AP Calclulus Ch. 2 Test Concept Review: Implicit Differentiation

For each problem, use implicit differentiation to find $\frac{d^2y}{dx^2}$ in terms of x and y.

1)

$$x^2 + y^2 = 4$$

2)

$$x^2 - y^2 = 36$$

3) $1 - xy = x - y$

For each problem, use implicit differentiation to find $\frac{d^2y}{dx^2}$ in terms of x and y.

4) $y^2 = 10x$

5) $4y^2 + 2 = 3x^2$

6)

The slope of the tangent line to the graph of $x^3 - 2y^3 + xy = 0$ at the point $(1,1)$ is

(A) $-\frac{4}{5}$ (B) $\frac{3}{2}$ (C) $-\frac{5}{4}$ (D) $\frac{5}{4}$ (E) $\frac{4}{5}$ (F) $-\frac{2}{3}$

Key

AP Calclulus Ch. 2 Test Concept Review: Implicit Differentiation

For each problem, use implicit differentiation to find $\frac{d^2y}{dx^2}$ in terms of x and y.

1) *Find 2nd derivative

$x^2 + y^2 = 4$

$$2x + 2y\left(\frac{dy}{dx}\right) = 0$$

$$2y\left(\frac{dy}{dx}\right) = -2x$$

$$\frac{dy}{dx} = \frac{-2x}{2y}$$

$$\boxed{\frac{dy}{dx} = \frac{-x}{y}}$$

$$\frac{d^2y}{dx^2} = \frac{(-1)(y) - (-x)\left(\frac{dy}{dx}\right)}{y^2}$$

$$\frac{d^2y}{dx^2} = \frac{-y + x\left(\frac{dy}{dx}\right)}{y^2}$$

$$\frac{d^2y}{dx^2} = \frac{-y + x\left(\frac{-x}{y}\right)}{y^2}$$

$$\frac{d^2y}{dx^2} = \frac{-y - \frac{x^2}{y}}{y^2}(y)$$

$$\frac{d^2y}{dx^2} = \frac{-y^2 - x^2}{y^3} = \frac{-(y^2 + x^2)}{y^3} = \boxed{\frac{-4}{y^3}}$$

2)

$x^2 - y^2 = 36$

$$2x - 2y\left(\frac{dy}{dx}\right) = 0$$

$$-2y\left(\frac{dy}{dx}\right) = -2x$$

$$\frac{dy}{dx} = \frac{-2x}{-2y} = \boxed{\frac{x}{y}}$$

$$\frac{d^2y}{dx^2} = \frac{(1)(y) - x\left(\frac{dy}{dx}\right)}{y^2}$$

$$\frac{d^2y}{dx^2} = \frac{y - x\left(\frac{x}{y}\right)}{y^2}$$

$$\frac{d^2y}{dx^2} = \frac{y - \frac{x^2}{y}}{y^2}(y)$$

$$\frac{d^2y}{dx^2} = \frac{y^2 - x^2}{y^3}$$

$$\frac{d^2y}{dx^2} = \frac{-(-y^2 + x^2)}{y^3}$$

$$\boxed{\frac{d^2y}{dx^2} = \frac{-36}{y^3}}$$

3) $1 - xy = x - y$

$$0 - \left[(1)(y) + x\left(\frac{dy}{dx}\right)\right] = 1 - \frac{dy}{dx}$$

$$-y - x\left(\frac{dy}{dx}\right) = 1 - \frac{dy}{dx}$$

$$-x\left(\frac{dy}{dx}\right) + \frac{dy}{dx} = y + 1$$

$$\frac{dy}{dx}(-x + 1) = y + 1$$

$$\boxed{\frac{dy}{dx} = \frac{y+1}{1-x}}$$

$$\frac{d^2y}{dx^2} = \frac{\left(\frac{dy}{dx}\right)(1-x) - (y+1)(-1)}{(1-x)^2}$$

$$\frac{d^2y}{dx^2} = \frac{\left(\frac{y+1}{\cancel{1-x}}\right)(\cancel{1-x}) - (y+1)(-1)}{(1-x)^2} =$$

$$\frac{d^2y}{dx^2} = \frac{y+1+y+1}{(1-x)^2} = \boxed{\frac{2y+2}{(1-x)^2}}$$

For each problem, use implicit differentiation to find $\frac{d^2y}{dx^2}$ in terms of x and y.

4) $y^2 = 10x$

$2y\left(\frac{dy}{dx}\right) = 10$

$\frac{dy}{dx} = \frac{10}{2y} = \frac{5}{y}$

$\frac{dy}{dx} = \frac{5}{y}$

$\frac{dy}{dx} = 5y^{-1}$

$\frac{d^2y}{dx^2} = -5y^{-2}\left(\frac{dy}{dx}\right)$

$\frac{d^2y}{dx^2} = \frac{-5}{y^2}\left(\frac{dy}{dx}\right)$

$\frac{d^2y}{dx^2} = \frac{-5}{y^2}\left(\frac{5}{y}\right)$

$\boxed{\frac{d^2y}{dx^2} = \frac{-25}{y^3}}$

5) $4y^2 + 2 = 3x^2$

$8y\left(\frac{dy}{dx}\right) + 0 = 6x$

$\frac{dy}{dx} = \frac{6x}{8y}$ $\quad$ $\frac{dy}{dx} = \frac{3x}{4y}$

$\frac{d^2y}{dx^2} = \frac{(3)(4y)-(3x)(4\frac{dy}{dx})}{(4y)^2}$

$\frac{d^2y}{dx^2} = \frac{12y - 12x\left(\frac{3x}{4y}\right)}{16y^2}$

$\frac{d^2y}{dx^2} = \frac{12y - \frac{9x^2}{y}}{16y^2}$ (y)

$\boxed{\frac{d^2y}{dx^2} = \frac{12y^2 - 9x^2}{16y^3}}$

6)

The slope of the tangent line to the graph of $x^3 - 2y^3 + xy = 0$ at the point $(1,1)$ is

(A) $-\frac{4}{5}$ $\quad$ (B) $\frac{3}{2}$ $\quad$ (C) $-\frac{5}{4}$ $\quad$ (D) $\frac{5}{4}$ $\quad$ $\boxed{\text{(E) } \frac{4}{5}}$ $\quad$ (F) $-\frac{2}{3}$

*product rule

$x^3 - 2y^3 + (x)(y) = 0$

$3x^2 - 6y\left(\frac{dy}{dx}\right) + (1)(y) + (x)\left(\frac{dy}{dx}\right) = 0$

$3(1)^2 - 6(1)\left(\frac{dy}{dx}\right) + 1(1) + (1)\left(\frac{dy}{dx}\right) = 0$

$3 - 6\left(\frac{dy}{dx}\right) + 1 + 1\left(\frac{dy}{dx}\right) = 0$

$4 - 5\left(\frac{dy}{dx}\right) = 0$ $\quad$ $-5\left(\frac{dy}{dx}\right) = -4$

$\frac{dy}{dx} = \frac{-4}{-5}$

$\boxed{\frac{dy}{dx} = \frac{4}{5}}$