Key ①

Ch. 4.2a Notes

I. Sigma Notation

$$\sum_{i=2}^{5} a_i = a_2 + a_3 + a_4 + a_5$$

upper bound

index

lower bound

argument expression (formula rule)

Ex. **1** $\sum_{i=2}^{4} i^2 + 1 = (2^2+1) + (3^2+1) + (4^2+1) = 5+10+17 = \boxed{32}$

II. Summation Formulas:

1) $\sum_{i=1}^{n} 1 = n$

2) $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$

3) $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$

4) $\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$

5) $\sum_{i=1}^{n} c\boldsymbol{a_i} = c\sum_{i=1}^{n} \boldsymbol{a_i}$

Example 2

$$\sum_{i=1}^{8}(3i^2+2) =$$

$$\sum_{i=1}^{8} 3i^2 + \sum_{i=1}^{8} 2$$

$$3\sum_{i=1}^{8} i^2 + 2\sum_{i=1}^{8} 1$$

$$3 \cdot \frac{8(8+1)(16+1)}{6} + 2(8)$$

$$= 612 + 16 = \boxed{628}$$

Example 3

$$\sum_{i=1}^{10}(i+2)^2 =$$

$$\sum (i+2)(i+2)$$

$$\sum i^2 + 4i + 4$$

$$\sum_{i=1}^{10} i^2 + 4\sum_{i=1}^{10} i + 4\sum_{i=1}^{10} 1$$

$$\frac{10(10+1)(20+1)}{6} + 4 \cdot \frac{10(11)}{2} + 4 \cdot 10$$

$$= 385 + 220 + 40 = \boxed{645}$$

Example 4

$$\sum_{k=1}^{n} \frac{1}{n}(k^2 - 1) = \sum \frac{1}{n}k^2 - \frac{1}{n}(1)$$

$$\frac{1}{n}\sum_{k=1}^{n} k^2 - \frac{1}{n}\sum_{k=1}^{n} 1$$

$$\frac{1}{n} \cdot \frac{n(n+1)(2n+1)}{6} - \frac{1}{n} \cdot n$$

$$\frac{(n+1)(2n+1)}{6} - 1$$

$$\frac{2n^2+3n+1}{6} - \frac{6}{6} = \boxed{\frac{2n^2+3n-5}{6}}$$

2

III. Limits as n approaches infinity

*Think back about finding horizontal asymptotes (*compare degrees between numerator and denominator)

Example 5: *If* $S(n) = \frac{1}{n^2}\left[\frac{n(n+1)}{2}\right]$, *then find* $\lim_{n\to\infty} S(n)$

$$\lim_{n\to\infty} \frac{1}{n^2}\cdot\frac{n(n+1)}{2}$$

$$\lim_{n\to\infty} \frac{n(n+1)}{2n^2}$$

$$\lim_{n\to\infty} \frac{n^2+2n}{2n^2} = \boxed{\frac{1}{2}}$$

Example 6: *Find* $\lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{2i}{n}\right)\left(\frac{2}{n}\right)$

$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{4i}{n^2}$$

$$\lim_{n\to\infty} \frac{4}{n^2}\sum_{i=1}^{n} i$$

$$\lim_{n\to\infty} \frac{4}{n^2}\cdot\frac{n(n+1)}{2}$$

$$\lim_{n\to\infty} \frac{4n^2+4n}{2n^2} = \frac{4}{2} = \boxed{2}$$

Example 7: *Find* $\lim_{n\to\infty} \sum_{i=1}^{n} \left(1+\frac{2i}{n}\right)^2\left(\frac{2}{n}\right) \rightarrow \frac{2}{n}\left(1+\frac{2i}{n}\right)\left(1+\frac{2i}{n}\right) \rightarrow \frac{2}{n}\left(1+\frac{4i}{n}+\frac{4i^2}{n^2}\right)$

$$\lim_{n\to\infty} \sum_{i=1}^{n}\left(\frac{2}{n}+\frac{8i}{n^2}+\frac{8i^2}{n^3}\right)$$

$$\lim_{n\to\infty}\left(\sum_{i=1}^{n}\frac{2}{n}+\sum_{i=1}^{n}\frac{8i}{n^2}+\sum_{i=1}^{n}\frac{8i^2}{n^3}\right)$$

$$\lim_{n\to\infty}\left[\frac{2}{n}\sum_{i=1}^{n}1+\frac{8}{n^2}\sum_{i=1}^{n}i+\frac{8}{n^3}\sum_{i=1}^{n}i^2\right]$$

$$\lim_{n\to\infty}\left[\frac{2}{n}\cdot n+\frac{8}{n^2}\cdot\frac{n(n+1)}{2}+\frac{8}{n^3}\cdot\frac{n(n+1)(2n+1)}{6}\right]$$

$$\lim_{n\to\infty} \frac{2n}{n}+\frac{8n^2+8n}{2n^2}+\frac{16n^3+24n^2+8n}{6n^3}$$

$$= 2+\frac{8}{2}+\frac{16}{6}$$

$$= 2+4+\frac{8}{3} = \boxed{\frac{26}{3}}$$

Formula Sheet:

Summation Formulas:

1) $\sum_{i=1}^{n} 1 = n$

2) $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$

3) $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$

4) $\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$

5) $\sum_{i=1}^{n} ca_i = c\sum_{i=1}^{n} a_i$

Area using Limit Definition

$$\lim_{n\to\infty}\sum_{i=1}^{n}(width) * f(left\ endpoint + width * i)$$

$$\text{width} = \frac{b-a}{n}$$

Trapezoid Area:

$$Area = \frac{1}{2}w(h_1 + h_2)$$

Integral Formulas:

Power Rule:

$$\int u^n du = \frac{u^{n+1}}{n+1} + C$$

Trig Integrals:

$$\int \sin u\, du = -\cos u + C \qquad \int \cos u\, du = \sin u + C$$

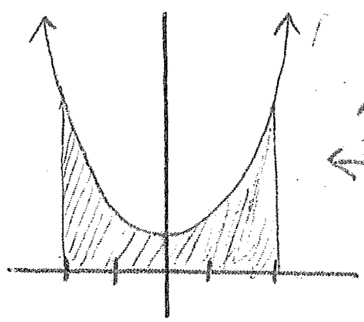
$$\int \sec^2 u\, du = \tan u + C \qquad \int \sec u \tan u\, du = \sec u + C$$

$$\int \csc^2 u\, du = -\cot u + C \qquad \int \csc u \cot u\, du = -\csc u + C$$

4.2b - Riemann Sums

Riemann Sums - Using rectangles to estimate area of region.
(Area under a curve)

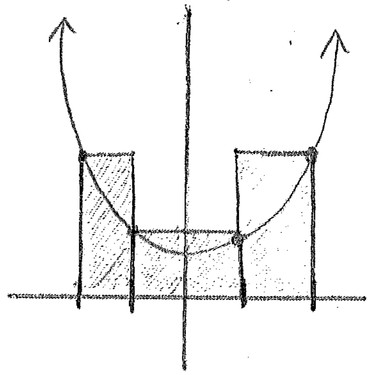
Consider the function
$f(x) = x^2 + 1 \quad [-2, 2]$



Area under the curve

Suppose we want to estimate the area of the shaded region using a given number of rectangles.

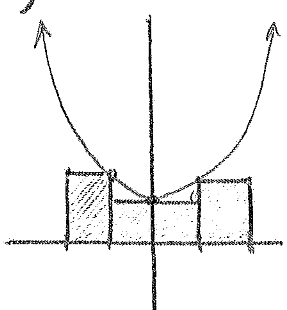
1) Upper rectangles or Circumscribed rectangles



* Using these rectangles will provide an overestimation of the area under the curve.

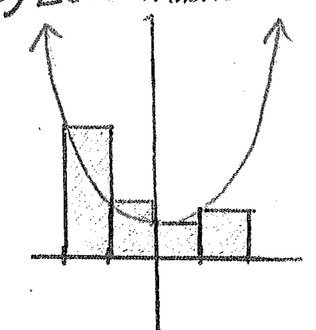
*Notice that one corner of each rectangle is on the graph. This ensures that the height of the rectangle is the same as the value of the function at the point where they connect.

2) Lower or Inscribed rectangles



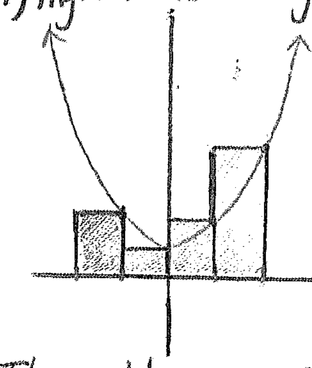
These rectangles provide an underestimation of the area under the curve

3) Left-handed Rectangles



The left corner of each rectangle attaches to the graph.

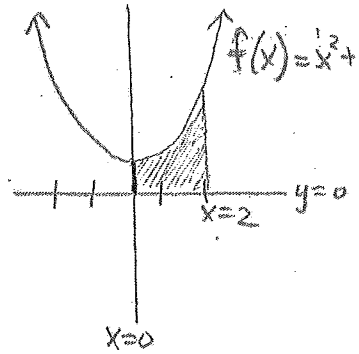
4) Right-handed Rectangles



The right corner of each rectangle attaches to the graph.

④ 4.2b (continued)

Ex. 1 Use 4 rectangles to estimate ⓐ upper and ⓑ lower sums for the area bounded by $x=0$, $y=0$, $x=2$, and $y=x^2+1$



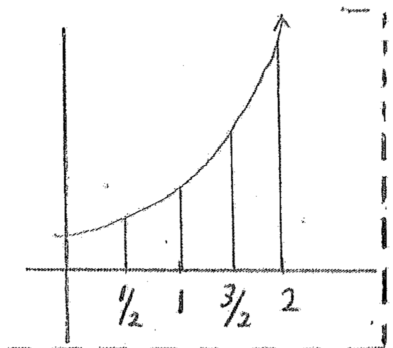
Step 1: Determine width of each rectangle.

$$\boxed{\text{width} = \frac{b-a}{n}}$$

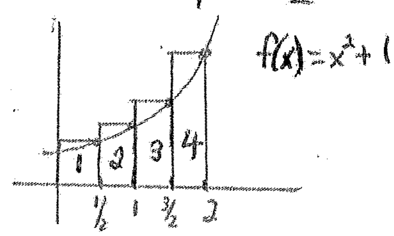
a = left endpoint
b = right endpoint
n = number of rectangles

Step 2: Draw the graph. Section off each interval.

Step 3: Find sum of areas of appropriate rectangles.



Since $\text{width} = \frac{b-a}{n}$, $b=2$, $a=0$, $n=4$ $\text{width} = \frac{2-0}{4} = \frac{1}{2}$



a) Find upper sum

Area = width × height

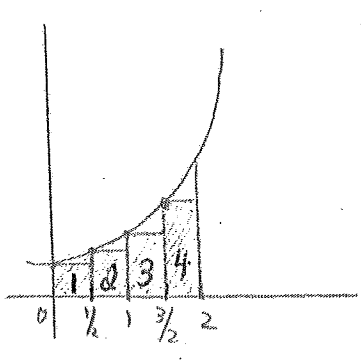
Rectangle #1: Area = $(1/2) \cdot f(1/2)$

$$S(4) = \underbrace{\left(\tfrac{1}{2}\right)\left(f(1/2)\right)}_{\text{Area of Rectangle \#1}} + \underbrace{\tfrac{1}{2}f(1)}_{\text{Area of Rectangle \#2}} + \underbrace{\tfrac{1}{2}f(3/2)}_{\text{Area of Rectangle \#3}} + \underbrace{\tfrac{1}{2}f(2)}_{\text{Area of Rectangle \#4}}$$

$f(0.5) = (0.5)^2+1 = 1.25$
$f(1) = 1^2+1 = 2$
$f(1.5) = 1.5^2+1 = 3.25$
$f(2) = 2^2+1 = 5$

$$S(4) = 0.625 + 1 + 1.625 + 2.5 = \boxed{5.75 \text{ or } 23/4}$$

b) Find lower sum



$$S(4) = \tfrac{1}{2}f(0) + \tfrac{1}{2}f(0.5) + \tfrac{1}{2}f(1) + \tfrac{1}{2}f(1.5)$$
$$= 0.5 + 0.625 + 1 + 1.625$$
$$= \boxed{3.75 \text{ or } 15/4}$$

$f(0) = 0^2+1 = 1$

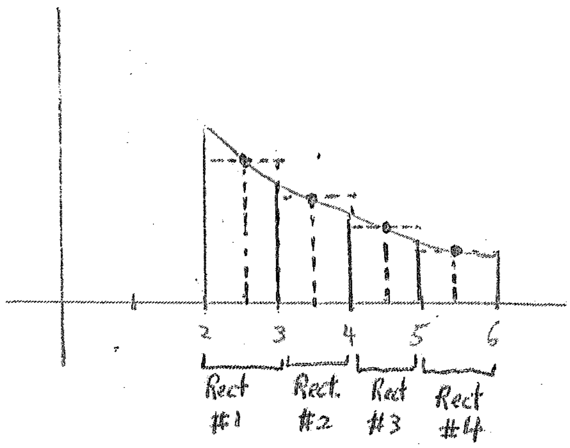
* Good approximation for the actual area is the average between upper and lower sum: $\frac{5.75+3.75}{2} = \boxed{4.75}$

4.2b (continued)

Midpoint Rule : Similar to upper/lower sum but use the midpoint of each rectangle to calculate rectangle's height.

$\boxed{\text{Ex. 2}}$ Estimate area under curve $f(x) = \frac{8}{x^2+1}$ from $[2,6]$
Use midpoint rule with 4 subintervals.

$$\text{width} = \frac{b-a}{n} = \frac{6-2}{4} = \frac{4}{4} = 1$$



$$\text{Midpoint Sum} = \underbrace{1 \cdot f(2.5)}_{\text{Area Rect \#1}} + \underbrace{1 f(3.5)}_{\text{Area Rect \#2}} + \underbrace{1 f(4.5)}_{\text{Area Rect \#3}} + \underbrace{f(5.5)}_{\text{Area Rect \#4}}$$

$f(2.5) = \frac{8}{2.5^2+1} = 1.103$ $\qquad f(4.5) = \frac{8}{4.5^2+1} = 0.376$

$f(3.5) = \frac{8}{3.5^2+1} = 0.604$ $\qquad f(5.5) = \frac{8}{5.5^2+1} = 0.256$

$$\text{Midpt Sum} = 1.103 + 0.604 + 0.376 + 0.256 = \boxed{2.34}$$

- Why would midpoint sum be a better approximation of area than upper or lower sum?

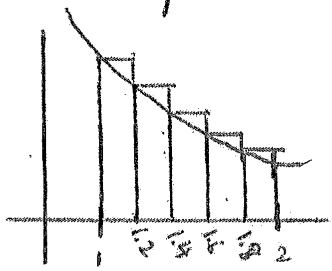
  This is because each rectangle has portions above and below the graph.

* Note: Midpoint sum is <u>not</u> the average between upper and lower sum!

(6)

4.2b Selected HW p.268-269 #23, 25, 27, 29, 63, 65

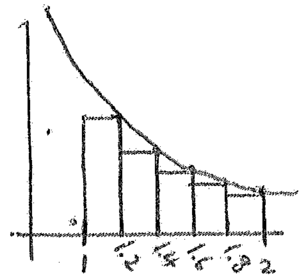
29) $y = \frac{1}{x}$



upper sum:
$f(1) = 1$
$f(1.2) = 1/1.2$
$f(1.4) = 1/1.4$
$f(1.6) = 1/1.6$
$f(1.8) = 1/1.8$

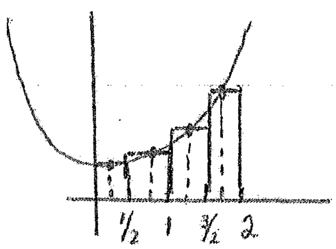
upper sum:
width $= \frac{b-a}{n} = \frac{2-1}{5} = \frac{1}{5} = 0.2$

$S = (0.2)f(1) + (0.2)f(1.2) + (0.2)f(1.4) + (0.2)f(1.6) + (0.2)f(1.8)$
$\approx \boxed{0.746}$



lower sum:
$s = (0.2)f(1.2) + (0.2)f(1.4) + (0.2)f(1.6) + (0.2)f(1.8) + (0.2)f(2)$
$\approx \boxed{0.646}$

63) Use Midpoint Rule: $n=4$
$f(x) = x^2 + 3 \quad [0,2]$



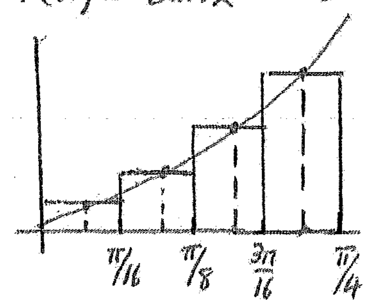
width $= \frac{b-a}{n} = \frac{2-0}{4} = \frac{1}{2}$

$f(1/4) = \frac{1}{16} + 3$
$f(3/4) = 9/16 + 3$
$f(5/4) = 25/16 + 3$
$f(7/4) = 49/16 + 3$

Area $\approx \frac{1}{2}f(1/4) + \frac{1}{2}f(3/4) + \frac{1}{2}f(5/4) + \frac{1}{2}f(7/4)$
$\approx \boxed{69/8}$

65) Midpt Rule: $n=4$
$f(x) = \tan x \quad 0 \le x \le \pi/4$

width $= \frac{\pi/4 - 0}{4} = \frac{\pi}{16}$



$f(\pi/32) = \tan(\pi/32)$
$f(3\pi/32) = \tan(3\pi/32)$
$f(5\pi/32) = \tan(5\pi/32)$
$f(7\pi/32) = \tan(7\pi/32)$

Area $\approx \frac{\pi}{16}f(\pi/32) + \frac{\pi}{16}f\left(\frac{3\pi}{32}\right) + \frac{\pi}{16}f\left(\frac{5\pi}{32}\right)$
$+ \frac{\pi}{16}f\left(\frac{7\pi}{32}\right)$

$\approx \boxed{0.345}$

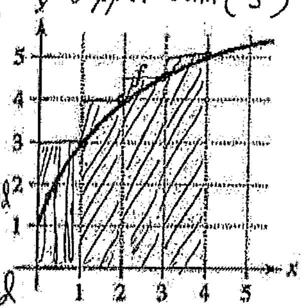
2017 Key 7

Ch. 4.2 Homework Problems

Using Upper and Lower Sums In Exercises 31 and 32, bound the area of the shaded region by approximating the upper and lower sums. Use rectangles of width 1.

31. Upper Sum (S)

Since $f(x)$ is always increasing upper sum is right-handed rectangles (circumscribed rectangles)



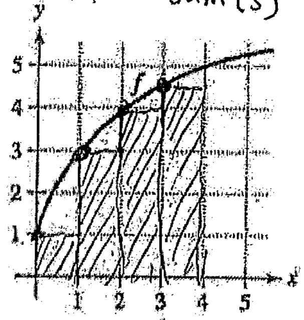
$S = 1f(1)+1f(2)+1f(3)+1f(4)$

$= 1(3)+1(4)+1(4.5)+1(5)$

$S = \boxed{\frac{33}{2} \text{ or } 16.5}$

31. lower sum (s)

since $f(x)$ is increasing, lower sum is left-handed rectangles (inscribed rectangles)



$s = 1f(0)+1f(1)+1f(2)+1f(3)$

$s = 1(1)+1(3)+1(4)+1(4.5)$

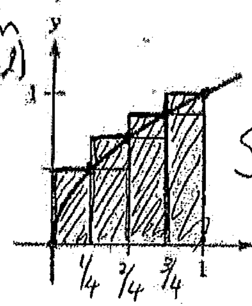
$s = \boxed{\frac{25}{2} = 12.5}$

Finding Upper and Lower Sums for a Region In Exercises 33–36, use upper and lower sums to approximate the area of the region using the given number of subintervals (of equal width).

width $= \frac{b-a}{n} = \frac{1-0}{4} = 1/4$

33. $y = \sqrt{x}$

upper sum (right-handed)



$S = \frac{1}{4}\cdot y(1/4) + \frac{1}{4}\cdot y(2/4) + \frac{1}{4}y(3/4) + 1/4\, y(1)$

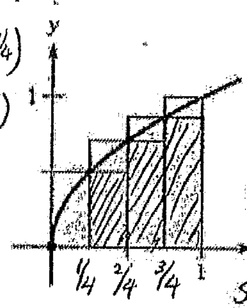
$S = \frac{1}{4}\left[\sqrt{1/4}+\sqrt{2/4}+\sqrt{3/4}+\sqrt{1}\right]$

$S = \frac{1}{4}\left[\frac{1}{2}+\frac{\sqrt{2}}{2}+\frac{\sqrt{3}}{2}+1\right]$

$\boxed{S \approx 0.768}$

33. $y = \sqrt{x}$

lower sum (left-handed)



$s = \frac{1}{4}\left[y(0)+y(1/4)+y(2/4)+y(3/4)\right]$

$s = \frac{1}{4}\left[0+\sqrt{1/4}+\sqrt{2/4}+\sqrt{3/4}\right]$

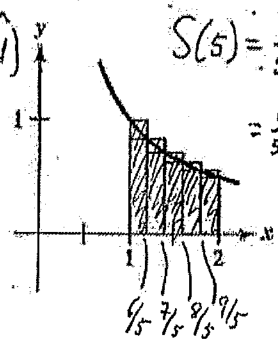
$s = \frac{1}{4}\left[\frac{1}{2}+\frac{\sqrt{2}}{2}+\frac{\sqrt{3}}{2}\right]$

$\boxed{s \approx 0.518}$

35. $y = \frac{1}{x}$

width $= \frac{b-a}{n} = \frac{2-1}{5} = \frac{1}{5}$

upper sum (left-handed)



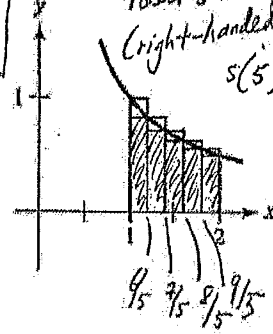
$S(5) = \frac{1}{5}\left[y(1)+y(6/5)+y(7/5)+y(8/5)+y(9/5)\right]$

$= \frac{1}{5}\left[\frac{1}{1}+\frac{1}{6/5}+1/7/5+1/8/5+1/9/5\right]$

$\boxed{S(5) \approx 0.746}$

35. $y = \frac{1}{x}$

lower sum (right-handed)



$s(5) = \frac{1}{5}\left[y(6/5)+y(7/5)+y(8/5)+y(9/5)+y(1)\right]$

≈ 0.646

$\boxed{s(5) \approx 0.646}$

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2017

Ch. 4.2 Continued

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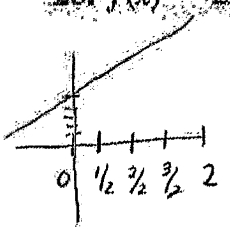
Approximating the Area of a Plane Region In Exercises 25–30, use left and right endpoints and the given number of rectangles to find two approximations of the area of the region between the graph of the function and the x-axis over the given interval.

width $= \frac{b-a}{n} = \frac{2-0}{4} = \frac{1}{2}$

25. $f(x) = 2x + 5$, $[0, 2]$, 4 rectangles (a, b; $n = 4$)

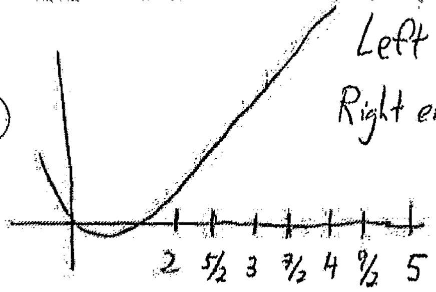


Left endpt: $A = \frac{1}{2}\left[f(0) + f(\tfrac{1}{2}) + f(1) + f(\tfrac{3}{2})\right] = \frac{1}{2}\left[5 + 6 + 7 + 8\right] = \frac{26}{2} = \boxed{13}$

Right endpt: $A = \frac{1}{2}\left[f(\tfrac{1}{2}) + f(1) + f(\tfrac{3}{2}) + f(2)\right] = \frac{1}{2}(6 + 7 + 8 + 9) = \frac{30}{2} = \boxed{15}$

27. $g(x) = 2x^2 - x - 1$, $[2, 5]$, 6 rectangles (a, b; $n = 6$)

width $= \frac{b-a}{n} = \frac{5-2}{6} = \frac{3}{6} = \frac{1}{2}$



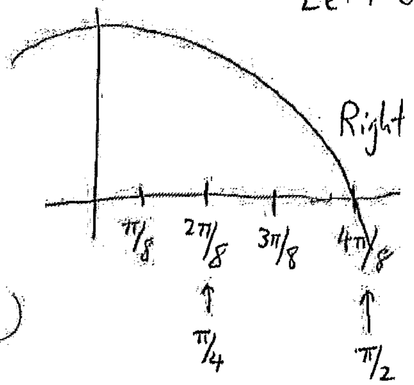
Left endpt: $A = \frac{1}{2}\left[f(2) + f(\tfrac{5}{2}) + f(3) + f(\tfrac{7}{2}) + f(4) + f(\tfrac{9}{2})\right] = \boxed{55}$

Right endpt: $A = \frac{1}{2}\left[f(\tfrac{5}{2}) + f(3) + f(\tfrac{7}{2}) + f(4) + f(\tfrac{9}{2}) + f(5)\right] = \boxed{74.5}$

29. $f(x) = \cos x$, $\left[0, \frac{\pi}{2}\right]$, 4 rectangles ($a, b$; $n = 4$)

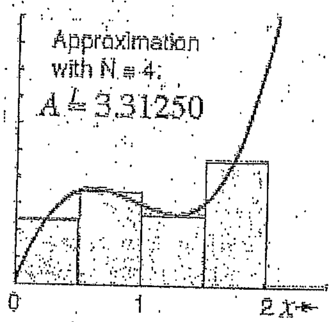
width $= \frac{b-a}{n} = \frac{\pi/2 - 0}{4} = \frac{\pi}{8}$

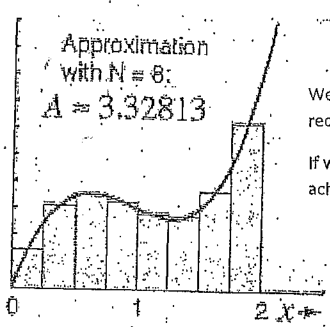
Left endpt: $\frac{\pi}{8}\left[f(0) + f(\tfrac{\pi}{8}) + f(\tfrac{\pi}{4}) + f(\tfrac{3\pi}{8})\right] \approx \boxed{1.1835}$



Right endpt: $\frac{\pi}{8}\left[f(\tfrac{\pi}{8}) + f(\tfrac{\pi}{4}) + f(\tfrac{3\pi}{8}) + f(\tfrac{\pi}{2})\right] \approx \boxed{0.7908}$

4.2c Finding Exact Area using limits





We can continually improve the Area Approximation under the curve by increasing the number of rectangles : above (n = 4) and n = 8, ...n = 16...

If we let n go out to infinity, (using limits) we will have something better than an approximation, we will achieve the actual area under the curve.

$$\text{Exact Area} = \lim_{n\to\infty} \sum_{i=1}^{n} \left[\left(\frac{b-a}{n}\right) \cdot f\left(a + \left(\frac{b-a}{n}\right)i\right)\right]$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} (\text{width}) \cdot f(\underline{\text{left endpoint}} + \underline{\text{width} \cdot i})$$

Memorize → "width f left plus width times i"

$\boxed{\text{Ex. 1}}$ Find exact area between $f(x) = 4 - x^2$ and x-axis from $[-2, 2]$

$$\text{width} = \frac{b-a}{n} = \frac{2--2}{n} = \frac{4}{n}$$

$$A = \lim_{n\to\infty} \sum_{i=1}^{n} \frac{4}{n} \cdot f\left(-2 + \frac{4}{n}i\right)$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \frac{4}{n} \cdot \left[4 - \left(-2 + \frac{4}{n}i\right)^2\right]$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \frac{4}{n} \cdot \left[4 - \left(4 - \frac{8i}{n} - \frac{8i}{n} + \frac{16}{n^2}i^2\right)\right]$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \frac{4}{n} \cdot \left[\cancel{4} - \cancel{4} + \frac{16i}{n} - \frac{16}{n^2}i^2\right]$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \frac{4}{n} \cdot \left[\frac{16i}{n} - \frac{16}{n^2}i^2\right]$$

$$= \lim_{n\to\infty} \left[\frac{64}{n^2} \sum_{i=1}^{n} i - \frac{64}{n^3} \sum_{i=1}^{n} i^2\right]$$

$$= \lim_{n\to\infty} \left[\frac{64}{n^2}\left(\frac{n^2}{2} + \frac{n}{2}\right) - \frac{64}{n^3}\left(\frac{n^3}{3} + \frac{n^2}{2} + \frac{n}{6}\right)\right]$$

$$= \lim_{n\to\infty} \left[\frac{64n^2}{2n^2} + \cancel{\frac{64n}{2n^2}} - \frac{64n^3}{3n^3} - \cancel{\frac{64n^2}{2n^3}} - \cancel{\frac{64n}{6n^3}}\right]$$

$$= \frac{64}{2} - \frac{64}{3}$$

$$= 32 - \frac{64}{3}$$

$$= \boxed{\frac{32}{3}}$$

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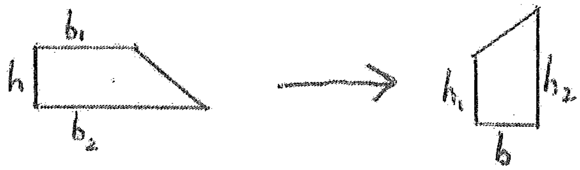
4.6 Trapezoids

→ * Better approximation than inscribed, circumscribed, or midpoint rectangles.

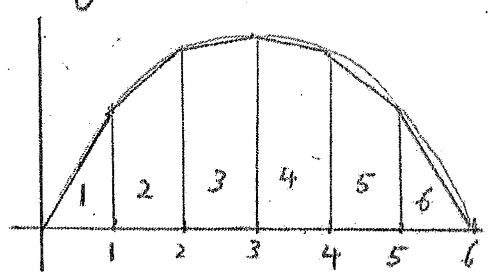
Trapezoidal Rule: Approximate Area of region using areas of trapezoids.

Review: Area of Trapezoid $= \frac{1}{2}h(b_1+b_2)$ or $\frac{h}{2}(b_1+b_2)$



$\boxed{\text{Area} = \frac{b}{2}(h_1+h_2)}$ or $\frac{1}{2}w(h_1+h_2)$

Ex. 1 Estimate area bounded by $f(x) = 6x - x^2$ and the x-axis using 6 trapezoids.



Set $6x - x^2 = 0$ to find bounds for graph

$x(6-x) = 0$ $\boxed{x=0, x=6}$

width $= \frac{b-a}{n} = \frac{6-0}{6} = \frac{6}{6} = \boxed{1}$

$f(0) = 6(0) - 0^2 = 0$

$f(1) = 6(1) - 1^2 = 5$

$f(2) = 6(2) - 2^2 = 8$

$f(3) = 6(3) - 3^2 = 9$

$f(4) = 6(4) - 4^2 = 8$

$f(5) = 6(5) - 5^2 = 5$

$f(6) = 6(6) - 6^2 = 0$

$\text{Area}_1 = \frac{1}{2}(f(0)+f(1)) = \frac{1}{2}(0+5) = 5/2$

$\text{Area}_2 = \frac{1}{2}(f(1)+f(2)) = \frac{1}{2}(13) = 13/2$

$\text{Area}_3 = \frac{1}{2}(f(2)+f(3)) = \frac{1}{2}(17) = 17/2$

$\text{Area}_4 = \frac{1}{2}(f(3)+f(4)) = \frac{1}{2}(17) = 17/2$

$\text{Area}_5 = \frac{1}{2}(f(4)+f(5)) = \frac{1}{2}(13) = 13/2$

$\text{Area}_6 = \frac{1}{2}(f(5)+f(6)) = \frac{1}{2}(5) = 5/2$

$\text{Area} = \frac{70}{2} = \boxed{35}$

* Integral Notation

$$\int_0^6 (6x - x^2)\,dx$$

6 ← upper bound (b)

$(6x-x^2)dx$ ← function

0 ← lower bound (a)

or

$$A = \frac{1}{2}\left[f(0) + 2f(1) + 2f(2) + 2f(3) + 2f(4) + 2f(5) + f(6)\right] = \frac{1}{2}(70) = \boxed{35}$$

4.2c/4.6 Selected HW p.269 #47-53 odd

p.314 #1, 5, 9, 13, 17

Use limit process to find area of the region

49) $y = x^2 + 2 \quad [0,1]$ width $= \frac{b-a}{n} = \frac{1-0}{n}$ $\lim_{n\to\infty} \sum_{i=1}^{n}$ width$\cdot f$(left + width$\cdot i$)

$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{1}{n} f\left(0 + \frac{1}{n} i\right)$$

$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{1}{n} f\left(\frac{1}{n} i\right) = \sum_{i=1}^{n} \frac{1}{n}\left[\left(\frac{i}{n}\right)^2 + 2\right] = \sum_{i=1}^{n} \frac{i^2}{n^3} + \frac{2}{n}$$

$$\lim_{n\to\infty} \left[\frac{1}{n^3}\sum_{i=1}^{n} i^2 + \frac{2}{n}\sum_{i=1}^{n} 1\right] = \lim_{n\to\infty}\left[\frac{1}{n^3}\left(\frac{n^3}{3} + \frac{n^2}{2} + \frac{n}{6}\right) + \frac{2}{n}(n)\right]$$

$$\lim_{n\to\infty} \frac{n^3}{3n^3} + \cancel{\frac{n^2}{2n^3}} + \cancel{\frac{n}{6n^3}} + \frac{2n}{n} = \frac{1}{3} + 2 = \boxed{\frac{7}{3}}$$

51) $y = 16 - x^2 \quad [1,3]$ width $= \frac{3-1}{n} = \frac{2}{n}$

$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{2}{n} f\left(1 + \frac{2}{n} i\right) = \sum_{i=1}^{n} \frac{2}{n}\left[16 - \left(1 + \frac{2}{n} i\right)^2\right] = \sum_{i=1}^{n} \frac{2}{n}\left[16 - \left(1 + \frac{2}{n} i\right)\left(1 + \frac{2}{n} i\right)\right]$$

$$= \sum_{i=1}^{n} \frac{2}{n}\left[16 - \left(1 + \frac{4i}{n} + \frac{4}{n^2} i^2\right)\right] = \sum_{i=1}^{n} \frac{2}{n}\left[15 - \frac{4i}{n} - \frac{4i^2}{n^2}\right]$$

$$\lim_{n\to\infty}\left[\frac{30}{n}\sum_{i=1}^{n} 1 - \frac{8}{n^2}\sum_{i=1}^{n} i - \frac{8}{n^3}\sum_{i=1}^{n} i^2\right]$$

$$\lim_{n\to\infty}\left[\frac{30}{n}(n) - \frac{8}{n^2}\left(\frac{n^2}{2} + \frac{n}{2}\right) - \frac{8}{n^3}\left(\frac{n^3}{3} + \frac{n^2}{2} + \frac{n}{6}\right)\right]$$

$$\lim_{n\to\infty}\left[\frac{30n}{n} - \frac{8n^2}{2n^2} - \cancel{\frac{8n}{2n^2}} - \frac{8n^3}{3n^3} - \cancel{\frac{8n^2}{2n^3}} - \cancel{\frac{8n}{6n^3}}\right]$$

$$30 - 4 - \frac{8}{3} = \boxed{\frac{70}{3}}$$

Pascal's Triangle

$$\begin{array}{ccccccc} & & & 1 & & & \\ & & 1 & & 1 & & \\ & 1 & & 2 & & 1 & \\ 1 & & 3 & & 3 & & 1 \end{array}$$

$(a+b)^3 \longrightarrow 1 \quad 3 \quad 3 \quad 1$

$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

53) $y = 64 - x^3 \quad [1, 4] \quad \text{width} = \frac{4-1}{n} = \frac{3}{n}$

$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{3}{n} f\left(1 + \frac{3}{n}i\right)$$

$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{3}{n}\left[64 - \left(1+\frac{3}{n}i\right)^3\right] = \sum_{i=1}^{n} \frac{3}{n}\left[64 - \left(1 + 3\left(\frac{3i}{n}\right) + 3\left(\frac{3}{n}i\right)^2 + \left(\frac{3}{n}i\right)^3\right)\right]$$

$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{3}{n}\left[63 - \frac{9i}{n} - \frac{27}{n^2}i^2 - \frac{27}{n^3}i^3\right]$$

$$\lim_{n\to\infty}\left[\frac{189}{n}\sum_{i=1}^{n} 1 - \frac{27}{n^2}\sum_{i=1}^{n} i - \frac{81}{n^3}\sum_{i=1}^{n} i^2 - \frac{81}{n^4}\sum_{i=1}^{n} i^3\right]$$

$$\lim_{n\to\infty}\left[\frac{189}{n}(n) - \frac{27}{n^2}\left(\frac{n^2}{2}+\frac{n}{2}\right) - \frac{81}{n^3}\left(\frac{n^3}{3}+\frac{n^2}{2}+\frac{n}{6}\right) - \frac{81}{n^4}\left(\frac{n^4}{4}+\frac{n^3}{2}+\frac{n^2}{4}\right)\right]$$

exponent degree should never be greater in the numerator.

$$\lim_{n\to\infty}\left[\frac{189n}{n} - \frac{27n^2}{2n^2} - \cancel{\frac{27n}{2n^2}} - \frac{81n^3}{3n^3} - \cancel{\frac{81n^2}{2n^3}} - \cancel{\frac{81n}{6n^3}} - \frac{81n^4}{4n^4} - \cancel{\frac{81n^3}{2n^4}} - \cancel{\frac{81n^2}{4n^4}}\right]$$

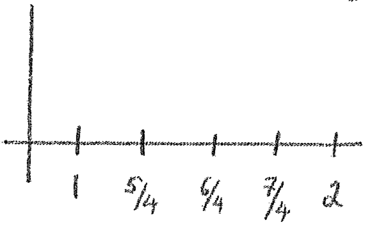
$$= 189 - \frac{27}{2} - \frac{81}{3} - \frac{81}{4} = \boxed{\frac{513}{4} = 128.25}$$

4.6 Trapezoid Rule: p.314 #1, 5, 9, 13, 17

Approximate the definite integral using Trapezoidal Rule with $n=4$

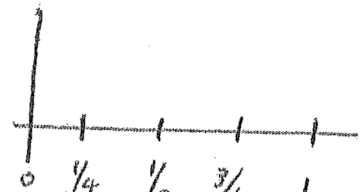
9) $\displaystyle\int_1^2 \frac{1}{(x+1)^2}\,dx$ width $= \frac{2-1}{4} = \frac{1}{4}$ $\frac{w}{2}\left[h_1 + 2h_2 + 2h_3 + \dots + h_n\right]$

$A \approx \frac{1}{4}\left(\frac{1}{2}\right)\left[f(1) + 2f(5/4) + 2f(6/4) + 2f(7/4) + f(2)\right]$



$= \frac{1}{8}\left[\frac{1}{4} + \frac{32}{81} + \frac{8}{25} + \frac{32}{121} + \frac{1}{9}\right] \approx \boxed{0.1676}$

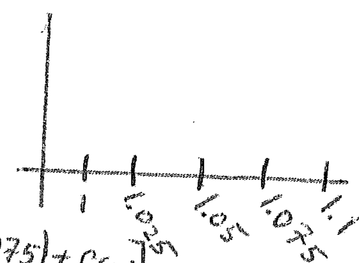
13) $\displaystyle\int_0^1 \sqrt{x}\,\sqrt{1-x}\,dx$ width $= \frac{1-0}{4} = \frac{1}{4}$



$A = \frac{1}{4}\left(\frac{1}{2}\right)\left[f(0) + 2f(1/4) + 2f(1/2) + 2f(3/4) + f(1)\right]$

$\approx \boxed{0.342}$

17) $\displaystyle\int_1^{1.1} \sin x^2\,dx$ width $= \frac{1.1-1}{4} = \frac{0.1}{4} = \frac{1}{40}$



$A = \frac{1}{40}\left(\frac{1}{2}\right)\left[f(1) + 2f(1.025) + 2f(1.05) + 2f(1.075) + f(1.1)\right]$

$A = \boxed{0.089}$

Limit Definition of Area Practice Problems WS

$$\text{Area} = \lim_{n\to\infty} \sum_{i=1}^{n} \left[\frac{b-a}{n} \cdot f\left[a + \frac{b-a}{n} i\right]\right]$$

1) $y = 2x^2 - 3x + 2 \qquad [1, 3]$

11b

2) $y = 1 - 2x - x^2$ $[-1, 4]$

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Use Limit Definition of Area:

$$Area = \lim_{n\to\infty} \sum_{i=1}^{n} \left[\frac{b-a}{n} \cdot f\left[a + \frac{b-a}{n} i\right]\right]$$

1) $y = 2x^2 - 3x + 2 \quad [1, 3]$

$$width = \frac{3-1}{n} = \frac{2}{n}$$

$$\lim_{n\to\infty} \sum \frac{2}{n} \cdot f\left[1 + \frac{2}{n} i\right]$$

$$\lim_{n\to\infty} \sum \frac{2}{n}\left[2\left(1+\frac{2}{n}i\right)^2 - 3\left(1+\frac{2}{n}i\right)+2\right]$$

$$\frac{2}{n}\left[2\left(1+\frac{2}{n}i\right)\left(1+\frac{2}{n}i\right) - 3\left(1+\frac{2}{n}i\right)+2\right]$$

$$\frac{2}{n}\left[2\left(1+\frac{2}{n}i+\frac{2}{n}i+\frac{4}{n^2}i^2\right) - 3\left(1+\frac{2}{n}i\right)+2\right]$$

$$\frac{2}{n}\left[2\left(1+\frac{4}{n}i+\frac{4}{n^2}i^2\right) - 3\left(1+\frac{2}{n}i\right)+2\right]$$

$$\frac{2}{n}\left[2+\frac{8}{n}i+\frac{8}{n^2}i^2 - 3 - \frac{6}{n}i+2\right]$$

$$\sum \frac{2}{n}\left[\frac{8}{n^2}i^2 + \frac{2}{n}i + 1\right]$$

$$\sum \frac{16}{n^3}i^2 + \frac{4}{n^2}i + \frac{2}{n}$$

$$\sum \frac{16}{n^3}i^2 + \sum \frac{4}{n^2}i + \sum \frac{2}{n}$$

$$\frac{16}{n^3}\boxed{\sum i^2} + \frac{4}{n^2}\boxed{\sum i} + \frac{2}{n}\boxed{\sum 1}$$

$$\lim_{n\to\infty} \frac{16}{n^3}\left[\frac{n(n+1)(2n+1)}{6}\right] + \frac{4}{n^2}\left[\frac{n(n+1)}{2}\right] + \frac{2}{n}[n]$$

$$\lim_{n\to\infty} \frac{32n^3+\ldots}{6n^3} + \frac{4n^2+\ldots}{2n^2} + \frac{2n}{n}$$

$$\frac{32}{6} + 2 + 2 = \boxed{\frac{28}{3}}$$

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2) $y = 1 - 2x - x^2 \qquad [-1, 4]$

$$\text{width} = \frac{4 - -1}{n} = \frac{5}{n}$$

$$\lim_{n\to\infty} \sum \frac{5}{n} \cdot f\left[-1 + \frac{5}{n}i\right]$$

$$\sum \frac{5}{n} \cdot \left[1 - 2\left(-1 + \frac{5}{n}i\right) - \left(-1 + \frac{5}{n}i\right)^2\right]$$

$$\sum \frac{5}{n} \left[1 + 2 - \frac{10}{n}i - \left(-1 + \frac{5}{n}i\right)\left(-1 + \frac{5}{n}i\right)\right]$$

$$\sum \frac{5}{n} \left[3 - \frac{10}{n}i - \left(1 - \frac{5}{n}i - \frac{5}{n}i + \frac{25}{n^2}i^2\right)\right]$$

$$\sum \frac{5}{n} \left[3 - \frac{10}{n}i - \left(1 - \frac{10}{n}i + \frac{25}{n^2}i^2\right)\right]$$

$$\sum \frac{5}{n} \left[3 - \frac{10}{n}i - 1 + \frac{10}{n}i - \frac{25}{n^2}i^2\right]$$

$$\sum \frac{5}{n} \left[2 - \frac{25}{n^2}i^2\right]$$

$$\sum \frac{10}{n} - \frac{125}{n^3}i^2 = \sum \frac{10}{n} - \sum \frac{125}{n^3}i^2$$

$$\frac{10}{n}\boxed{\sum 1} - \frac{125}{n^3}\boxed{\sum i^2}$$

$$\lim_{n\to\infty} \frac{10}{n}[n] - \frac{125}{n^3}\left[\frac{n(n+1)(2n+1)}{6}\right]$$

$$\lim_{n\to\infty} \frac{10n}{n} - \frac{250n^3 + \ldots}{6n^3}$$

$$10 - \frac{250}{6} = \boxed{-\frac{95}{3}}$$

Riemann Sum Worksheet **Name** ____________________ (13)

For each problem sketch the graph showing the appropriate region, then approximate the area bound by the curve and the x-axis on the given interval using 6 different Riemann sums Left, Right, Upper, Lower, midpoint, Trapezoidal, using the specified number of subintervals.

1. Function : $f(x) = -(x-3)^2+20$ on Interval [0, 5] using 5 subintervals

Graph:

Left Sum	Right Sum
Lower Sum	Upper Sum
Midpoint Sum	Trapezoidal Sum

136

2. Function : $f(x)$ =2sinx+3 on Interval $[0, 2\pi]$using 6 subintervals

Graph:

Left Sum	Right Sum
Lower Sum	Upper Sum
Midpoint Sum	Trapezoidal Sum

14

3. Function : $f(x) = \sqrt[3]{2x-1} + 5$ on Interval [-2, 2] using 4 subintervals

Graph:

Left Sum	Right Sum
Lower Sum	Upper Sum
Midpoint Sum	Trapezoidal Sum

145

4.

x	−4	−2	0	3	6	13	20
f(x)	8	12	18	4	9	31	12

Left Sum- 6 Subintervals	Right Sum- 6 Subintervals
Lower Sum- 6 Subintervals	Upper Sum- 6 Subintervals
Midpoint Sum- 3 Subintervals	Trapezoidal Sum- 6 Subintervals

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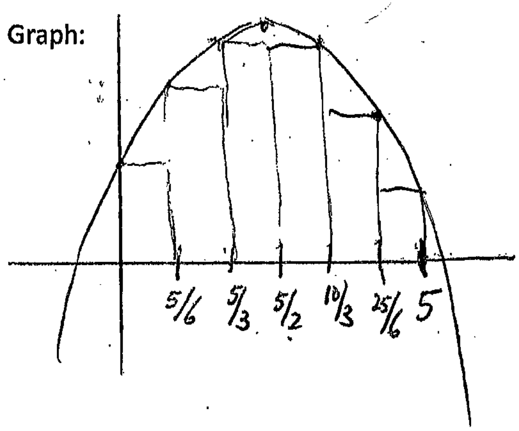
Riemann Sum Worksheet **Name** Key

For each problem sketch the graph showing the appropriate region, then approximate the area bound by the curve and the x-axis on the given interval using 6 different Riemann sums Left, Right, Upper, Lower, midpoint, Trapezoidal, using the specified number of subintervals.

1. Function : $f(x) = -(x-3)^2+20$ on Interval [0, 5] using 5 subintervals

Graph:



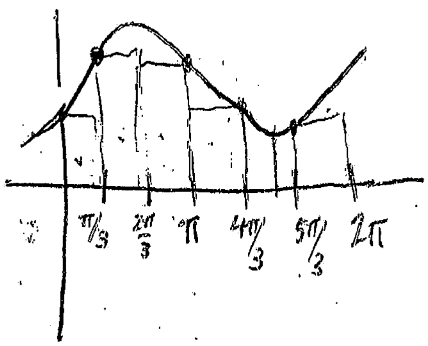
width $= \frac{b-a}{n} = \frac{5-0}{6} = 5/6$

Left Sum	Right Sum
$\frac{5}{6}\left[f(0)+f(5/6)+f(5/3)+f(5/2)+f(10/3)+f(25/6)\right]$	$\frac{5}{6}\left[f(5/6)+f(5/3)+f(5/2)+f(10/3)+f(25/6)+f(5)\right]$
Lower Sum	**Upper Sum**
$\frac{5}{6}\left[f(0)+f(5/6)+f(5/3)+f(10/3)+f(25/6)+f(5)\right]$	$\frac{5}{6}\left[f(5/6)+f(5/3)+f(5/2)+f(5/2)+f(10/3)+f(25/6)\right]$
Midpoint Sum	**Trapezoidal Sum** $\frac{w}{2}\left[h_1+2h_2+2h_3+\ldots h_n\right]$
$\frac{5}{6}\left[f(5/12)+f(5/4)+f(25/12)+f(35/12)+f(15/4)+f(55/12)\right]$	$\frac{5}{6}\cdot\frac{1}{2}\left[f(0)+2f(5/6)+2f(5/3)+2f(5/2)+2f(10/3)+2f(25/6)+f(5)\right]$

15b

2. Function : $f(x)$ =2sinx+3 on Interval $[0, 2\pi]$using 6 subintervals $\frac{b-a}{n} = \frac{2\pi - 0}{6} = \frac{\pi}{3}$

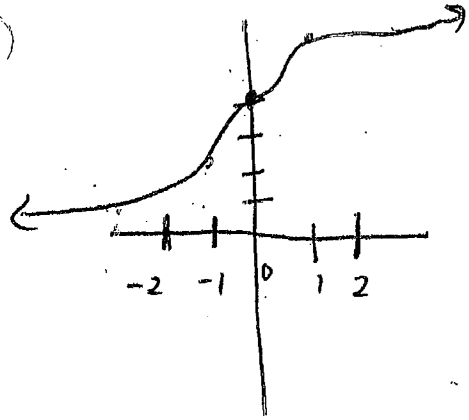
Graph:



Left Sum	Right Sum
$\frac{\pi}{3}\left[f(0)+f(\pi/3)+f(2\pi/3)+f(\pi)+f(\frac{4\pi}{3})+f(5\pi/3)\right]$	$\frac{\pi}{3}\left[f(\pi/3)+f(\frac{2\pi}{3})+f(\pi)+f(4\pi/3)+f(5\pi/3)+f(2\pi)\right]$
Lower Sum	**Upper Sum**
$\frac{\pi}{3}\left[f(0)+f(\pi/3)+f(\pi)+f(\frac{4\pi}{3})+f(5\pi/3)+f(2\pi)\right]$	$\frac{\pi}{3}\left[f(\pi/3)+f(\frac{2\pi}{3})+f(2\pi/3)+f(\pi)+f(\frac{4\pi}{3})+f(2\pi)\right]$
Midpoint Sum	**Trapezoidal Sum**
$\pi/3\left[f(\pi/6)+f(\pi/2)+f(7\pi/6)+f(3\pi/2)+f(11\pi/6)\right]$	$\pi/3\left[f(0)+2f(\pi/3)+2f(2\pi/3)+2f(\pi)+2f(\frac{4\pi}{3})+2f(5\pi/3)+f(2\pi)\right]$

3. Function : $f(x) = \sqrt[3]{2x-1} + 5$ on Interval [-2, 2] using 4 subintervals $W = \frac{b-a}{n} = \frac{2--2}{4} = 1$ (16)

Graph:



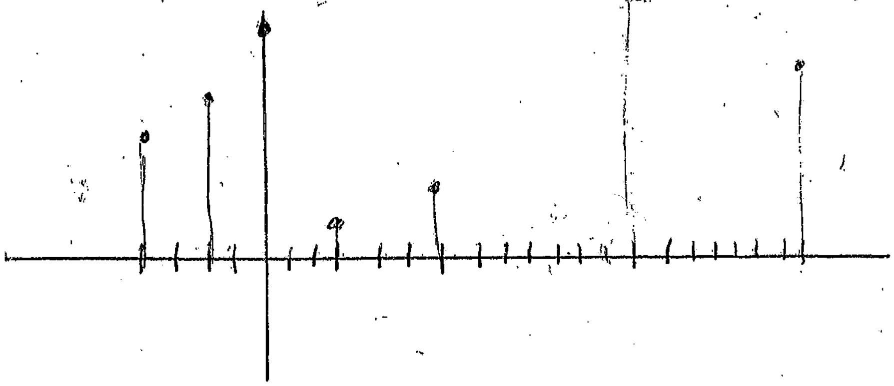
Left Sum $1\left[f(-2)+f(-1)+f(0)+f(1)\right]$	Right Sum $1\left[f(-1)+f(0)+f(1)+f(2)\right]$
Lower Sum $1\left[f(-2)+f(-1)+f(0)+f(1)\right]$	Upper Sum $1\left[f(-1)+f(0)+f(1)+f(2)\right]$
Midpoint Sum $1\left[f(-1.5)+f(-\frac{1}{2})+f(\frac{1}{2})+f(1.5)\right]$	Trapezoidal Sum $\frac{1}{2}\cdot 1\left[f(-2)+2f(-1)+2f(0)+2f(1)+f(2)\right]$

11b

4.

1 2 3 4 5 6

x	−4	−2	0	3	6	13	20
f(x)	8	12	18	4	9	31	12



Left Sum- 6 Subintervals

$2f(-4)+2(f(-2))+3f(0)+3f(3)+7f(6)+7f(13)$

$2(8)+2(12)+3(18)+3(4)+7(9)+7(31)$

Right Sum- 6 Subintervals

$2f(-2)+2f(0)+3f(3)+3f(6)+7f(13)+7f(20)$

Lower Sum- 6 Subintervals

$2f(-4)+2f(-2)+3f(3)+3f(3)+7f(6)+7f(20)$

Upper Sum- 6 Subintervals

$2f(-2)+2f(0)+3f(0)+3f(6)+7f(13)+7f(13)$

Midpoint Sum- 3 Subintervals

$4f(-2)+6f(3)+14f(13)$

$4(12)+6(4)+14(13)$

Trapezoidal Sum- 6 Subintervals

$\frac{2}{2}\left[f(-4)+f(-2)\right]+\frac{2}{2}\left[f(-2)+f(0)\right]$

$+\frac{3}{2}\left[f(0)+f(3)\right]+\frac{3}{2}\left[f(3)+f(6)\right]$

$+\frac{7}{2}\left[f(6)+f(13)\right]+\frac{7}{2}\left[f(13)+f(20)\right]$

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Non-AP Calculus **4-2, 4-6 Riemann Sums WS: Using Tables of Values**

1) Selected values of a function, f, are given in the table below:

x	0	5	8	9	12	18	20
f(x)	4	2	3	7	3	6	10

a) Give the middle approximation with 3 subintervals for f on the interval [0, 20]

b) Use right-handed rectangles to approximate the area with 3 subintervals for f on the interval [0, 20]

c) Use left-handed rectangles to approximate the area with 3 subintervals for f on the interval [0, 9]

d) Use trapezoids to approximate the area with 2 subintervals for f on the interval [0, 20]

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2) Selected values of a function, *f*, are given in the table below:

x	1	3	7	10	12	13	16	17	20
f(x)	3	6	1	9	15	2	4	5	6

a) Give the middle approximation with 2 subintervals for *f* on the interval [1, 20]

b) Use right-handed rectangles to approximate the area with 3 subintervals for *f* on the interval [3, 17]

c) Use left-handed rectangles to approximate the area with 4 subintervals for *f* on the interval [1, 12]

d) Use trapezoids to approximate the area with 3 subintervals for *f* on the interval [3, 17]

Non-AP Calculus **4-2, 4-6 Riemann Sums WS: Using Tables of Values**

Key

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1) Selected values of a function, f, are given in the table below:

x	0	5	8	9	12	18	20
f(x)	4	2	3	7	3	6	10

a) Give the middle approximation with 3 subintervals for f on the interval [0, 20]

$$\text{Area} \approx 8(2) + 4(7) + 8(6)$$

$$16 + 28 + 48 = \boxed{92}$$

b) Use right-handed rectangles to approximate the area with 3 subintervals for f on the interval [0, 20]

$$\text{Area} \approx 8(3) + 4(3) + 8(10)$$

$$24 + 12 + 80 = \boxed{116}$$

c) Use left-handed rectangles to approximate the area with 3 subintervals for f on the interval [0, 9]

$$\text{Area} \approx 5(4) + 3(2) + 1(3)$$

$$20 + 6 + 3 = \boxed{29}$$

d) Use trapezoids to approximate the area with 2 subintervals for f on the interval [0, 20]

$$\text{Area} \approx \frac{9}{2}[4+7] + \frac{11}{2}[7+10]$$

$$\frac{w}{2}[h_1+h_2] \qquad \frac{99}{2} + \frac{187}{2} = \boxed{143}$$

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2) Selected values of a function, *f*, are given in the table below:

x	1	3	7	10	12	13	16	17	20
f(x)	3	6	1	9	15	2	4	5	6

a) Give the middle approximation with 2 subintervals for *f* on the interval [1, 20]

$$\text{Area} \approx 11(1) + 8(4)$$

$$11 + 32 = \boxed{43}$$

b) Use right-handed rectangles to approximate the area with 3 subintervals for *f* on the interval [3, 17]

$$\text{Area} \approx 7(9) + 3(2) + 4(5)$$

$$63 + 6 + 20 = \boxed{89}$$

c) Use left-handed rectangles to approximate the area with 4 subintervals for *f* on the interval [1, 12]

$$\text{Area} \approx 2(3) + 4(6) + 3(1) + 2(9)$$

$$6 + 24 + 3 + 18 = \boxed{51}$$

d) Use trapezoids to approximate the area with 3 subintervals for *f* on the interval [3, 17]

$$\text{Area} \approx \frac{7}{2}[6+9] + \frac{3}{2}[9+2] + \frac{4}{2}[2+5]$$

$$52.5 + 16.5 + 14 = \boxed{83}$$

$$\frac{w}{2}[h_1+h_2]$$

Review #3
Key WS#3

Use sigma notation to write sum

1) $7\left[\frac{3}{6}+4\right]+7\left[\frac{6}{6}+8\right]+7\left[\frac{9}{6}+12\right]+ \dots 7\left[\frac{18}{6}+24\right]$

$$\boxed{\sum_{i=1}^{6} 7\left[\frac{3i}{6}+4i\right]}$$

2) Use Limit Definition to find area: $h(x)=3x-x^2 \; [-1,2]$

$$A=\lim_{n\to\infty}\sum_{i=1}^{n}\left[\frac{b-a}{n}\cdot f\left[a+\frac{b-a}{n}i\right]\right]$$

width $=\frac{b-a}{n}=\frac{2-(-1)}{n}=\frac{3}{n}$

$$A=\lim_{n\to\infty}\sum_{i=1}^{n}\left[\frac{3}{n}\cdot f\left[-1+\frac{3}{n}i\right]\right]$$

$$A=\lim_{n\to\infty}\sum_{i=1}^{n}\left[\frac{3}{n}\cdot\left[3\left(-1+\frac{3}{n}i\right)-\left(-1+\frac{3}{n}i\right)^2\right]\right]$$

$$=\lim_{n\to\infty}\sum_{i=1}^{n}\frac{3}{n}\cdot\left[-3+\frac{9}{n}i-\left(1-\frac{6}{n}i+\frac{9}{n^2}i^2\right)\right]$$

$$=\lim_{n\to\infty}\sum_{i=1}^{n}\frac{3}{n}\cdot\left[-4+\frac{15}{n}i-\frac{9}{n^2}i^2\right]$$

$$=\lim_{n\to\infty}\sum_{i=1}^{n}-\frac{12}{n}+\frac{45}{n^2}i-\frac{27}{n^3}i^2$$

$$=\lim_{n\to\infty}\sum_{i=1}^{n}\frac{-12}{n}+\sum_{i=1}^{n}\frac{45}{n^2}i-\sum_{i=1}^{n}\frac{27}{n^3}i^2$$

$$=\lim_{n\to\infty}-\frac{12}{n}\sum_{i=1}^{n}1+\frac{45}{n^2}\sum_{i=1}^{n}i-\frac{27}{n^3}\sum_{i=1}^{n}i^2$$

$$=\lim_{n\to\infty}-\frac{12}{n}(n)+\frac{45}{n^2}\left[\frac{n(n+1)}{2}\right]-\frac{27}{n^3}\left[\frac{n(n+1)(2n+1)}{6}\right]$$

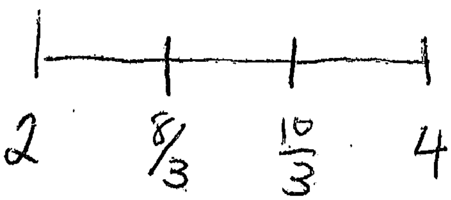
$$=\lim_{n\to\infty}\frac{-12n}{n}+\frac{45n^2+45n}{2n^2}-\frac{54n^3+\dots}{6n^3}$$

$$=-12+\frac{45}{2}-\frac{54}{6}$$

$$=\boxed{\frac{3}{2}\text{ or }1.5}$$

3) Use 3 right-handed rectangles to approximate area of $f(x) = 1 + 3x^2$, x-axis, $x = 2$, $x = 4$

$$\text{width} = \frac{b-a}{n} = \frac{4-2}{3} = \frac{2}{3}$$



$$A = \frac{2}{3} \cdot f\left(\frac{8}{3}\right) + \frac{2}{3} \cdot f\left(\frac{10}{3}\right) + \frac{2}{3} f(4)$$

$$A = \frac{2}{3}\left[22.33 + 34.33 + 49\right] = \boxed{70.44}$$

4) Use 2 trapezoids to approximate area [3, 20]

x	2	3	6	9	10	11	13	17	19	20	33
f(x)	8	4	1	5	6	9	3	11	4	17	19

$$A = \frac{w}{2}\left[h_1 + h_2\right]$$

$$A = \frac{8}{2}\left[f(3) + f(11)\right] + \frac{9}{2}\left[f(11) + f(20)\right]$$

$$= 4(4+9) + \frac{9}{2}(9+17)$$

$$= 4(13) + 4.5(26)$$

$$= 52 + 117 = \boxed{169}$$

2C

AP Calculus AB 4-2, 4-6 Quiz Review **Name** Solution Key

Calculators permitted.

1. Find the sum: $\sum_{i=2}^{4}\left[(i+1)^2-(2-i)^3\right]$

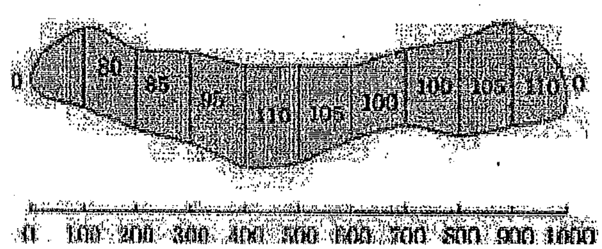
$(2+1)^2-(2-2)^3+(3+1)^2-(2-3)^3+(4+1)^2-(2-4)^3$

$9 - 0 + 16 - (-1) + 25 - (-8) = \boxed{59}$

2. Use Sigma notation to write the sum: $\frac{2}{\sqrt[3]{5-2}}+\frac{4}{\sqrt[3]{5-4}}+\frac{6}{\sqrt[3]{5-6}}+\frac{8}{\sqrt[3]{5-8}}$

$\sum_{i=1}^{4}\frac{2i}{\sqrt[3]{5-2i}}$

3. The width, in feet, at various points along the fairway of a hole on a golf course is given to the right. If one pound of fertilizer covers 200 square feet, estimate the amount of fertilizer needed to fertilize the fairway using trapezoids.



$A = \frac{w}{2}\left[h_1+2h_2+2h_3+2h_3+\dots+h_n\right]$

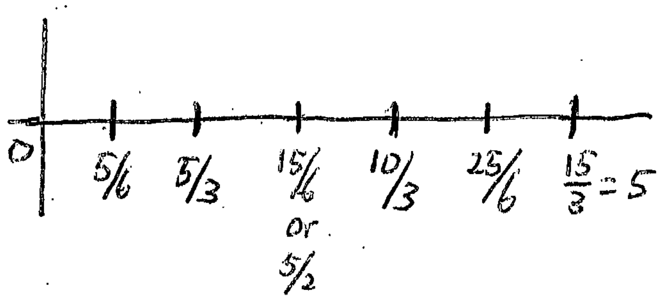
$A \approx \frac{100}{2}\left[0+2(80)+2(85)+2(95)+2(110)+2(105)+2(100)+2(100)+2(105)+2(110)+0\right]$

$A \approx 50(1780) = 89000\ ft^2$

fertilizer needed $= 89000\ ft^2 \cdot \frac{1\text{ lb fertilizer}}{200\ ft^2} = \boxed{\text{445 pounds of fertilizers}}$

4. Use 3 midpoint rectangles to approximate the area of the region bounded by $f(x) = x^2 + 3$, the x-axis, $x = 0$, and $x = 5$.

$W = \frac{b-a}{n} = \frac{5-0}{3} = \frac{5}{3}$



$A \approx \frac{5}{3}f(5/6) + \frac{5}{3}f(5/2) + \frac{5}{3}f(25/6)$

$\approx \frac{5}{3}(33.3) = \boxed{55.5}$

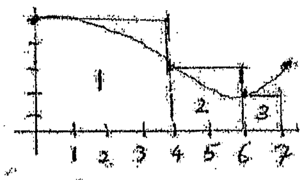
(23)

5. Use the table of values on the right to estimate the below:

x	0	4	6	7	10
$f(x)$	5	3	2	3	5

a. Use 3 left-handed rectangles with intervals indicated by the table to estimate the area between the curve and x-axis on [0, 7]

x	0	4	6	7
f(x)	5	3	2	3



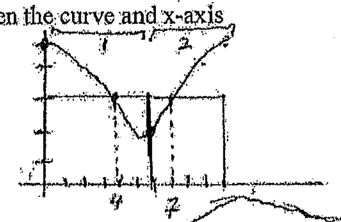
$A = 4\cdot f(0) + 2\cdot f(4) + 1\cdot f(6)$

$= 4(5) + 2(3) + 1(2)$

$= \boxed{28}$

b. Use 2 middle rectangles with intervals indicated by the table to estimate the area between the curve and x-axis on [0, 10]

x	0	4	6	7	10
f(x)	5	3	2	3	5



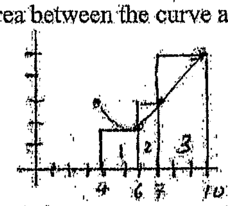
$A = 6\cdot f(4) + 4\cdot f(7)$

$= 6(3) + 4(3)$

$= \boxed{30}$

c. Use 3 right-handed rectangles with intervals indicated by the table to estimate area between the curve and x-axis on [4, 10]

X	4	6	7	10
f(x)	3	2	3	5



$A = 2f(6) + 1f(7) + 3f(10)$

$= 2(2) + 1(3) + 3(5)$

$= 4 + 3 + 15 = \boxed{22}$

d. Use 3 trapezoids with interval indicated by the table to estimate area between the curve and x-axis on [0, 7]

X	0	4	6	7
f(x)	5	3	2	3

$A = \frac{w}{2}[h_1 + h_2]$

$\frac{1}{2}[f(6) + f(7)]$

$A = \frac{4}{2}[f(0) + f(4)] + \frac{2}{2}[f(4) + f(6)] + \frac{1}{2}[f(6) + \ldots]$

$= 2(5+3) + 1(3+2) + \frac{1}{2}(2+3)$

$= 2(8) + 1(5) + \frac{1}{2}(5)$

$= 16 + 5 + 5/2 = \boxed{23.5 \text{ or } 47/2}$

6. Given the region bounded by $g(x) = 6 - x^2$, the x-axis, $x = -1$, and $x = 2$. Use the limit definition to find the exact area of the region.

$A = \lim_{n\to\infty} \sum_{i=1}^{n} \text{width} \cdot f(\text{left} + \text{width}\cdot i)$ $\quad$ width $= \frac{b-a}{n} = \frac{2-(-1)}{n} = \frac{3}{n}$

$= \lim_{n\to\infty} \sum_{i=1}^{n} \frac{3}{n} \cdot f\left(-1 + \frac{3}{n}i\right)$

$= \lim_{n\to\infty} \sum_{i=1}^{n} \frac{3}{n} \cdot \left[6 - \left(-1 + \frac{3i}{n}\right)^2\right]$

$= \lim_{n\to\infty} \sum_{i=1}^{n} \frac{3}{n}\left[6 - \left(-1 + \frac{3i}{n}\right)\left(-1 + \frac{3i}{n}\right)\right]$

$= \lim_{n\to\infty} \sum_{i=1}^{n} \frac{3}{n}\left[6 - \left(1 - \frac{6i}{n} + \frac{9i^2}{n^2}\right)\right]$

$= \lim_{n\to\infty} \sum_{i=1}^{n} \frac{3}{n}\left[5 + \frac{6i}{n} - \frac{9i^2}{n^2}\right]$

$= \lim_{n\to\infty} \sum_{i=1}^{n} \left[\frac{15}{n} + \frac{18}{n^2}i - \frac{27i^2}{n^3}\right]$

$= \lim_{n\to\infty} \left[\frac{15}{n}\sum_{i=1}^{n} 1 + \frac{18}{n^2}\sum_{i=1}^{n} i - \frac{27}{n^3}\sum_{i=1}^{n} i^2\right]$

$= \lim_{n\to\infty} \left[\frac{15}{n}(n) + \frac{18}{n^2}\left(\frac{n^2}{2} + \frac{n}{2}\right) - \frac{27}{n^3}\left(\frac{n^3}{3} + \frac{n^2}{2} + \frac{n}{6}\right)\right]$

$= \lim_{n\to\infty} \left[\frac{15n}{n} + \frac{18n^2}{2n^2} + \cancel{\frac{18n}{2n^2}} - \frac{27n^3}{3n^3} - \cancel{\frac{27n^2}{2n^3}} - \cancel{\frac{27n}{6n^3}}\right]$

$= 15 + \frac{18}{2} - \frac{27}{3} = 15 + 9 - 9 = \boxed{15}$

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4.2/4.6 Quiz Review WS #3

Use sigma notation to write sum 4.2, 4.6 Quiz Review #3

1) $7\left[\frac{3}{6}+4\right]+7\left[\frac{6}{6}+8\right]+7\left[\frac{9}{6}+12\right]+\ldots 7\left[\frac{18}{6}+24\right]$

2) Use Limit Definition to find area: $h(x)=3x-x^2\ [-1,2]$.

25b

3) Use 3 right-handed rectangles to approximate area of $f(x) = 1 + 3x^2$, x-axis, $x = 2$, $x = 4$

4) Use 2 trapezoids to approximate area $[3, 20]$

x	2	3	6	9	10	11	13	17	19	20	33
f(x)	8	4	1	5	6	9	3	11	4	17	19

Review 4.2 4.6 Formulas and Definitions:

2C

Summation Formulas:

1) $\sum_{i=1}^{n} 1 =$

2) $\sum_{i=1}^{n} i =$

3) $\sum_{i=1}^{n} i^2 =$

4) $\sum_{i=1}^{n} i^3 =$

5) Area of Trapezoid: ______________________

6) Width formula: ________________________

7) Limit Definition of Area under Curve

Review 4.2 4.6 Formulas and Definitions:

Summation Formulas:

1) $\sum_{i=1}^{n} 1 =$ $\sum_{i=1}^{n} 1 = n$

2) $\sum_{i=1}^{n} i =$ $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$

3) $\sum_{i=1}^{n} i^2 =$ $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$

4) $\sum_{i=1}^{n} i^3 =$ $\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$

5) Area of Trapezoid: ________________________________ $Area = \frac{w}{2}(h_1 + h_2)$

6) Width formula: ________________________________ $width = \frac{b-a}{n}$

7) Limit Definition of Area under Curve $\lim_{n\to\infty} \sum_{i=1}^{n} (width) * f(a + width * i)$