GEORGIA MATHEMATICS LEAGUE

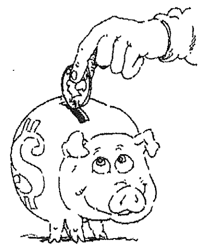
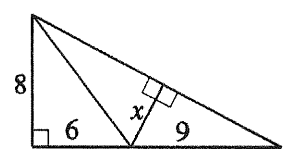
P.O. Box 12014, Columbus, Georgia 31917-2014

All official participants must take this contest at the same time.

Contest Number 1 *Any calculator without a QWERTY keyboard is allowed.* Answers must be exact *or* have 4 (or more) significant digits, correctly rounded. **October 13, 2015**

Name ________________ **Teacher** __________ **Grade Level** ____ **Score** ____

Time Limit: 30 minutes NEXT CONTEST: NOV. 10, 2015 *Answer Column*

1-1. Each vertex of a square is assigned a different positive integer. If the numbers on the endpoints of each diagonal have the same sum, what is the least possible value of this sum?	1-1.
1-2. Every coin in my piggy bank has a face value of 50¢, 25¢, 10¢, 5¢, or 1¢. The bank contains many coins of each type. At most how much money can I withdraw from my piggy bank without being able to make change for $1? 	1-2.
1-3. What are all integers x for which $\lvert x^2 - 26x + 88\rvert$ is a prime?	1-3.
1-4. In the big right triangle shown, the lengths of the legs are 8 and 15. How long is the line segment whose length is marked x? 	1-4.
1-5. Which integer > 1 leaves the same remainder when divided into each of the numbers 1108, 1453, 1844, and 2281?	1-5.
1-6. What is the largest value of c for which exactly three different pairs of positive integers (x,y) satisfy $5x + 7y = c$?	1-6.

Contest # 1 — *Answers & Solutions* — 10/13/15

Problem 1-1

The sum of the smallest four positive integers is $1+2+3+4 = 10$, and since $10/2 = 5$, the sum we seek is 5. Finally, $1+4 = 5 = 2+3$, so it is possible to get two sets of two integers each whose sum is $\boxed{5}$.

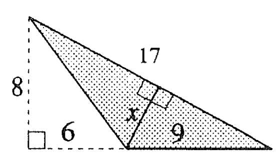
Problem 1-2

We can take at most one 50¢ coin and, with that, at most 1 quarter. Continuing, we can take at most 4 dimes, no nickels, and at most 4 pennies. We can't add more coins without being able to make change for $1, but we can exchange some coins for lower denomination coins. By doing that, we get three sets of coins, {1×50¢, 1×25¢, 4×10¢, 4×1¢}, {3×25¢, 4×10¢, 4×1¢}, and {1×25¢, 9×10¢, 4×1¢}, each with a sum of $\boxed{\$1.19}$.

Problem 1-3

If the expression $|x^2-26x+88| = |x-22| \times |x-4|$ is prime, then one of its factors must be 1. If $|x-22| = 1$, then $x = 23$ or 21. Also, if $|x-4| = 1$, then $x = 5$ or 3. For each of these four values of x, the value of $|x-22| \times |x-4|$ is a prime, so $x = \boxed{3, 5, 21, 23}$.

Problem 1-4



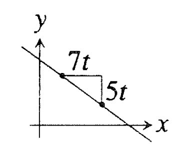
Method I: In the diagram, there are two ways to get the area of the entire shaded triangle. If we use 9 as the base, the altitude is 8. If we use 17 as the base, the altitude is x. Therefore, $9\times 8 = 17x$, so $x = \boxed{\frac{72}{17}}$.

Method II: Using the diagram from Method I, since the rightmost of the two smaller shaded right triangles is similar to the entire 8-15-17 right triangle, we get $9:x = 17:8$. Solving, $x = 72/17$.

Problem 1-5

Any divisor which leaves the same remainder when divided into two different numbers must divide the difference of those two numbers, and so forth. Subtracting, $1453-1108 = 345$, $1844-1453 = 391$, and $2281-1844 = 437$. Continuing, $437-391 = 46$ and $391-345 = 46 = 2(23)$. By inspection, 2 does not work. The required divisor is $\boxed{23}$, which is easily confirmed to leave the same remainder when divided into each of the original numbers.

Problem 1-6



A point with two integral coordinates is called a *lattice point*. Consider the line $5x+7y = c$ for some integer c. One lattice point that lies on any such line is $(3c,-2c)$. Since the slope of the line is $-5/7$, we can move from any lattice point on the line to another lattice point on the line by moving 7 units to the right and 5 units down (or 7 left, 5 up); and every such line passes through an infinite number of lattice points. If (a,b) is a lattice point on the line, then 5 consecutive lattice points that lie on the line are (a,b), $(a+7,b-5)$, $(a+14,b-10)$, $(a+21,b-15)$, and $(a+28,b-20)$. In fact, every lattice point on the line is of the form $(a+7t,b-5t)$, where t is any integer. The middle 3 points will lie in the first quadrant, and the other 2 will not, if and only if every one of the following conditions is met: $a \le 0$, $a+7 > 0$, $b-20 \le 0$, $b-15 > 0$. We conclude that $-7 < a \le 0$ and $15 < b \le 20$. The largest possible values of a and b (which correspond to the largest possible value of c) that satisfy these inequalities are $a = 0$ and $b = 20$. For $(a,b) = (0,20)$, the 5 consecutive lattice points above are $(0,20)$, $(7,15)$, $(14,10)$, $(21,5)$, and $(28,0)$. We can get the value of c by computing $c = 5x+7y$ for any of these points. This value of c is $\boxed{140}$.

