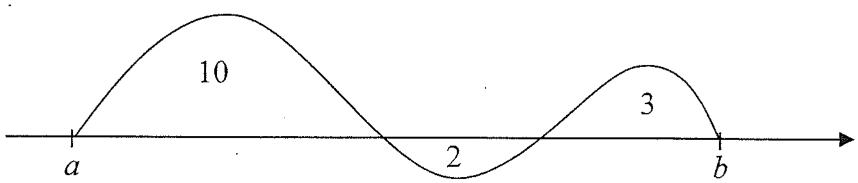
AP CALCULUS AB Integral Unit Additional Notes

Displacement is how far you are from where you started. displacement = $\int_a^b v(t)dt$ where $v\,(t)$ is the velocity function. (*displacement = integral of velocity*)

Distance is the total amount a particle has traveled. distance = $\int_a^b |v(t)|dt$.
(distance = integral of absolute value of velocity)

Ex) Suppose the graph below represents an object's velocity function. The numbers inside of each region represent the area of that region.



Based on this graph the object's displacement from *a* to *b* would be the integral of this function from *a* to *b* :

The object's distance traveled would be the integral of the absolute value of this function :

Position, Velocity, Acceleration

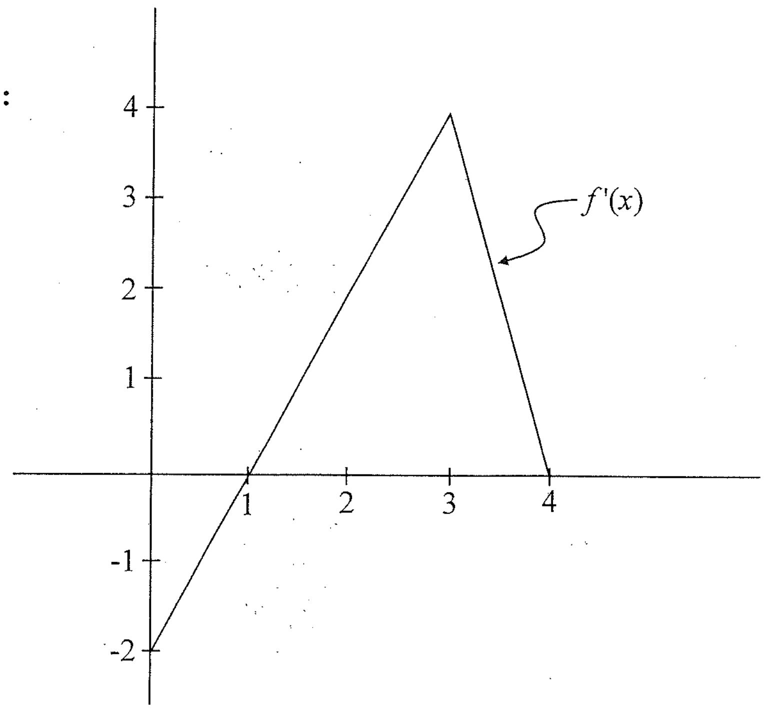
the derivative of position is velocity, the derivative of velocity is acceleration

So, the antiderivative of acceleration is velocity and the antiderivative of velocity is position

Acceleration s''(t)	+	+	-	-
Velocity s'(t)	+	-	+	-
Speed	increasing	decreasing	decreasing	increasing

Remember: Speed is increasing when s''(t) and s'(t) have the same signs
Speed is decreasing when s''(t) and s'(t) have opposite signs

Example 2:



The derivative of $f(x)$ is graphed above. Use this graph to answer the following:

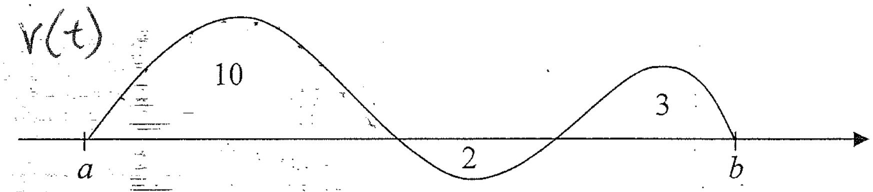
a) Find the x-coordinates of all relative extrema. Justify your answer.	b) Find the x-coordinates of all points of inflection. Justify your answer.
c) If $f(0) = 1$, find $f(1), f(2), f(3)$, and $f(4)$.	d) State the absolute extrema and the x-values where they occur.
e) Find total distance traveled from $t = 0$ to $t = 4$	f) Find total displacement from $t = 0$ to $t = 4$
g) On the interval $1 < t < 3$, is speed increasing or decreasing? State reason:	h) On the interval $0 < t < 1$, is speed increasing or decreasing? State reason

i) Using the data from parts a, b, and c, sketch a graph of $f(x)$ below

Diplacement is how far you are from where you started. displacement = $\int_a^b v(t)dt$ where $v(t)$ is the velocity function. (*displacement = integral of velocity*)

Distance is the total amount a particle has traveled. distance = $\int_a^b |v(t)|dt$.
(distance = integral of absolute value of velocity)

Ex) Suppose the graph below represents an object's velocity function. The numbers inside of each region represent the area of that region.



Based on this graph the object's displacement from a to b would be the integral of this function from a to b :

$$\int v(t)dt = 10 - 2 + 3 = 11$$

The object's distance traveled would be the integral of the absolute value of this function :

$$\int |v(t)|dt = 10 + 2 + 3 = 15$$

$s(t)$ = position function
$v(t)$ = velocity function
$a(t)$ = acceleration function

Position, Velocity, Acceleration

the derivative of position is velocity, the derivative of velocity is acceleration

$$\frac{d}{dx} s(t) = s'(t) = v(t) \qquad \frac{d}{dx} v(t) = v'(t) = a(t)$$

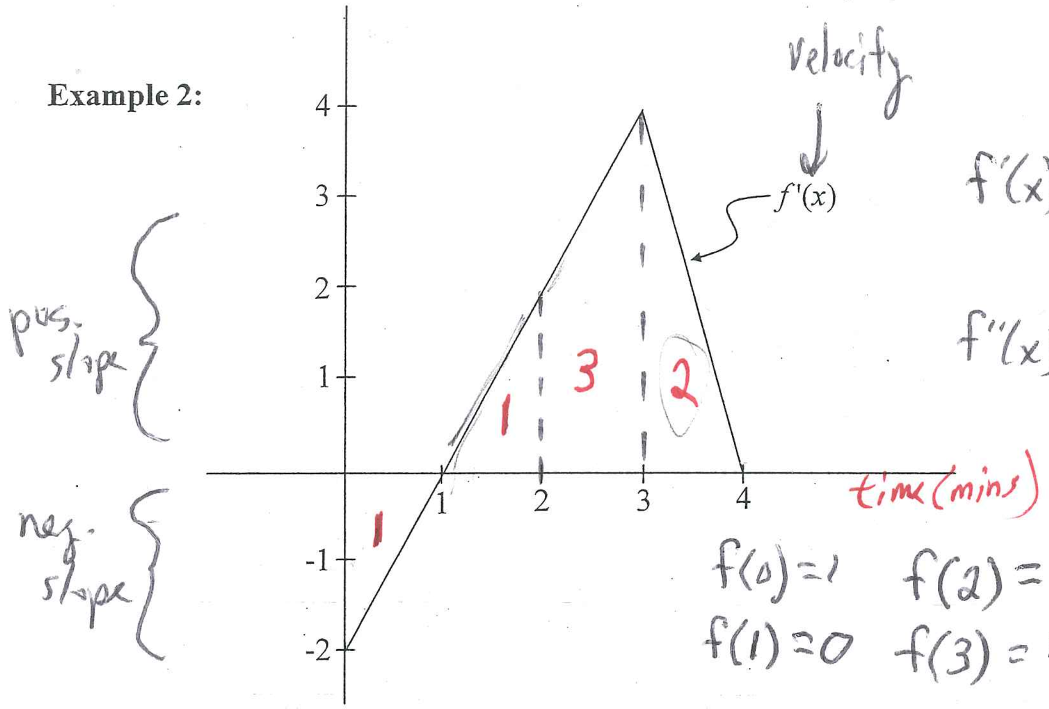
So, the antiderivative of acceleration is velocity and the antiderivative of velocity is position

$$\int a(t)dt = v(t) + c \qquad \int v(t)dt = s(t) + c$$

Acceleration s''(t)	+	+	-	-
Velocity s'(t)	+	-	+	-
Speed	increasing	decreasing	decreasing	increasing

Remember: Speed is increasing when s''(t) and s'(t) have the same signs
Speed is decreasing when s''(t) and s'(t) have opposite signs

Example 2:



$f'(x)$: $\searrow$ on $(0,1)$, $-$; $\nearrow$ on $(1,4)$, $+$

$f''(x)$: $\cup$ on $(0,3)$, $+$; $\cap$ on $(3,4)$, $-$

$f(0)=1$ $f(2)=1$ $f(4)=6$

$f(1)=0$ $f(3)=4$

The derivative of $f(x)$ is graphed above. Use this graph to answer the following:

a) Find the x-coordinates of all relative extrema. Justify your answer.

$x=1$ (min) b/c $f'(x)$ changes from $-$ to $+$

b) Find the x-coordinates of all points of inflection. Justify your answer.

$x=3$ b/c $f''(x)$ changes signs

c) If $f(0) = 1$, find $f(1)$, $f(2)$, $f(3)$, and $f(4)$.

$f(1) = f(0) + \int_0^1 f'(x)dx = 1-1=0$

$f(2) = f(1) + \int_1^2 f'(x)dx = 0+1=1$

$f(3) = f(2) + \int_2^3 f'(x)dx = 1+3=4$

$f(4) = f(3) + \int_3^4 f'(x)dx = 4+2=6$

d) State the absolute extrema and the x-values where they occur.

$f(1)=0$ $f(4)=6$

e) Find total distance traveled from $t = 0$ to $t = 4$

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f) Find total displacement from $t = 0$ to $t = 4$

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g) On the interval $1 < t < 3$, is speed increasing or decreasing? State reason:

increasing speed b/c $f'(x)>0$, $f''(x)>0$

h) On the interval $0 < t < 1$, is speed increasing or decreasing? State reason

decreasing speed b/c $f'(x)<0$, $f''(x)>0$

i) Using the data from parts a, b, and c, sketch a graph of $f(x)$ below