All official participants must take this contest at the same time.

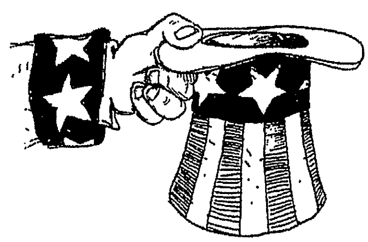
Contest Number 3 *Any calculator without a QWERTY keyboard is allowed.* Answers must be exact *or* have 4 (or more) significant digits, correctly rounded. **December 8, 2015**

Name ____________ **Teacher** ________ **Grade Level** ____ **Score** ___

Time Limit: 30 minutes NEXT CONTEST: JAN. 12, 2016 *Answer Column*

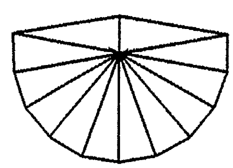
3-1. What is the largest possible degree-measure of an angle of a triangle if the degree-measures of all three angles are positive integers? 3-1.

3-2. A magical hat takes any number fed into it and divides 1492 by that number. If 2015 is fed into the machine and the first output is fed back into the machine, what is the value of the second output? 3-2.



3-3. In a certain sequence of numbers, each number after the first is the sum of all the preceding numbers. If this sequence's 100th term is 2015, what is its 101st term? 3-3.

3-4. Twelve congruent isosceles triangles share a common vertex, as shown. If the sum of the measures of all the angles that share the common vertex is 360°, what is the measure of each triangle's smallest angle? 3-4.



3-5. For what integer $k > 0$ can $(\sqrt{2}-1)^5$ be written as $\sqrt{k+1} - \sqrt{k}$, the difference between the square roots of two consecutive integers? 3-5.

3-6. There are an infinite number of ordered pairs of positive integers (m,n) such that $m^3 = n^2$ and $m+n$ is a perfect square. One such pair is $(m,n) = (9,27)$. What is the largest value of $m < 1000$ for which such an ordered pair exists? 3-6.

Contest # 3 — *Answers & Solutions* — 12/8/15

Problem 3-1

To maximize the largest angle, minimize the two smallest angles. If the two smallest angles have measures of 1 and 1, the measure of the largest angle of the triangle will be $\boxed{178 \text{ or } 178°}$.

Problem 3-2

The first output is $\frac{1492}{2015}$. This output is fed back in and the second output is 1492 divided by $\frac{1492}{2015}$. That output is $\boxed{2015}$.

Problem 3-3

Method I: By inspection, the sequence is a, a, $2a$, $4a$, $8a$, . . . , so for $n > 2$, each term is twice the preceding term. Therefore, is the 100th term is 2015, the 101st term is $\boxed{4030}$.

Method II: We'll let S_n represent the sum of the first n terms of the sequence, and a_n represent its nth term. We're told that $a_n = S_{n-1}$, so $2015 = a_{100} = S_{99}$, so $a_{101} = a_1 + \ldots + a_{99} + a_{100} = S_{99} + a_{100} = 2015 + 2015 = 4030$.

Problem 3-4

In each isosceles triangle, let each vertex angle be $x°$ and let each base angle be $y°$. In each triangle, $x° + 2y° = 180°$. In the diagram, the 12 triangles surround the center, so $10x° + 2y° = 360°$. Subtracting the first equation from the second, $9x° = 180°$, so $x° = \boxed{20°}$.

Problem 3-5

Method I: From the binomial theorem, $(\sqrt{2}-1)^5 = (\sqrt{2})^5 - 5(\sqrt{2})^4 + 10(\sqrt{2})^3 - 10(\sqrt{2})^2 + 5\sqrt{2} - 1 = 4\sqrt{2} - 5(4) + 10(2\sqrt{2}) - 10(2) + 5\sqrt{2} - 1 = 29\sqrt{2} - 41 = \sqrt{1682} - \sqrt{1681}$, so $k = \boxed{1681}$.

Method II: Use the table feature on your calculator, using $y = \sqrt{x+1} - \sqrt{x}$. Use "ask" for the "independent variable" and "auto" for the "dependent variable." By underestimating and overestimating, you should be able to zero in on $x = 1681$ in not too many tries.

Problem 3-6

Each pair (m,n) has the form (t^2, t^3), where t is a positive integer. Since $m+n = t^2+t^3 = t^2(t+1)$ is a square, $t+1$ must also be a square. Let $t+1 = k^2$. [Conversely, if $m = t^2$ and $n = t^3$, where $t = k^2-1$, then $m+n = t^2+t^3 = (1+t)(t^2) = k^2(k^2-1)^2 = (k^3-k)^2$ is a square.] Since $m = t^2$, $t = k^2-1$, and $m < 1000$, it follows that $(k^2-1)^2 < 1000$. Taking square roots, $k^2-1 < 31.6\ldots$, so $k^2 < 32.6\ldots$. The largest such k^2 is 25, so $t+1 = k^2 = 25$ and $t = 24$. Finally, $m = t^2 = 24^2 = \boxed{576}$.

[Note: $m+n = 24^2+24^3 = 576+13824 = 14400 = 120^2$.]

