GEORGIA MATHEMATICS LEAGUE

P.O. Box 12014, Columbus, Georgia 31917-2014

All official participants must take this contest at the same time.

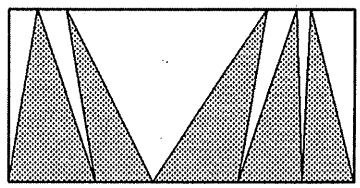
Contest Number 4 *Any calculator without a QWERTY keyboard is allowed.* Answers must be exact *or* have 4 (or more) significant digits, correctly rounded. **January 12, 2016**

Name ________________ **Teacher** __________ **Grade Level** ____ **Score** ____

Time Limit: 30 minutes NEXT CONTEST: FEB. 9, 2016 *Answer Column*

4-1. For all real numbers x, the function f is defined by $f(x) = 2016$. What is the value of $f(x+2016)$? 4-1.

4-2. What is the sum of the areas of the five shaded triangles shown at the right that are drawn interior to a 3 by 6 rectangle? 4-2.



4-3. If $A^{2x} = 4$ and $A > 0$, what is the numerical value of $\frac{A^{3x} - A^{-3x}}{A^{x} - A^{-x}}$, written as a ratio of positive integers in lowest terms? 4-3.

4-4. Al runs three times as fast as he walks. It takes Al 21 minutes to get to work from home if he walks for twice the amount of time that he runs. How many minutes does it take Al to get to work from home if he runs for twice the amount of time that he walks? 4-4.

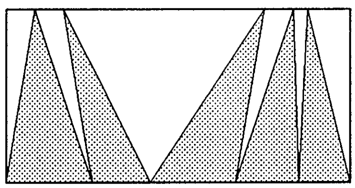
4-5. At most how many of the first 100 positive integers can be chosen if no two of the chosen numbers have a sum divisible by 5? 4-5.

4-6. What is the area of quadrilateral $ABCD$ whose vertices have polar coordinates $A(0,0)$, $B(4,0)$, $C(3,\frac{\pi}{8})$, $D(1,\frac{3\pi}{8})$? 4-6.

Problem 4-1

The statement "$f(x) = 2016$ for all x" means that, for EVERY real number input, the function f assigns the output 2016. Therefore $f(x+2016) = \boxed{2016}$.

Problem 4-2



There are 5 triangles pictured. Let their bases have lengths b_1, b_2, b_3, b_4, and b_5. Let h be the altitude of each triangle [as well as the width of the rectangle]. Hence, the sum of the areas of the 5 triangles is $\frac{1}{2}h(b_1+b_2+b_3+b_4+b_5) = \frac{1}{2}w\ell = \frac{1}{2}(18) = \boxed{9}$.

Problem 4-3

Since $A^{2x} = 4$, $A^x = 2$ and $A^{3x} = 8$. Therefore,

$$\frac{A^{3x}-A^{-3x}}{A^x-A^{-x}} = \frac{8-\frac{1}{8}}{2-\frac{1}{2}} = \frac{63/8}{3/2} = \frac{63}{12} = \boxed{\frac{21}{4}}.$$

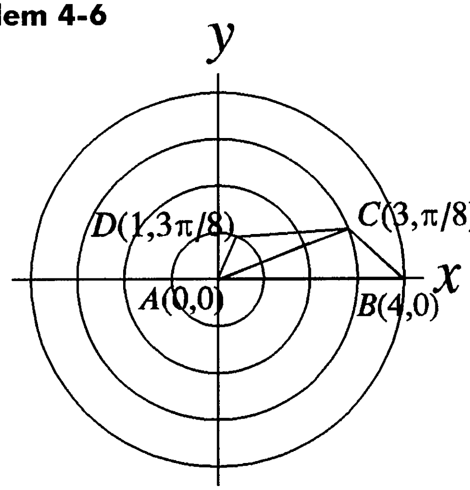
Problem 4-4

On his 21-minute trip, Al spent 7 minutes running and 14 minutes walking. In km/min, if his walking rate was w, then his running rate was $3w$. Since rate $\times$ time = distance, the distance to work, in km, was $14w + 21w = 35w$. Had he walked for m minutes and run for twice that time, the km distance would have been $wm + (3w)(2m) = 7wm$. Since $35w = 7wm$, $m = 5$. The total time, in minutes, would have been $m + 2m = \boxed{15}$.

Problem 4-5

If we split the 100 integers into 5 disjoint 20-element sets according to the remainder we get when we divide each integer by 5, we get $\{1,6, \ldots ,91,96\}$, $\{2,7, \ldots ,92,97\}$, $\{3,8, \ldots ,93,98\}$, $\{4,9, \ldots ,94,99\}$, and $\{5,10, \ldots , 95,100\}$, where the remainders respectively 1, 2, 3, 4, and 0. To choose as many integers as possible, choose every number from the first and second sets (or first and third sets, or second and fourth sets, or third and fourth sets) and a single number from the fifth set. If we chose any additional number, then, for some number pair, the sum of the remainders would be 0 or 5. We can choose at most $20 + 20 + 1 = \boxed{41}$ of the first 100 positive integers.

Problem 4-6



Area of quadrilateral $ABCD$

$= $ area of $\triangle BAC$ + area of $\triangle DAC$

$= \frac{1}{2}AB \times AC \sin \angle BAC + \frac{1}{2}AD \times AC \sin \angle DAC$

$= \frac{1}{2}(4)(3)(\sin\frac{\pi}{8}) + \frac{1}{2}(1)(3)(\sin\frac{2\pi}{8})$

$= 6\sin\frac{\pi}{8} + \frac{3}{2}\sin\frac{\pi}{4}$

$= 3.35676076597\ldots$

$\approx \boxed{3.357}$.

[Note: $6\sin\frac{\pi}{8} + \frac{3}{2}\sin\frac{\pi}{4}$ is an **acceptable** answer.]

