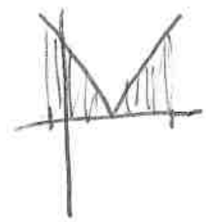
Ch.4 (Non-calculator) Integrals Unit test topics:

1) Power Rule (Integrals)
2) U-substitution (and change of variable)
3) Trig Integrals
4) Definite Integrals (Unit circle values) (Trig)
5) Definite Integrals (Absolute Value Function)



6) Even/Odd Functions
7) 2nd Fundamental Theorem of Calculus
8) Avg. Value Theorem
9) Given $f''(x)$, find $f(x)$ (Antiderivative)
   (Given $a(t)$, find $x(t)$)

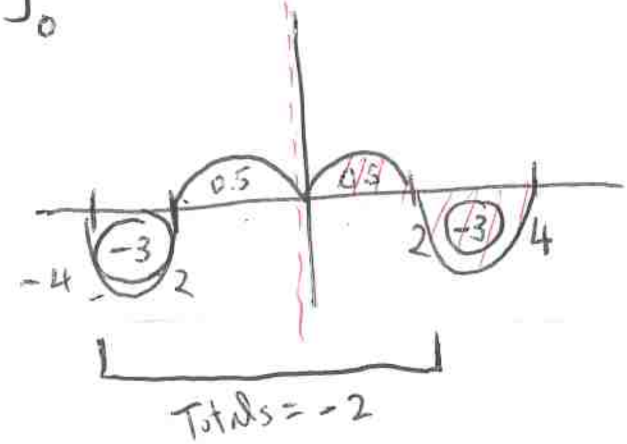
Help Session #2:

Even/odd functions:

Ex. 1 $\int_4^2 f(x)\,dx = 3 \qquad \int_{-4}^{2} f(x)\,dx = -2$

a) Given $f(x)$ is even: Find $\int_0^4 f(x)\,dx = \boxed{-2.5}$

$$\boxed{\int_2^4 f(x)\,dx = -3}$$

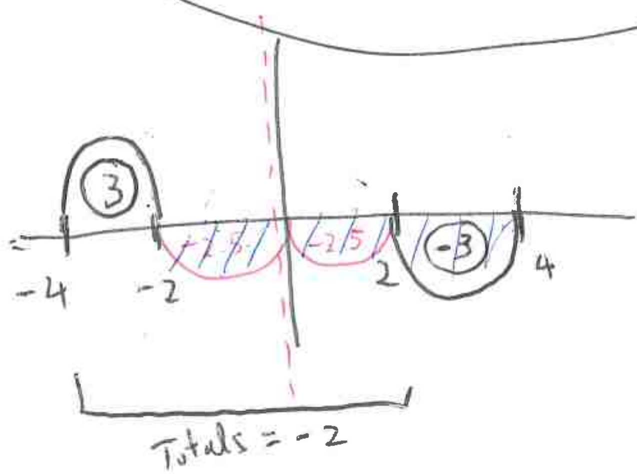


b) Given $f(x)$ is odd, find $\int_4^{-2} f(x)\,dx$

$$\downarrow$$

$$-\int_{-2}^{4} f(x)\,dx$$

$$= -(-8) = \boxed{8}$$



$\boxed{\text{Ex. 2}}$ 2nd Fundamental Theorem of Calculus (SFTC)

$$\frac{d}{dx}\int_{q(x)}^{p(x)} f(x)\,dx = f(p(x))\cdot p'(x) - f(q(x))\cdot q'(x)$$

$\boxed{\text{Ex. 2}}$

Find $\displaystyle \frac{d}{dx}\int_{e}^{\sqrt{x}} \frac{4t^2}{5-3t}\,dt = \frac{4(\sqrt{x})^2}{5-3\sqrt{x}}\cdot\frac{1}{2}x^{-1/2}$

$\sqrt{x} \to x^{1/2}$

$$= \frac{4x}{5-3\sqrt{x}}\cdot\frac{1}{2\sqrt{x}} \to \boxed{\frac{4x}{2\sqrt{x}(5-3\sqrt{x})}}$$

$$\downarrow$$

$$\boxed{\frac{4x}{10\sqrt{x}-6x}}$$

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b) $\displaystyle \frac{d}{dx}\int_{3x}^{2x^2} \frac{5\sqrt{t}}{1+2t^3}\,dt$

$$\frac{5\sqrt{2x^2}}{1+2(2x^2)^3}\cdot 4x - \frac{5\sqrt{3x}}{1+2(3x)^3}\cdot 3$$

$$\boxed{\frac{20x\sqrt{2x^2}}{1+16x^6} - \frac{15\sqrt{3x}}{1+54x^3}}$$

Avg. Value Theorem

* height = $\frac{\text{Area}}{\text{width}}$

$$f(c) = \frac{1}{b-a}\int_a^b f(x)\,dx$$

Ex. 3 Given $f(x) = \sec^2 x$ $\quad [\overset{a}{-\pi/4}, \overset{b}{\pi/4}]$

$$f(c) = \frac{1}{\frac{\pi}{4} - (-\pi/4)} \int_{-\pi/4}^{\pi/4} \sec^2 x\,dx \rightarrow \frac{1}{\pi/2}\int_{-\pi/4}^{\pi/4} \sec^2 x\,dx$$

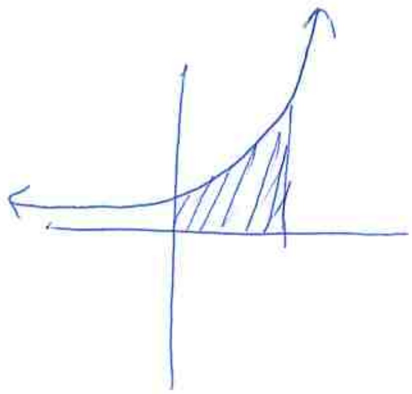
$$\tan x \Big]_{-\pi/4}^{\pi/4} = \tan(\pi/4) - \tan(-\pi/4)$$

$$= 1 - (-1) = 2$$

$$f(c) = \frac{2}{\pi}(2)$$

$$\boxed{f(c) = \frac{4}{\pi}}$$

2) $y = e^x \quad y = 0 \quad x = 0 \quad x = 1$



$$A = \int_0^1 e^x - 0\,dx = e^x\Big]_0^1 = e^1 - e^0$$

$$\boxed{e - 1}$$

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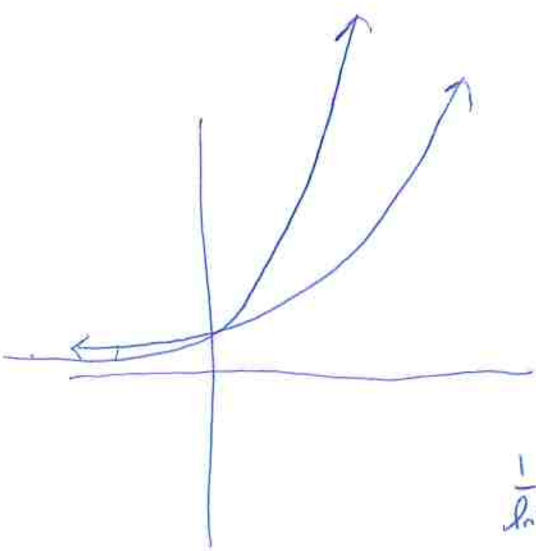
3) $y = 2^x \quad y = 5^x \quad x = -1 \quad x = 1 \qquad \int a^u = \frac{1}{\ln a}a^u + C$



$$\int_{-1}^{0} 2^x - 5^x\,dx + \int_0^1 5^x - 2^x\,dx$$

$$\frac{1}{\ln 2}(2^x) - \frac{1}{\ln 5}(5^x)\Big]_{-1}^{0} \qquad \frac{1}{\ln 5}5^x - \frac{1}{\ln 2}2^x\Big]_0^1 =$$

$$\frac{5}{\ln 5} - \frac{2}{\ln 2} - \left(\frac{1}{\ln 5} - \frac{1}{\ln 2}\right)$$

$$\frac{1}{\ln 2}2^0 - \frac{1}{\ln 5}5^0 - \left(\frac{1}{\ln 2}2^{-1} - \frac{1}{\ln 5}5^{-1}\right)$$

$$\cancel{\frac{1}{\ln 2}} - \frac{1}{\ln 5} - \frac{1}{2\ln 2} + \frac{1}{5\ln 5} + \frac{5}{\ln 5} - \cancel{\frac{2}{\ln 2}} - \frac{1}{\ln 5} + \cancel{\frac{1}{\ln 2}}$$

$$\frac{3}{\ln 5} + \frac{1}{5\ln 5} - \frac{1}{2\ln 2} = \boxed{\frac{16}{5\ln 5} - \frac{1}{2\ln 2} = 1.2669}$$

$$\frac{15}{5\ln 5} + \frac{1}{5\ln 5}$$

$\boxed{\text{Ex. 4}}$ PVA:

Given: $a(t) = 3t - \frac{1}{\sqrt{t}}$ $v(0)=1$ $x(0)=3$

Find $x(t)$

$$v(t) = \int 3t - t^{-1/2}\, dt$$

$$v(t) = \frac{3t^2}{2} - \frac{t^{1/2}}{1/2} + C$$

$$v(t) = \frac{3}{2}t^2 - 2t^{1/2} + C$$

$$1 = \frac{3}{2}(0)^2 - 2(0) + C$$

$$1 = C$$

$$\underline{v(t) = \frac{3}{2}t^2 - 2t^{1/2} + 1}$$

$$x(t) = \int \frac{3}{2}t^2 - 2t^{1/2} + 1\, dt$$

$$x(t) = \frac{3}{2}\cdot\frac{t^3}{3} - \frac{2t^{3/2}}{3/2} + 1t + C_2$$

$$x(t) = \frac{1}{2}t^3 - \frac{4}{3}t^{3/2} + 1t + C_2$$

plug in $x(0)=3$

$$3 = \frac{1}{2}(0)^3 - \frac{4}{3}(0)^{3/2} + 1(0) + C_2 \qquad 3 = C_2$$

$$\boxed{x(t) = \frac{1}{2}t^3 - \frac{4}{3}t^{3/2} + 1t + 3}$$