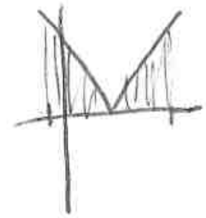
Ch. 4 (Non-calculator) Integrals Unit test topics:

1) Power Rule (Integrals)
2) U-substitution (and change of variable)
3) Trig Integrals
4) Definite Integrals (Unit circle values) (Trig)
5) Definite Integrals (Absolute Value Function)



6) Even/Odd Functions
7) 2nd Fundamental Theorem of Calculus
8) Avg. Value Theorem
9) Given $f''(x)$, find $f(x)$ (Antiderivative)
   (Given $a(t)$, find $x(t)$)

3) SFTC: $\frac{d}{dx}\int_a^{p(x)} f(t)\,dt = f(p(x)) \cdot p'(x)$

$$\frac{d}{dx}\int_{-2}^{-2x^2} \frac{t}{4-t^3}\,dt = \frac{-2x^2}{4-(-2x^2)^3} \cdot -4x = \boxed{\frac{8x^3}{4+8x^6}}$$

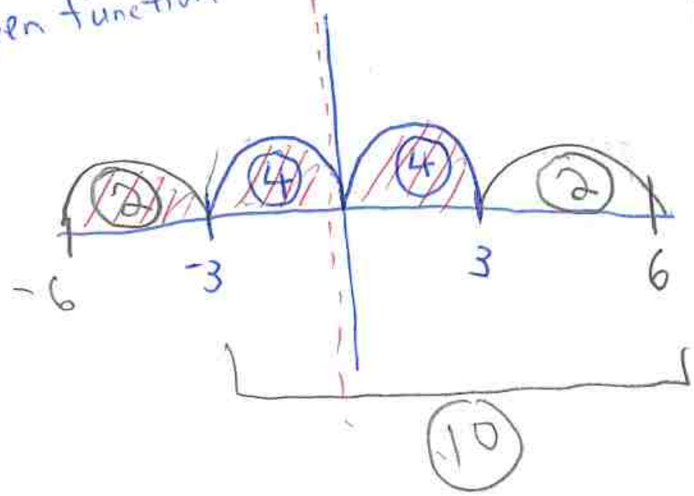
SFTC(2): $\frac{d}{dx}\int_{q(x)}^{p(x)} f(t)\,dt = f(p(x)) \cdot p'(x) - f(q(x)) \cdot q'(x)$

4) $\frac{d}{dx}\int_{-x}^{3\sqrt{x} \Rightarrow 3x^{1/2}} 1-2t\,dt = (1-2(3\sqrt{x})) \cdot \frac{3}{2}x^{-1/2} - \left[(1-2(-x)) \cdot -1\right]$

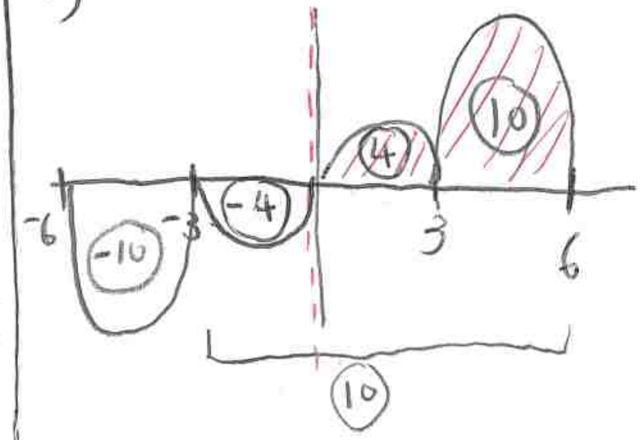
$$(1-6\sqrt{x}) \cdot \frac{3}{2\sqrt{x}} + 1 + 2x$$

$$\boxed{\frac{3-18\sqrt{x}}{2\sqrt{x}} + 1 + 2x}$$

a) Even function



b) odd function



7) $\int \frac{2}{x^2} \sec\left(\frac{3}{x}\right) \tan\left(\frac{3}{x}\right) dx$

u-substitution:

$u = \frac{3}{x} \rightarrow 3x^{-1}$

$\frac{du}{dx} = -3x^{-2}$

$\frac{du}{dx} = \frac{-3}{x^2}$

$-3dx = x^2 du$

$dx = \frac{x^2 du}{-3}$

$\int \frac{2}{\cancel{x^2}} \sec u \tan u \cdot \frac{\cancel{x^2} du}{-3}$

$-\frac{2}{3} \int \sec u \tan u du$

$-\frac{2}{3} \sec u + C$

$\boxed{-\frac{2}{3} \sec\left(\frac{3}{x}\right) + C}$

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8) $\int 5x\sqrt{2-x}\, dx \rightarrow \int 5x(2-x)^{1/2} dx$

u-sub

$u = 2-x$

$\frac{du}{dx} = -1$

$dx = -1du$

$\int 5x \cdot u^{1/2} \cdot -1du$

$x = \boxed{2-u}$

$\int 5(2-u)u^{1/2}(-1du)$

$\int -5u^{1/2}(2-u)du$

$\int -10u^{1/2} + 5u^{3/2} du$

$-10\frac{u^{3/2}}{3/2} + 5\frac{u^{5/2}}{5/2} + C$

$-10 \cdot \frac{2}{3} u^{3/2} + 5 \cdot \frac{2}{5} u^{5/2} + C$

$\boxed{-\frac{20}{3}(2-x)^{3/2} + 2(2-x)^{5/2} + C}$

Definite Integrals (Trig) (Unit circle)

$$\int_{-\pi/3}^{\pi/6} \sin^2 x \cos x \, dx \rightarrow \int (\underline{\sin x})^2 \cos x \, \underline{dx}$$

$u = \sin x$

$\frac{du}{dx} = \frac{\cos x}{1}$

$dx \cos x = du$

$dx = \frac{du}{\cos x}$

$$\int u^2 \cdot \cancel{\cos x} \cdot \frac{du}{\cancel{\cos x}} \rightarrow \int u^2 \, du \rightarrow \frac{u^3}{3}$$

$$\left.\frac{(\sin x)^3}{3}\right]_{-\pi/3}^{\pi/6} \rightarrow \frac{1}{3}\left(\sin\left(\frac{\pi}{6}\right)\right)^3 - \frac{1}{3}\left(\sin\left(-\frac{\pi}{3}\right)\right)^3$$

$$\frac{1}{3}\left(\frac{1}{2}\right)^3 - \frac{1}{3}\left(\frac{-\sqrt{3}}{2}\right)^3$$

$$\boxed{\frac{1}{3}\left(\frac{1}{8}\right) + \frac{3\sqrt{3}}{24}} = \boxed{\frac{1+3\sqrt{3}}{24}}$$

* U-sub (change of variable)

$$\int x^2 \sqrt{4-x}\,dx$$

$$\int x^2 (4-x)^{1/2}\,dx$$

$u = 4-x$

$\frac{du}{dx} = -1$

$dx = -1du$

$$\int x^2 \cdot u^{1/2} \cdot (-1du)$$

$x = \boxed{4-u}$

$$\int (4-u)^2 u^{1/2} (-1du)$$

$(4-u)(4-u)$

$16-4u-4u+u^2$

$$\int -u^{1/2}(16-8u+u^2)\,du$$

$$\int -16u^{1/2} + 8u^{3/2} - u^{5/2}\,du$$

$$-16\frac{u^{3/2}}{3/2} + \frac{8u^{5/2}}{5/2} - \frac{u^{7/2}}{7/2} + C$$

$$-16 \cdot \frac{2}{3}u^{3/2} + 8 \cdot \frac{2}{5}u^{5/2} - \frac{2}{7}u^{7/2} + C$$

$$\boxed{-\frac{32}{3}(4-x)^{3/2} + \frac{16}{5}(4-x)^{5/2} - \frac{2}{7}(4-x)^{7/2} + C}$$

5) $\int x^2\sqrt{7-x}\,dx$

$\int x^2(7-x)^{1/2}dx$

$u=7-x$

$\frac{du}{dx}=-1$

$-1dx=du$

$dx=-1du$

$\int x^2\cdot u^{1/2}\cdot -1du$

$x=\boxed{7-u}$

$\int(7-u)^2\cdot u^{1/2}(-1du)$

$(7-u)(7-u)$

$49-7u-7u+u^2$

$\int -u^{1/2}(49-14u+u^2)du$

$\int -49u^{1/2}+14u^{3/2}-u^{5/2}du$

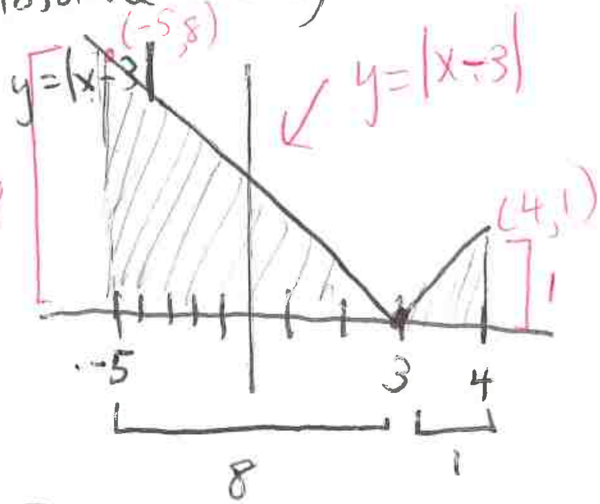
8) SFTC $\frac{d}{dx}\int_a^{p(x)} f(t)\,dt = f(p(x))\cdot p'(x)$

$$\frac{d}{dx}\int_{\pi}^{3x^2} -\sqrt{1-t^2}\,dt = -\sqrt{1-(3x^2)^2}\cdot 6x$$

$$= \boxed{-6x\sqrt{1-9x^4}}$$

Definite Integrals (Absolute Value)

$$\int_{-5}^{4} |x-3|\,dx$$



$A = \frac{1}{2}bh$

$\frac{1}{2}(8)(8) + \frac{1}{2}(1)(1)$

$\frac{64}{2} + \frac{1}{2} = \boxed{\frac{65}{2}}$