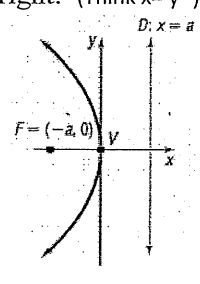
Axis of Symmetry(AOS) : x = h

Focal Width = $|4p|$

If graph opens left or right: (Think $x = y^2$)

$(y-k)^2 = 4p\,(x-h)$





Vertex (h, k)

p: distance from vertex to focus (inside parabola)

p: distance from vertex to directrix (line outside parabola)

Axis of Symmetry(AOS) : y = k

Focal Width = $|4p|$

Parabola: Converting from general form to standard form steps:

1. Group the variable with the squared term on the left. The other variable is on the right.
2. Complete the square for the squared variable, using $\left(\frac{b}{2}\right)^2$ be sure to **balance** the equation
3. Write equation in standard form

Solving Systems of equations

Substitution Steps:

1.Choose one equation and solve for one variable on one side (either the x or the y)
2.Substitute the solution from step 1 into the second equation and solve for the variable in the equation.
3.Using the value found in step 2, substitute it into the first equation and solve for the second variable.
4. Substitute the values for both variables into both equations to show they are correct.

Unit 7: Probability Unit

1. **The Multiplication Counting Principle:** this is to find the total number of combinations given a number of different events:

If there are **m** ways to make a first selection and **n** ways to make a second selection, there are **m** × **n** ways to make the two selections.

2. **Pick one (card)** - Add and/or subtract probabilities (do not change denominator)

$\frac{_}{52} + \frac{_}{52}$ or $\frac{_}{52} + \frac{_}{52} - \frac{_}{52}$

a. Mutually exclusive: no overlaps

b. Overlapping events: There are shared events so we need to subtract overlapping events

3. **Pick more than one (card)**

Multiply probabilities

**Don't need to look for overlaps!

These are Different events!

Independent	a) With replacement - $\frac{_}{52} * \frac{_}{52}$
Dependent	b)Without replacement $\frac{_}{52} * \frac{_}{51}$

Independent P(A|B) = P(B|A)

Dependent P(A|B) ≠ P(B|A)

CCGPS Analytic Geometry | Factoring Quiz Review | Jan 20, 2015 (Tues)

Factor each polynomial and solve

1. $x^2 - 16x + 15 = 0$

2. $14x^3 = 2x$

3. $2x^2 - x - 28 = 0$

4. $6x^3 + 15x^2 - 9x = 0$

5. $2x^2 + 16x = 40$

6. $9x^2 = 4$

7. $2x^2 - 98 = 0$

8. $3x^2y - 6xy - 45y = 0$

9. $2x^2 = -8x + 42$

10. $6bx^2 + 7bx + 2b = 0$

11. $4x^2 + 6x - 4 = 0$

12. $12x^2 - 2x = 30$

CCGPS Analytic Geometry **Unit 5A Test Review #2**

Solve equation by factoring

1. $48x^3 = 9x + 6x^2$

Factored Form: _____

Solution: _____

Solve equation by Completing the Square

2. $16 + 4x^2 = 32x$

3. a) Solve equation by quadratic formula
   b) Find the discriminant
   c) Based on the discriminant, find the number and the type of solutions

   $3 - 4x^2 = 7x$

4. Solve equation choosing a method (this may be solved multiple ways)

   $4x^2 + 12x^3 = 12x$

Solve equation by factoring

5. $20x^4 - 60x^2 - 65x^3 = 0$

Factored Form: _____

Solution: _____

Solve equation by Completing the Square

6. $2x^2 + 10 = 16x$

7. a) Solve equation by quadratic formula
   b) Find the discriminant
   c) Based on the discriminant, find the number and the type of solutions

$2x^2 + 7x = -9$

8. Create a quadratic function that has solutions at $x = -3/4$, and $x = 2/3$. Make sure that it is written in standard form $ax^2 + bx + c = 0$

Analytic Geometry Unit 5B Practice

1. $y = x^2 - 4x - 12$

Form: _____ a = _____ Opens: _____

Vertex: _____ AOS: _____

x-int: _____ y-int: _____

Domain: _____ Range: _____

End Behavior: As $x \to \infty, f(x) \to$ _____

As $x \to -\infty, f(x) \to$ _____

Interval of Increase: _____

Interval of Decrease: _____

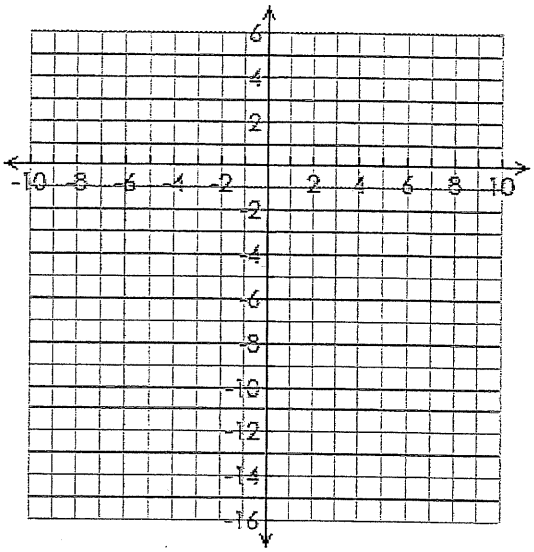
Interval(s) of Positive: _____

Interval(s) of Negative: _____

Min: _____

Max: _____

Average Rate of Change from [4, 6] = _____



2. $y = 2(x + 3)(x - 1)$

Form: _____ a = _____ Opens: _____

Vertex: _____ AOS: _____

x-int: _____ y-int: _____

Domain: _____ Range: _____

End Behavior: As $x \to \infty, f(x) \to$ _____

As $x \to -\infty, f(x) \to$ _____

Interval of Increase: _____

Interval of Decrease: _____

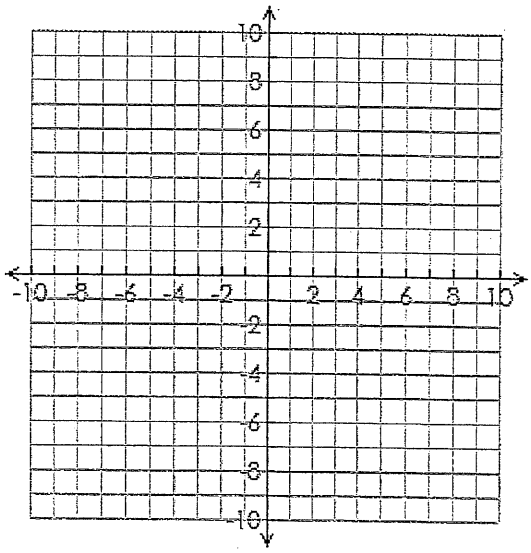
Interval(s) of Positive: _____

Interval(s) of Negative: _____

Min: _____

Max: _____

Average Rate of Change from [-4, 0] = _____



3. $y = -(x-4)^2 + 16$

Form: _____ a = _____ Opens: _____

Vertex: _____ AOS: _____

x-int: _____ y-int: _____

Domain: _____ Range: _____

End Behavior: As $x \to \infty, f(x) \to$ _____

As $x \to -\infty, f(x) \to$ _____

Interval of Increase: _____

Interval of Decrease: _____

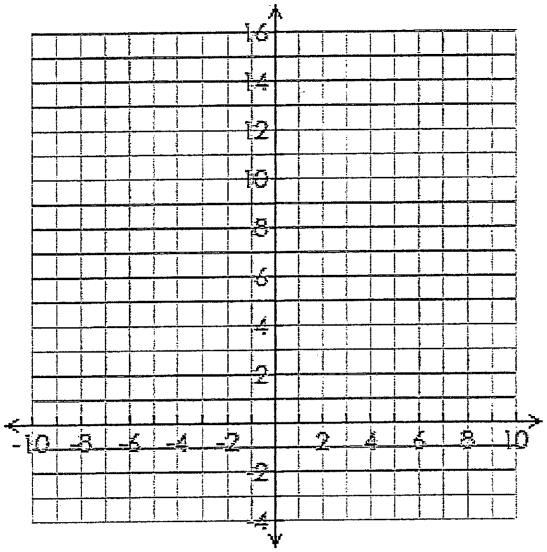
Interval(s) of Positive: _____

Interval(s) of Negative: _____

Min: _____

Max: _____

Average Rate of Change from [0, 2] = _____



4. $y = -x^2 - 10x - 21$

Form: _____ a = _____ Opens: _____

Vertex: _____ AOS: _____

x-int: _____ y-int: _____

Domain: _____ Range: _____

End Behavior: As $x \to \infty, f(x) \to$ _____

As $x \to -\infty, f(x) \to$ _____

Interval of Increase: _____

Interval of Decrease: _____

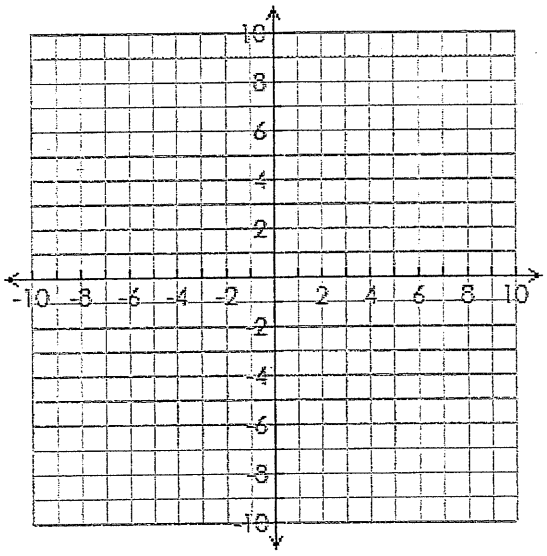
Interval(s) of Positive: _____

Interval(s) of Negative: _____

Min: _____

Max: _____

Average Rate of Change from [-6, -2] = _____



For the following problems, either state the transformation that are occurring or write the equation based on the transformations given.

5. $y = -2(x-4)^2 + 7$

Vertex: _____ Horizontal translation: _____

Vertical translation: _____ Stretch/compress/reflect: _____

6. $y = -\frac{1}{2}(x-3)^2 - 9$

Vertex: _____ Horizontal translation: _____

Vertical translation: _____ Stretch/compress/reflect: _____

7. $y = -(x+5)^2$

Vertex: _____ Horizontal translation: _____

Vertical translation: _____ Stretch/compress/reflect: _____

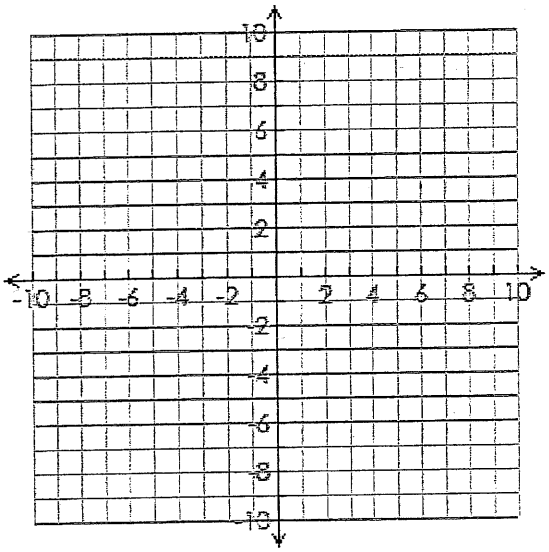
8. The function $y = x^2$, is translated 3 to the right and 4 down.

9. The function $y = x^2$ is reflected over the $x-$axis and translated 8 up.

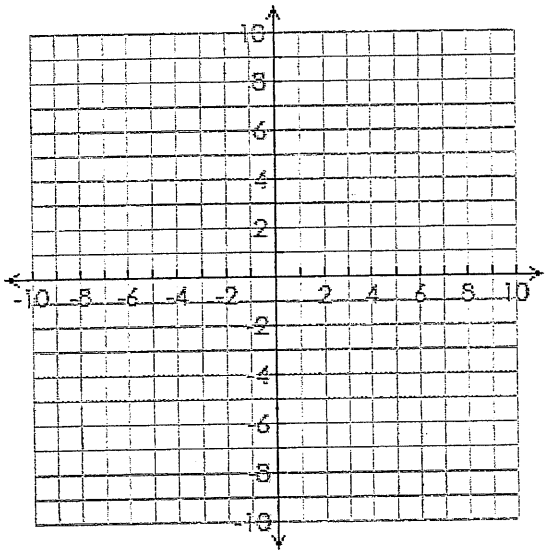
10. The function $y = x^2$ is stretched by a factor of 3 and translated 3 to the left.

For the following problems, sketch a graph of the inequality and write the solutions in interval notation.

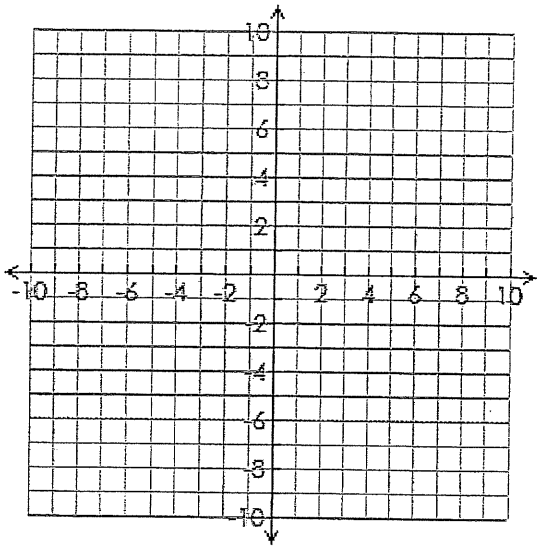
11. $-4x^2 - 24x - 32 \geq 0$ _____



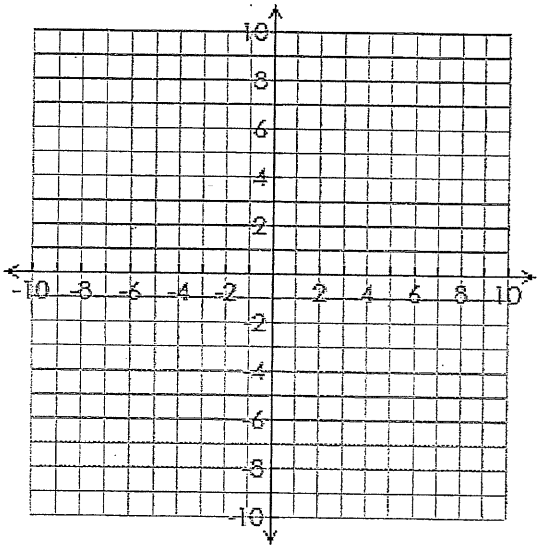
12. $-2x^2 + 4x < 0$ Solutions: _____



13. $x^2 + 4x + 3 \geq 0$ Solutions: _____



14. $2(x-7)(x+1) < 0$ Solutions: _____



15. The product of two numbers is 640. Their difference is 12. Find the two numbers.

16. Jason jumped off of a cliff that is 480 feet high, into the ocean in Acapulco with initial velocity of 16 ft/s, while vacationing with some friends. His height as a function of time could be modeled by the function $h(t) = -16t^2 + v_i t + h_i$, where t is the time in seconds and h is the height in feet.

a. How long did it take for Jason to reach the water?

b. What was the highest point that Jason reached? (think about how to find the vertex of this parabola)

17. The length of a rectangle is 6 less than 3 times the width. The area of the rectangle is 144 square inches. Find the length and the width.

CCGPS Analytic Geometry
Circle Quiz Review WS #1

Name_____

Circle equation: $(x-h)^2 + (y-k)^2 = r^2$

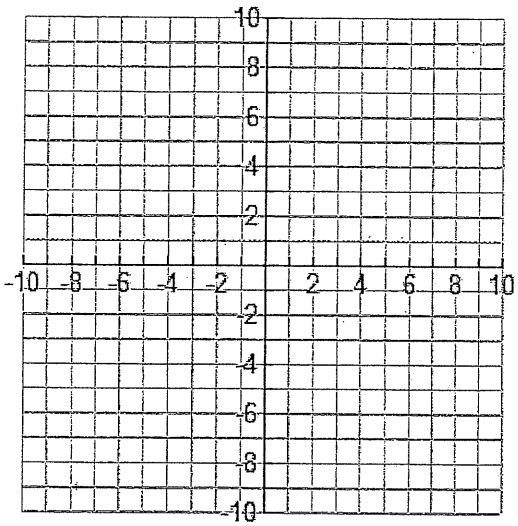
Midpoint: $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

1. Given the circle: $x^2 + (y+3)^2 = 36$

a. Identify the center:_____

b. Identify the radius:_____

c. Graph the circle.



2. Given the circle whose center is the origin and goes through the point (-2, 7)

Write the equation of the circle in standard form:_____

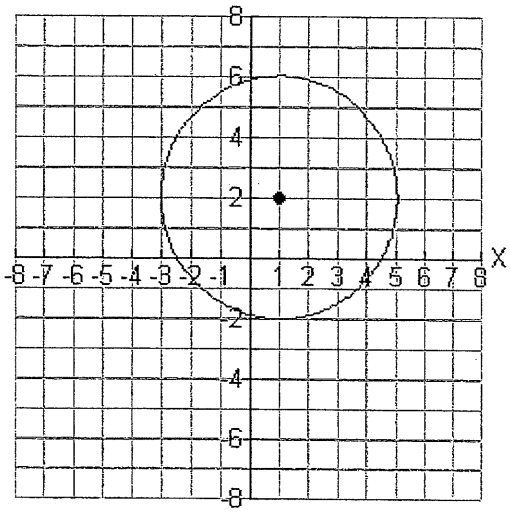
3. Given the following graph of the circle:

Identify the center:_____

Identify the radius:_____

Write the equation in standard form:

_____



Circle equation: $(x-h)^2 + (y-k)^2 = r^2$ Midpoint: $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

4. Given the equation below:

$$y^2 + 2x + x^2 = 24y - 120$$

A. Write this equation in standard form:_____

B. Identify the center and radius of the circle. Center:_____ Radius:_____.

5. Given the circle with a center at (1, -7) and a radius of 15.

Write the equation of the circle in standard form:_____

6. The circle passes through the point (5, 6) and has its center at (2, 3).

Write the equation of the circle in standard form:_____

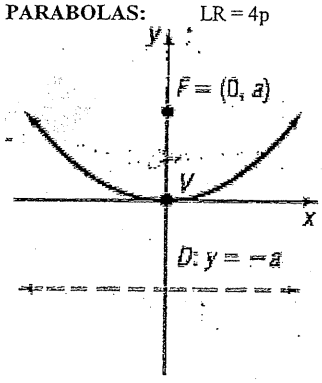
7. The endpoints of a diameter are at (2, 3) and at (−6, −5).

Write the equation of the circle in standard form:_____

Analytic Geometry **Conics: Parabola Day 1** March 9, 2015 (Mon)

Parabola: a conic section where the distance from 1 fixed point (focus) and a line is equal.

PARABOLAS: LR = 4p



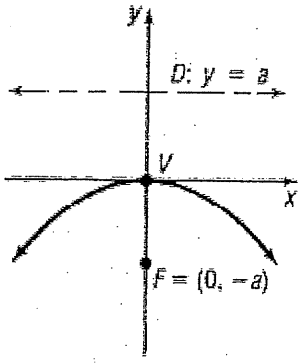
Graph opens up
(Think $y = x^2$)

$(x - h)^2 = 4p\,(y - k)$

Vertex (h, k)
p: distance from vertex to focus (inside parabola)
p: distance from vertex to directrix (line outside parabola)

Axis of Symmetry(AOS) : x = h

LR = $|4p|$



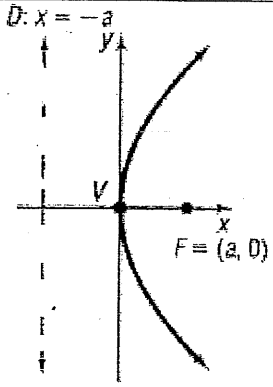
Graph opens down
(Think $y = -x^2$)

$(x - h)^2 = 4p\,(y - k)$

Vertex (h, k)
p: distance from vertex to focus (inside parabola)
p: distance from vertex to directrix (line outside parabola)

AOS: x = h

LR = $|4p|$



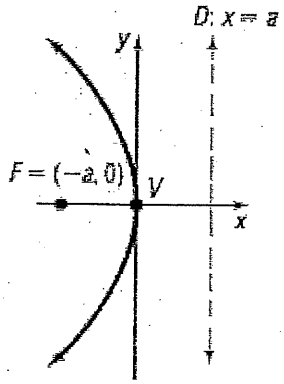
Graph opens right
(Think $x = y^2$)

$(y - k)^2 = 4p\,(x - h)$

Vertex (h, k)
p: distance from vertex to focus (inside parabola)
p: distance from vertex to directrix (line outside parabola)

AOS: y = k

LR = $|4p|$



Graph opens left
(Think $x = -y^2$)

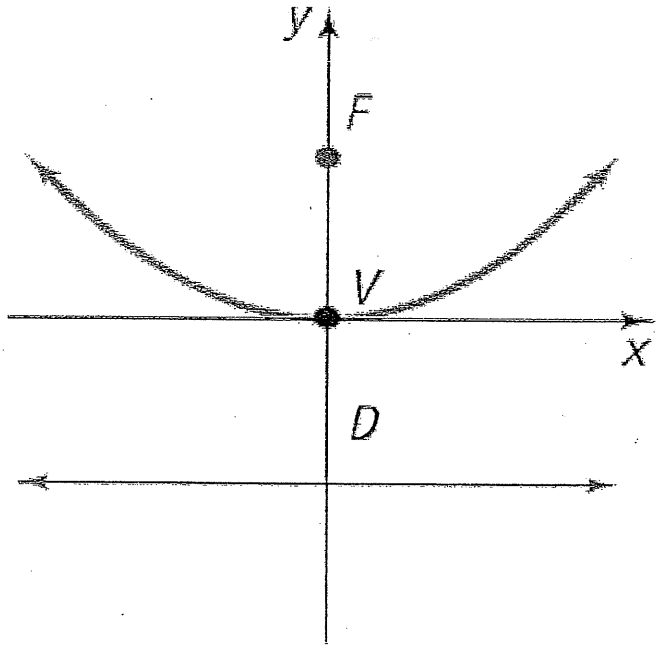
$(y - k)^2 = 4p\,(x - h)$

Vertex (h, k)
p: distance from vertex to focus (inside parabola)
p: distance from vertex to directrix (line outside parabola)

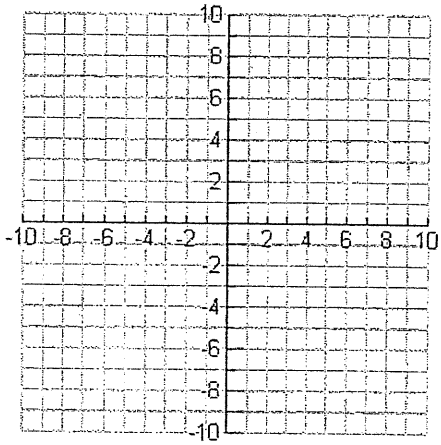
AOS: y = k

LR = $|4p|$

Graph the parabola: State the vertex, AOS, focus, directrix, and LR



1. $(x - 2)^2 = 8(y + 1)$



Vertex: _____

AOS: _____

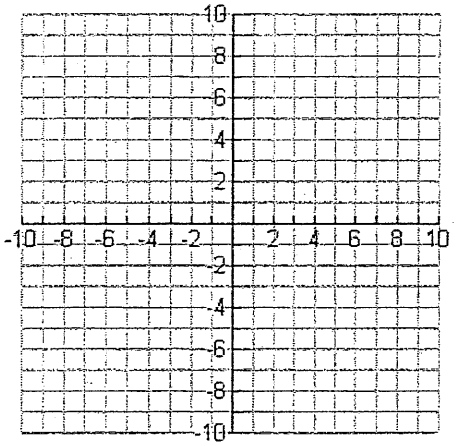
Focus: _____

Directrix: _____

LR: _____

Graph the parabola: State the vertex, AOS, focus, directrix, and LR

2. $(y + 3)^2 = -20x$



Vertex: _____

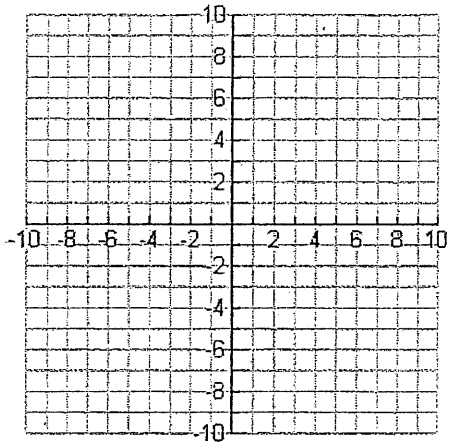
AOS: _____

Focus: _____

Directrix: _____

LR: _____

3. $(x + 4)^2 = 16(y - 2)$



Vertex: _____

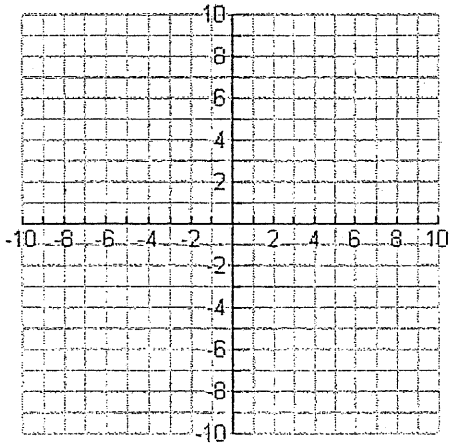
AOS: _____

Focus: _____

Directrix: _____

LR: _____

4. $(y - 3)^2 = 12(x - 1)$



Vertex: _____

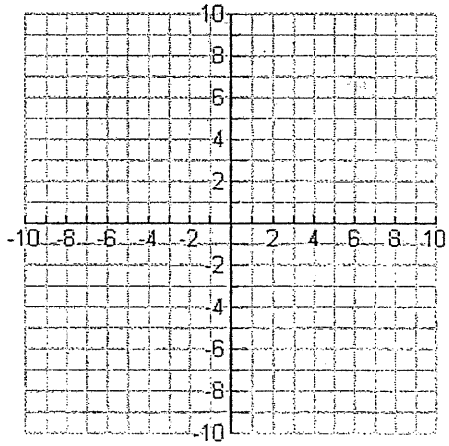
AOS: _____

Focus: _____

Directrix: _____

LR: _____

5. $(x - 5)^2 = -4(y + 5)$



Vertex: _____

AOS: _____

Focus: _____

Directrix: _____

LR: _____

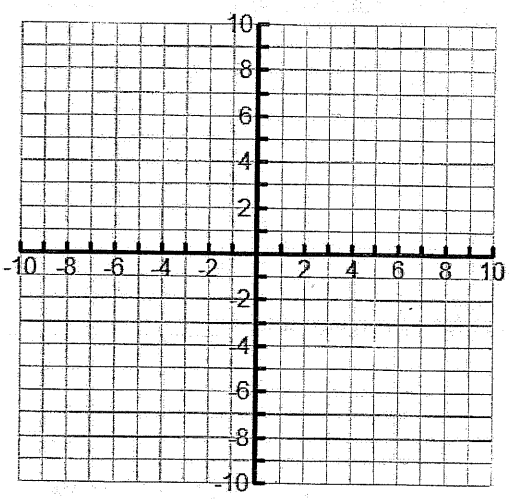
CCGPS Analytic Geometry **Unit 6 Review 2**

Circles, Parabolas, Systems of Equations

Equations: $(x-h)^2 + (y-k)^2 = r^2$ $(x-h)^2 = 4p(y-k)$ $(y-k)^2 = 4p(x-h)$

Graph the equation and identify the important characteristics

1. $8y + x^2 - 4x + 1 = 5$



Standard Form: _____

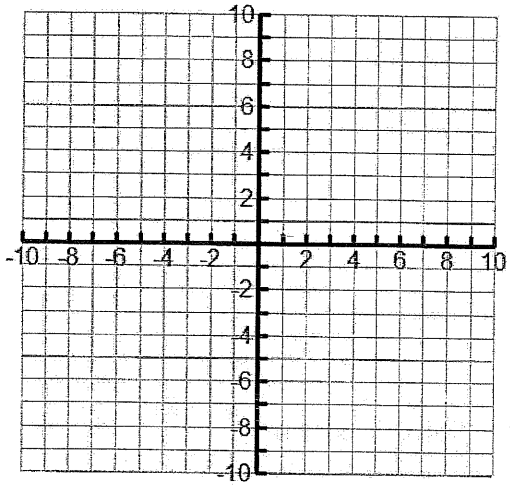
Opens: _____ Vertex: _____ p = _____

Focus: _____ Directrix: _____

Focal Width: _____ Axis of Symmetry: _____

Write the equation for each circle in <u>standard form</u>. Then identify the center and radius.

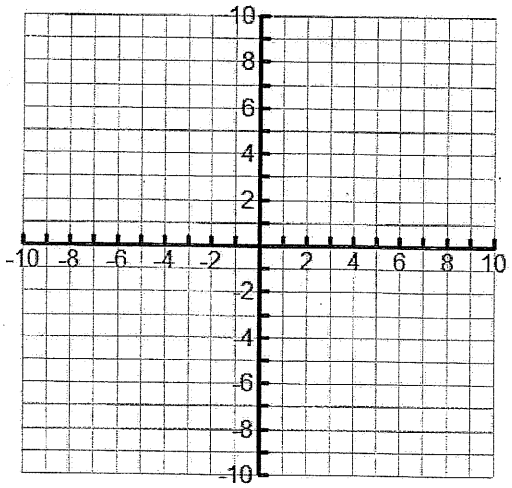
2. The endpoints of a diameter are at (-4, 4) and at (-8, 2).



Equation: _____

Center: _____ Radius: _____

3. $x^2 + y^2 + 2x - 4y - 11 = 0$



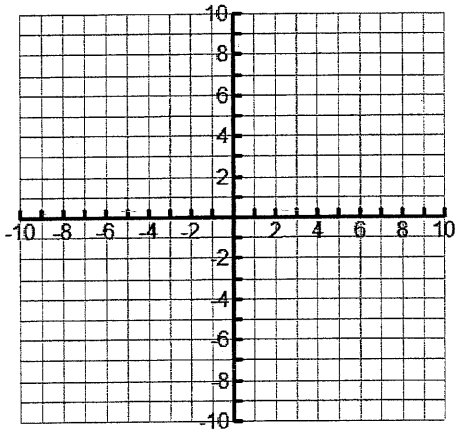
Standard Form Equation : _____

Center: _____ Radius: _____

6. Given the circle with a center at (-2, 6) and a radius of $5\sqrt{2}$.

Write the equation of the circle in standard form:_____

7. Find the standard form of the equation for the parabola with a focus located at (-1, –2) and directrix at $x=5$



Standard form equation _____

8. Solve the system $\begin{cases} x^2 + y^2 = 36 \\ x + y + 6 = 0 \end{cases}$ using algebra. SHOW ALL WORK.

9. Solve the system $\begin{cases} y^2 + 8x - 6y + 9 = -8 \\ 2x + y = 1 \end{cases}$ using algebra. SHOW ALL WORK.

CCGPS Analytic Geometry Probability Test Review 2

1. The math club is electing new officers. There are 4 candidates for president, 5 candidates for vice-president, 2 candidates for secretary, and 1 candidate for treasurer. How many different combinations of officers are possible?

2. A piggybank contains 2 quarters, 3 dimes, 4 nickels, and 5 pennies. One coin is removed at random.

   a) What is the probability that the coin is a dime?

   b) What is the probability that the coin is a dime **or** a nickel?

   c) What is the probability that you choose a nickel **and** then a nickel? (without replacement)

   d) What is the probability that the coin is not a quarter?

3. Each of the letters of the word "ALGEBRA " is on a separate card. The cards have been mixed and placed in a box. If you select one card at random, what is the probability that its letter will be "A"?

4. A card is randomly selected from a standard deck of 52 cards. Find the indicated probability. **Hint: There are 4 jacks, 4 queens, 4 kings, 4 aces, 13 diamonds, 13 spades, 13 clubs, 13 hearts, 36 numbered cards, 26 red cards, and 26 black cards.**

   a) P(Face card)

   b) P(Ace or a Diamond)

   c) P(Black and Ace)

   d) P(Black card or Face card)

5. Two cards are randomly selected from a standard deck of 52 cards (WITH REPLACEMENT). Find the indicated probability. **Hint: There are 4 jacks, 4 queens, 4 kings, 4 aces, 13 diamonds, 13 spades, 13 clubs, 13 hearts, 36 numbered cards, 26 red cards, and 26 black cards.**

a) P(Jack and Heart)

b) P(Diamond and Diamond and Diamond)

c) P(Red and King)

d) P(Black card and Numbered card)

6. Two cards are randomly selected from a standard deck of 52 cards (**WITHOUT** REPLACEMENT). Find the indicated probability. **Hint: There are 4 jacks, 4 queens, 4 kings, 4 aces, 13 diamonds, 13 spades, 13 clubs, 13 hearts, 36 numbered cards, 26 red cards, and 26 black cards.**

a) P(Jack of Hearts and Heart)

b) P(Face card and Face card and Ace)

d) P(Black Card and Red King)

d) P(Black Jack and Numbered card)

7. The probability that a student plays tennis is 47%. The probability that a student plays tennis and Lacrosse is 16%. What is the probability that student plays Lacrosse, given that they play tennis?

8. The probability that a high school senior drives to school is .81. The probability that a high school senior having a job and driving to school is .52. What is the probability that high school senior will have a job, given that they drive to school?

For #9 - 13, refer to the following table.

	Male	Female	Subtotal
Blue Eyes	40	20	
Green Eyes	10	80	
Subtotal			

9) P (Male) =

10. P (Green Eyes) =

11.P(Green Eyes | Male) =

12. P(Male | Green Eyes) =