

Calculus

Ch. 5.6 Inverse Trig Derivatives

Classwork Worksheet

THEOREM 5.16 Derivatives of Inverse Trigonometric Functions

Let u be a differentiable function of x .

$$\frac{d}{dx} [\arcsin u] = \frac{u'}{\sqrt{1-u^2}}$$

$$\frac{d}{dx} [\arccos u] = \frac{-u'}{\sqrt{1-u^2}}$$

$$\frac{d}{dx} [\arctan u] = \frac{u'}{1+u^2}$$

$$\frac{d}{dx} [\operatorname{arccot} u] = \frac{-u'}{1+u^2}$$

$$\frac{d}{dx} [\operatorname{arcsec} u] = \frac{u'}{|u|\sqrt{u^2-1}}$$

$$\frac{d}{dx} [\operatorname{arccsc} u] = \frac{-u'}{|u|\sqrt{u^2-1}}$$

Evaluating an Expression In Exercises 21–24, evaluate each expression without using a calculator. (*Hint:* See Example 3.)

21. (a) $\sin\left(\arctan \frac{3}{4}\right)$

22. (a) $\tan\left(\arccos \frac{\sqrt{2}}{2}\right)$

23. (a) $\cot\left[\arcsin\left(-\frac{1}{2}\right)\right]$

24. (a) $\sec\left[\arctan\left(-\frac{3}{5}\right)\right]$

Simplifying an Expression Using a Right Triangle In Exercises 25–32, write the expression in algebraic form. (*Hint:* Sketch a right triangle, as demonstrated in Example 3.)

25. $\cos(\arcsin 2x)$

26. $\sec(\arctan 4x)$

29. $\tan\left(\operatorname{arcsec} \frac{x}{3}\right)$

31. $\csc\left(\arctan \frac{x}{\sqrt{2}}\right)$

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$$\begin{aligned}\frac{d}{dx} [\arcsin u] &= \frac{u'}{\sqrt{1-u^2}} & \frac{d}{dx} [\arccos u] &= \frac{-u'}{\sqrt{1-u^2}} \\ \frac{d}{dx} [\arctan u] &= \frac{u'}{1+u^2} & \frac{d}{dx} [\operatorname{arccot} u] &= \frac{-u'}{1+u^2} \\ \frac{d}{dx} [\operatorname{arcsec} u] &= \frac{u'}{|u|\sqrt{u^2-1}} & \frac{d}{dx} [\operatorname{arccsc} u] &= \frac{-u'}{|u|\sqrt{u^2-1}}\end{aligned}$$

Finding a Derivative In Exercises 39–58, find the derivative of the function.

39. $f(x) = 2 \arcsin(x - 1)$

44. $f(x) = \arctan \sqrt{x}$

46. $h(x) = x^2 \arctan 5x$

47. $h(t) = \sin(\arccos t)$

50. $y = \ln(t^2 + 4) - \frac{1}{2} \arctan \frac{t}{2}$

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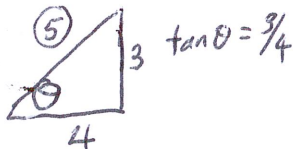
$$\frac{d}{dx} [\operatorname{arcsec} u] = \frac{u'}{|u|\sqrt{u^2-1}}$$

$$\frac{d}{dx} [\operatorname{arccsc} u] = \frac{-u'}{|u|\sqrt{u^2-1}}$$

Key

Evaluating an Expression In Exercises 21–24, evaluate each expression without using a calculator. (Hint: See Example 3.)

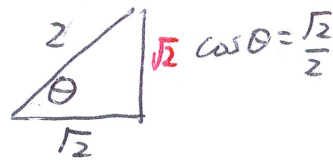
21. (a) $\sin\left(\arctan \frac{3}{4}\right)$



$\tan \theta = \frac{3}{4}$

$$= \frac{3}{5}$$

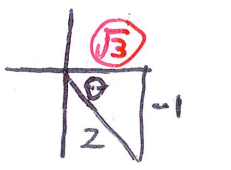
22. (a) $\tan\left(\arccos \frac{\sqrt{2}}{2}\right)$



$\cos \theta = \frac{\sqrt{2}}{2}$

$$= \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

23. (a) $\cot\left[\arcsin\left(-\frac{1}{2}\right)\right]$

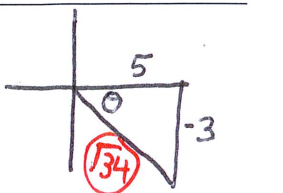


$\sin \theta = -\frac{1}{2}$

$$= \frac{\sqrt{3}}{-1} = -\sqrt{3}$$

$[-\pi/2, \pi/2]$

24. (a) $\sec\left[\arctan\left(-\frac{3}{5}\right)\right]$



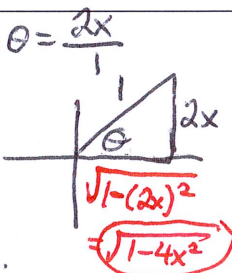
$\tan \theta = -\frac{3}{5}$

$$= \frac{\sqrt{34}}{5}$$

$[-\pi/2, \pi/2]$

Simplifying an Expression Using a Right Triangle In Exercises 25–32, write the expression in algebraic form. (Hint: Sketch a right triangle, as demonstrated in Example 3.)

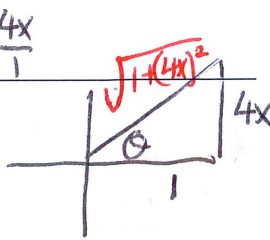
25. $\cos(\arcsin 2x)$



$\sin \theta = \frac{2x}{1}$

$$= \frac{\sqrt{1-4x^2}}{1} = \sqrt{1-4x^2}$$

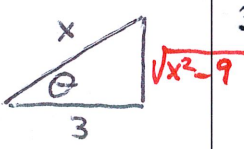
26. $\sec(\arctan 4x)$



$\tan \theta = \frac{4x}{1}$

$$= \frac{\sqrt{1+16x^2}}{1} = \sqrt{1+16x^2}$$

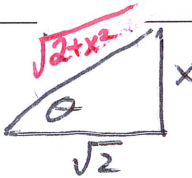
29. $\tan(\operatorname{arcsec} \frac{x}{3})$



$\sec \theta = \frac{x}{3}$

$$= \frac{\sqrt{x^2-9}}{x}$$

31. $\csc\left(\arctan \frac{x}{\sqrt{2}}\right)$



$\tan \theta = \frac{x}{\sqrt{2}}$

$$= \frac{\sqrt{2+x^2}}{x}$$

THEOREM 5.16 Derivatives of Inverse Trigonometric Functions

Let u be a differentiable function of x .

$$\begin{aligned} \frac{d}{dx} [\arcsin u] &= \frac{u'}{\sqrt{1-u^2}} & \frac{d}{dx} [\arccos u] &= \frac{-u'}{\sqrt{1-u^2}} \\ \frac{d}{dx} [\arctan u] &= \frac{u'}{1+u^2} & \frac{d}{dx} [\operatorname{arccot} u] &= \frac{-u'}{1+u^2} \\ \frac{d}{dx} [\operatorname{arcsec} u] &= \frac{u'}{|u|\sqrt{u^2-1}} & \frac{d}{dx} [\operatorname{arccsc} u] &= \frac{-u'}{|u|\sqrt{u^2-1}} \end{aligned}$$

Finding a Derivative In Exercises 39–58, find the derivative of the function.

39. $f(x) = 2 \arcsin(x - 1)$

$$f'(x) = 2 \cdot \left(\frac{1}{\sqrt{1-(x-1)^2}} \right) = \boxed{\frac{2}{\sqrt{1-(x-1)^2}}}$$

44. $f(x) = \arctan \sqrt{x} \quad \arctan(x^{1/2})$

$$f'(x) = \frac{\frac{1}{2}x^{-1/2}}{1+(\sqrt{x})^2} = \frac{\frac{1}{2\sqrt{x}}}{1+x}$$

$$= \boxed{\frac{1}{2\sqrt{x}(1+x)}}$$

46. $h(x) = x^2 \arctan 5x$

*product rule

$$h'(x) = \overbrace{2x}^{f'} \cdot \overbrace{\arctan(5x)}^g + \overbrace{x^2}^f \cdot \overbrace{\frac{5}{1+(5x)^2}}^{g'}$$

$$h'(x) = 2x \arctan(5x) + \frac{5x^2}{1+25x^2}$$

47. $h(t) = \sin(\arccos t)$

*chain rule

$$h'(t) = \underbrace{\cos(\arccos t)}_t \cdot \frac{-1}{\sqrt{1-t^2}}$$

out: $\sin(\)$

in: $\arccos t$

$$h'(t) = t \cdot \left(\frac{-1}{\sqrt{1-t^2}} \right)$$

$$h'(t) = \frac{-t}{\sqrt{1-t^2}}$$

50. $y = \ln(t^2 + 4) - \frac{1}{2} \arctan \frac{t}{2} \quad \leftarrow \frac{1}{2} \arctan\left(\frac{1}{2}t\right)$

*apply

$$\frac{d}{dx} \ln u = \frac{u'}{u}$$

$$y' = \frac{2t}{t^2+4} - \frac{1}{2} \cdot \frac{\frac{1}{2}}{1+\left(\frac{t}{2}\right)^2}$$

$$y' = \frac{2t}{t^2+4} - \frac{1}{4\left(1+\frac{t^2}{4}\right)}$$

$$y' = \frac{2t}{t^2+4} - \frac{1}{4+t^2} = \boxed{\frac{2t-1}{t^2+4}}$$