Key

Junior Varsity State Competition

hosted by Stratford Academy

March 19, 2016

No calculators are allowed on this exam. You will have 60 minutes to complete 40 questions. Each correct response on the written test is worth 5 points. There is no penalty for an incorrect answer.

1. Find the solution set for the equation.

$$\frac{x}{2} = \frac{3}{x-1}$$

(a) $x = \{0\}$ (b) $x = \{0, 1\}$ (c) $x = \{-2\}$ (d) $x = \{-2, 3\}$ (e) $x = \{1, 3\}$

$x^2 - x = 6$

$x^2 - x - 6 = 0$

$(x-3)(x+2) = 0$ $\boxed{x = 3, -2}$

2. Simplify the complex fraction into a simplified rational expression.

$$\frac{1 - \frac{x}{x+1}}{2 - \frac{x-1}{x}} = \frac{\frac{x+1-x}{x+1}}{\frac{2x-x+1}{x}} = \frac{\frac{1}{x+1}}{\frac{x+1}{x}} = \boxed{\frac{x}{(x+1)^2}}$$

(a) $\frac{x+1}{x-1}$ (b) $\frac{x}{x-1}$ (c) $\frac{x}{x+1}$ (d) $\frac{x}{(x-1)^2}$ (e) $\frac{x}{(x+1)^2}$

$\pi/6$ s

3. The arc that corresponds to a 30° central angle has length b. Find the area of a sector in the same circle that corresponds to a central angle of 45°.

(a) $\frac{b}{12\pi}$ (b) $\frac{b^2}{12\pi}$ (c) $\frac{9b}{4\pi}$ (d) $\frac{9b^2}{2\pi}$ (e) $\frac{16b^2}{9\pi}$

$s = r\theta$

$b = r(\pi/6)$

$\frac{6b}{\pi} = r$

$A_{sector} = \frac{1}{2}r^2\theta$

$A = \frac{1}{2}\left(\frac{6b}{\pi}\right)^2\left(\frac{\pi}{4}\right)$

$= \frac{1}{2} \cdot \frac{36b^2}{\pi^2} \cdot \frac{\pi}{4} = \frac{36b^2}{8\pi} = \boxed{\frac{9b^2}{2\pi}}$

4. In a group of three friends, the sums of any two of their ages are 20, 21, 23. How old is the youngest friend?

(a) 6 (b) 7 (c) 8 $\boxed{\text{(d) } 9}$ (e) 10

$x+y=20$
$x+z=21$
$y+z=23$

$2x+2y+2z=64$
$x+y+z=32$

*If x is assigned to be the youngest friend, $x+\underbrace{y+z}_{23}=32$

$x+23=32$

$\boxed{x=9}$

5. Solve the equation.

$$27^{x-2} = \left(\frac{1}{9}\right)^x$$

$3^{3(x-2)} = 3^{-2x}$

$3(x-2) = -2x$

$3x-6 = -2x$

$5x = 6$

$\boxed{x=\frac{6}{5}}$

(a) $x = \frac{3}{5}$

(b) $x = \frac{5}{6}$

$\boxed{\text{(c) } x = \frac{6}{5}}$

(d) $x = \frac{7}{5}$

(e) $x = \frac{7}{3}$

6. The linear function $f(x)$ satisfies the conditions $f(2) = 5$ and $f(f(2)) = 11$. What is the value of $f(11)$?

(a) 20 (b) 22 $\boxed{\text{(c) } 23}$ (d) 23 (e) 25

$f(2)=5$
$f(5)=11$

slope: $m=\frac{11-5}{5-2}=\frac{6}{3}=2$

point: $(2,5)$

$y-y_1=m(x-x_1)$
$y-5=2(x-2)$
$y-5=2x-4$
$y=2x+1$

$y(11)=2(11)+1$

$\boxed{y(11)=23}$

7. On a math test, the passing students averaged 83, while the failing students had an average of 55. If the overall average for the class was 76, what percent of the class passed the test?

$x=$ # of passing students $y=$ # of failing students

(a) 44% (b) 66% (c) 68% (d) 72% (e) $\boxed{75\%}$

$83x+55y=76(x+y)$ | $7x=21y$

$83x+55y=76x+76y$ | $x=3y$

8. Suppose that $\log_4 x = \frac{1}{3}$. What is the value of $\log_x 8$?

(a) $\sqrt[4]{8}$ (b) 2 (c) $\boxed{\frac{9}{2}}$ (d) 4 (e) $\sqrt[3]{4^8}$

$4^{1/3}=x$ | $\log_{4^{1/3}}(4^{3/2})$

$\log_{4^{1/3}}(4^{1/3})^{9/2}=\boxed{9/2}$

$\ln\left[\frac{3x^2-1}{5}\right]^{x^2-2x}=\ln 1$

$(x^2-2x)\cdot\ln\left(\frac{3x^2-1}{5}\right)=0$

9. How many real solutions does the equation $\left(\frac{3x^2-1}{5}\right)^{x^2-2x}=1$ have?

(a) 0 (b) 1 (c) 2 (d) 3 (e) $\boxed{4}$

$x^2-2x=0$

$x(x-2)=0$

$\boxed{x=0,2}$

$\ln\left(\frac{3x^2-1}{5}\right)=0$

$e^0=\frac{3x^2-1}{5}$

$1=\frac{3x^2-1}{5}$

$5=3x^2-1$

$5=3x^2$

$x^2=3/5$ $\boxed{x=\pm\sqrt{3/5}}$

10. Find the solution set for the equation below.

$\log a+\log b=\log(ab)$

$$\log_6(3x)+\log_6(x-4)=2$$

(a) ∅ (b) $\{-2,6\}$ (c) $\{6,18\}$ (d) $\{0,6\}$ (e) $\boxed{\{6\}}$

$\log_6[3x(x-4)]=2$

$6^2=3x(x-4)$

$36=3x^2-12x$

$0=3x^2-12x-36$

$0=3(x^2-4x-12)$

$0=3(x-6)(x+2)$

$x=6,-2$

$\boxed{x=6}$ $\boxed{x\neq-2}$

$* \; a^3+b^3=(a+b)(a^2-ab+b^2)$

11. If $\sin(x)+\cos(x)=\frac{2}{3}$, what is the value of $\sin^3(x)+\cos^3(x)$? $* \; \sin^2 x+\cos^2 x = 1$

(a) $\frac{8}{54}$ (b) $\frac{19}{54}$ (c) $\frac{8}{27}$ (d) $\frac{19}{27}$ (e) $\frac{23}{27}$

$\sin^3 x+\cos^3 x = (\sin x+\cos x)(\sin^2 x - \sin x\cos x + \cos^2 x)$

$= \left(\frac{2}{3}\right)\left[1-\left(\frac{-5}{18}\right)\right]$

$= \left(\frac{2}{3}\right)\left(\frac{23}{18}\right) = \boxed{\frac{23}{27}}$

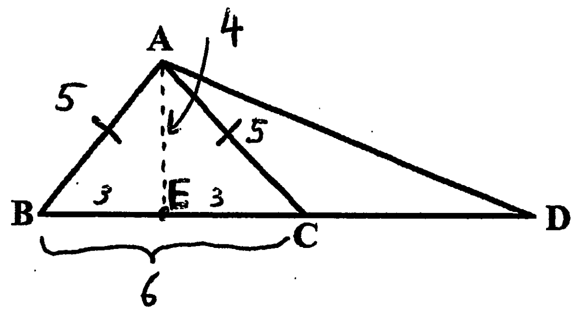
$(\sin x+\cos x)^2 = \sin^2 x + 2\sin x\cos x + \cos^2 x$

$\left(\frac{2}{3}\right)^2 = 1+2\sin x\cos x$

$\frac{4}{9}-1 = 2\sin x\cos x$

$\sin x\cos x = \left(\frac{-5}{9}\right)\left(\frac{1}{2}\right) = \boxed{\frac{-5}{18}}$

12. In $\triangle ABC$, the point C lies on the segment $\overline{BD}$. Also $AB=AC=5$ and $BC=6$. What is the area of $\triangle ABD$ if $CD=7$?



(a) 21 (b) 25 (c) 26 (d) 50 (e) 52

$\text{Area} = \frac{1}{2}\cdot AE\cdot BD$

$= \frac{1}{2}\cdot 4\cdot 13$

$\boxed{\text{Area} = 26}$

13. The two roots of the equation $x^2-85x+K=0$ are prime integers. What is the value of K?

(a) 78 (b) 80 (c) 154 (d) 166 (e) 858

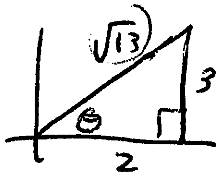
1, 84

2, 83

$2\times 83 = \boxed{166}$

14. Find $\sec\left(\arctan\left(\frac{3}{2}\right)\right)$. $\tan\theta = 3/2$

(a) $\frac{2\sqrt{13}}{13}$ (b) $\frac{3\sqrt{13}}{13}$ (c) $\frac{\sqrt{5}}{2}$ (d) $\boxed{\frac{\sqrt{13}}{2}}$ (e) $\frac{\sqrt{13}}{3}$



$h^2 = 2^2 + 3^2$
$h = \sqrt{13}$

$$\sec\left(\arctan(3/2)\right) = \boxed{\frac{\sqrt{13}}{2}}$$

15. How many diagonals does a regular 20 sided polygon have?

(a) 170
(b) 180
(c) 190
(d) 200
(e) None of the above.

$$\text{diagonals} = \frac{n(n-3)}{2}$$

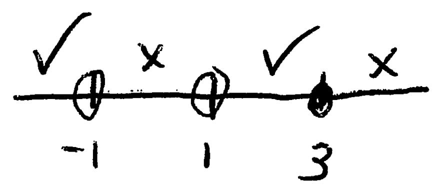
$n = 20$

$$\text{diagonals} = \frac{20(20-3)}{2} = \boxed{170}$$

16. Solve the inequality.

$x \neq 1, -1$

$$\frac{2}{x+1} \leq \frac{1}{x-1}$$



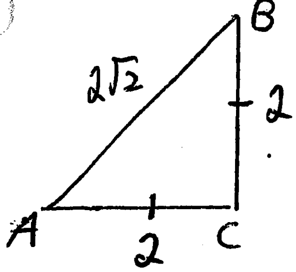
(a) $x < -1$ or $1 < x \leq 3$
(b) $-1 < x < 1$ or $x \geq 3$
(c) $1 < x \leq 3$
(d) $x < 1$ or $x \geq 3$
(e) $-1 < x < 1$ or $x \geq 3$

$$\frac{2}{x+1} = \frac{1}{x-1}$$

$2(x-1) = x + 1$
$2x - 2 = x + 1$
$x = 3$

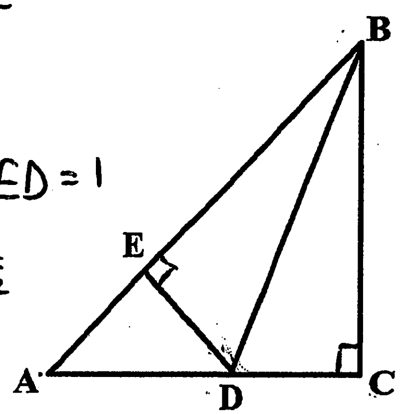
$\boxed{(-\infty, -1) \cup (1, 3]}$

17. In the picture below, $\triangle ABC$ is an isosceles triangle with a right angle at C. $\triangle ADB$ and $\triangle CDB$ both have area 1 in^2. $\overline{DE}$ is perpendicular to $\overline{AB}$. What is the length (in inches) of $\overline{DE}$?



$\frac{1}{2}(BC)^2 = 2$

$x = 2$



$\triangle ABD$ Area $= 1$

$\frac{1}{2}(AB)(ED) = 1$

$\frac{1}{2}(2\sqrt{2}) \cdot ED = 1$

$ED = \frac{1}{\sqrt{2}}$

$\boxed{ED = \frac{\sqrt{2}}{2}}$

(a) $\frac{\sqrt{5}}{5}$ (b) $\frac{2\sqrt{5}}{5}$ (c) $\frac{\sqrt{3}}{3}$ (d) $\frac{\sqrt{2}}{2}$ (e) $\frac{2\sqrt{2}}{5}$

18. Solve for x.

$$4^x + 4^x + 4^x + 4^x = 2^{2016}$$

*If $a^m = a^n$, then $m = n$

*$a^m \cdot a^n = a^{m+n}$

(a) $x = 504$ (b) $x = 1007$ (c) $x = \frac{2015}{2}$ (d) $x = 1008$ (e) $x = 2015$

$4(4^x) = 2^{2016}$

$2^2 \cdot 2^{2x} = 2^{2016}$

$2^{2+2x} = 2^{2016}$

$2x + 2 = 2016$

$2x = 2014$

$\boxed{x = 1007}$

19. The function $f(x) = 6x^5 + bx^3 + cx^2 - 35$ has integer values for b and c. Which of the following could *possibly* be a zero of $f(x)$?

(a) $x = 2$ (b) $x = \frac{7}{3}$ (c) $x = 3$ (d) $x = 4$ (e) $x = \frac{3}{5}$

*use rational root theorem

possible zeros $\left(\frac{p}{q}\right) = \pm \frac{1, 5, 7, 35}{1, 2, 3, 6}$

20. The function $f(x)$ is defined as

$$f(x) = \begin{cases} f(x+1) & \text{if } x < 3 \\ \left(\frac{1}{2}\right)^x & \text{if } x \geq 3 \end{cases}$$

≈ 0.5

Find $f\left(2 + \log_2\left(\frac{3}{2}\right)\right)$. $\rightarrow f(2.5) = f(3.5)$

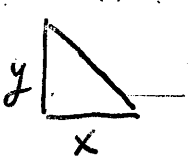
$\boxed{\text{(a) } \frac{1}{12}}$ (b) $\frac{1}{6}$ (c) $\frac{3}{16}$ (d) $\frac{3}{8}$ (e) $\frac{7}{16}$

$= \left(\frac{1}{2}\right)^{3+\log_2(3/2)}$ | $= \frac{1}{8} \cdot 2^{-1 \log_2(3/2)}$ | $= \frac{1}{8} \cdot \left(\frac{3}{2}\right)^{-1}$

$= \left(\frac{1}{2}\right)^3 \cdot \left(\frac{1}{2}\right)^{\log_2(3/2)}$ | $= 1/8 \cdot 2^{\log_2(3/2)^{-1}}$ | $= \frac{1}{8} \cdot \frac{2}{3} = \frac{2}{24} = \boxed{\frac{1}{12}}$

21. The base of a triangle is increased by 15% and the height is decreased by 20%. What is the change in the area?

(a) The area decreases by 5%.
$\boxed{\text{(b)}}$ The area decreases by 8%.
(c) The area decreases by 9.2%.
(d) The area increases by 8%.
(e) The area increases by 9.2%.



original area $= \frac{1}{2}xy$

new base $= x + 0.15x = 1.15x$
new height $= y - 0.20y = 0.8y$
new Area $= \frac{1}{2}(1.15x)(0.8y)$
$= \frac{1}{2}(0.92)xy$

This is 92% of the original area, which is a decrease by 8%

22. Find the equation of the line that passes through the centers of the following circles.

$$x^2 + y^2 - 8x + 2y = -1$$
$$x^2 + y^2 + 4x - 2y = 31$$

(a) $3x - y = 8$ $\boxed{\text{(b) } x + 3y = 1}$ (c) $x + 3y = -1$ (d) $x = 4$ (e) $3x + 8y = 4$

$x^2 - 8x + \underline{16} + y^2 + 2y + \underline{1} = -1 + \underline{16} + \underline{1}$
$(x-4)^2 + (y+1)^2 = 16$, center $(4, -1)$
$x^2 + 4x + \underline{4} + y^2 - 2y + \underline{1} = 31 + \underline{4} + \underline{1}$
$(x+2)^2 + (y-1)^2 = 36$, center $(-2, 1)$

slope: $\frac{1 - -1}{-2-4} = \frac{2}{-6} = -1/3$

$y - 1 = \frac{-1}{3}(x+2)$
$(y - 1 = \frac{-1}{3}x - 2/3)3$
$3y - 3 = -x - 2$

$\boxed{x + 3y = 1}$

23. Two real numbers x and y have the property that their difference is 6 and the difference of their square roots is 1. Find the sum of x and y.

(a) 16 (b) $\frac{37}{4}$ (c) $\frac{49}{4}$ (d) $\boxed{\frac{37}{2}}$ (e) $3+\sqrt{6}$

$x-y=6$

$\sqrt{x}-\sqrt{y}=1$

$\sqrt{x}=1+\sqrt{y}$

$(1+\sqrt{y})^2-y=6$

$1+2\sqrt{y}+y-y=6$

$2\sqrt{y}=5 \quad y=\frac{25}{4}$

$x-y=6$

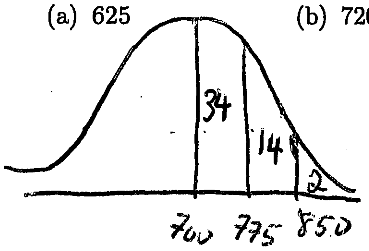
$x-\frac{25}{4}=6$

$x=49/4$

$49/4+\frac{25}{4}=\frac{74}{4}=\boxed{\frac{37}{2}}$

24. Scores on a national achievement test are normally distributed with a mean of 700 and a standard deviation of 75. The 85th percentile of test scores is closest to

(a) 625 (b) 720 (c) 740 $\boxed{\text{(d) 775}}$ (e) 800



25. The measure of an interior angle of a regular polygon is 9 times the measure of an exterior angle of that polygon. How many sides does the polygon have?

(a) 9 (b) 10 (c) 18 $\boxed{\text{(d) 20}}$ (e) 24

Interior $=\frac{180(n-2)}{n}$

Exterior $=\frac{360}{n}$

Interior $=9($Exterior$)$

$\frac{180(n-2)}{n}=9\left(\frac{360}{n}\right)$

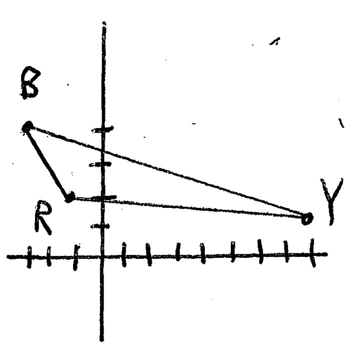
$180n-360=3240$

$180n=3600$

$\boxed{n=20 \text{ sides}}$

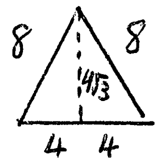
26. $\triangle RBY$ has vertices $R(-1,2)$, $B(-3,4)$, and $Y(8,1)$. Classify $\triangle RBY$ according to its sides and angles.

(a) Isosceles, obtuse
(b) Scalene, right
(c) Scalene, acute
(d) Isosceles, right
$\boxed{\text{(e) Scalene, obtuse}}$



27. The perimeter of an equilateral triangle is 24 units. What is the area?

(a) $8\sqrt{2}$ (b) $8\sqrt{3}$ (c) $16\sqrt{2}$ (d) $16\sqrt{3}$ (e) $32\sqrt{2}$



$A = \frac{1}{2}bh$

$A = \frac{1}{2}(8)(4\sqrt{3}) = \boxed{16\sqrt{3}}$

28. The quadratic equation $5x^2 - 48x + n = 0$, where n is a constant, has $x = 10$ as one solution. What is the other solution to the equation?

$ax^2+bx+c=0$

(a) $x = -38$ (b) $x = -\frac{8}{5}$ (c) $x = -\frac{2}{5}$ (d) $x = \frac{2}{5}$ (e) $x = \frac{8}{5}$

Quadratic nature of roots:

$r_1 + r_2 = -\frac{b}{a}$

$r_1 \cdot r_2 = \frac{c}{a}$

$10 + r_2 = -\left(\frac{-48}{5}\right)$

$r_2 = \frac{48}{5} - 10$

$\boxed{r_2 = \frac{-2}{5}}$

29. A restaurant offers a lunch menu with 3 different sandwiches, 2 ~~unique~~ salads, 6 varieties of drink, and 4 types of soup. How many combinations are possible if you choose exactly one sandwich, one drink, and one soup?

(a) 15 (b) 36 (c) 48 (d) 72 (e) 144

$3 \times 6 \times 4 = \boxed{72}$

30. Find the inverse function for $f(x) = \dfrac{2x-1}{x+3}$.

(a) $f^{-1}(x) = \dfrac{x+3}{2x-1}$

(b) $\boxed{f^{-1}(x) = -\dfrac{3x+1}{x-2}}$

(c) $f^{-1}(x) = \dfrac{2x+1}{x-3}$

(d) $f^{-1}(x) = -\dfrac{3x-1}{x-2}$

(e) $f^{-1}(x) = -\dfrac{2x-1}{x-3}$

$y = \dfrac{2x-1}{x+3}$

$x = \dfrac{2y-1}{y+3}$

$x(y+3) = 2y-1$

$xy + 3x = 2y - 1$

$xy - 2y = -3x - 1$

$y(x-2) = -3x-1$

$y = \dfrac{-3x-1}{x-2}$

$f^{-1}(x) = \dfrac{-(3x+1)}{x-2}$

$\boxed{f^{-1}(x) = -\dfrac{3x+1}{x-2}}$

31. Simplify. $i^{-11} =$

$i^{-4} = 1$

$i^{-3} = i \longrightarrow i^{-11} = \boxed{i}$

$i^{-2} = -1$

$i^{-1} = -i$

$i^{0} = 1$

$i^{1} = i$

$i^{2} = -1$

$i^{3} = -i$

(a) 0 (b) 1 (c) -1 $\boxed{\text{(d) } i}$ (e) $-i$

32. Use properties of logarithms to simplify the expression.

$\log_2 3 \cdot \log_3 6 \cdot \log_6 16 = \dfrac{\cancel{\ln 3}}{\ln 2} \cdot \dfrac{\cancel{\ln 6}}{\cancel{\ln 3}} \cdot \dfrac{\ln 16}{\cancel{\ln 6}} = \dfrac{\ln 16}{\ln 2}$

*use change of base properties: $\log_b a = \dfrac{\ln a}{\ln b}$

(a) $\log_2 25$ (b) $\log_6 25$ (c) $\log_6 324$ $\boxed{\text{(d) } 4}$ (e) 6

$\log_2 16 = x$

$2^x = 16$, $\boxed{x = 4}$

33. If a is a non-zero constant, how many real solutions will the equation $\frac{1}{x+a} = \frac{1}{x} + \frac{1}{a}$ have?

(a) 0 (b) 1 (c) 2 (d) 4 (e) Infinitely many

$\frac{1}{x+a} = \frac{a+x}{ax}$

$(x+a)^2 = ax$

$x^2 + 2ax + a^2 = ax$

$x^2 + ax + a^2 = 0$

$\frac{-a \pm \sqrt{a^2 - 4(1)(a^2)}}{2(a)}$

$\frac{-a \pm \sqrt{-3a^2}}{2a}$

no Real solutions

34. Which of the following is the base 2 equivalent of 25.25?

$2^4 + 2^3 + 2^0 = 25$ $2^{-2} = \frac{1}{4} = 0.25$

(a) 1011.01 (b) 10101.1 (c) 10011.01 (d) 11001.01 (e) 110011.1

$(1 \times 2^4) + (1 \times 2^3) + (0 \times 2^2) + (0 \times 2^1) + (1 \times 2^0) + (0 \times 2^{-1}) + (1 \times 2^{-2}) = 25.25$

$11001.01 = 25.25_2$

35. In the circle below, chord $\overline{CD}$ bisects chord $\overline{AB}$ at point E. If the measure of $\overline{CE}$ is nine less than twice the measure of $\overline{BE}$, and the measure of $\overline{ED}$ is six more than the measure of $\overline{EA}$, find the measure of chord $\overline{CD}$.
`Note: All units are in centimeters.`

$BE \times AE = CE \times ED$

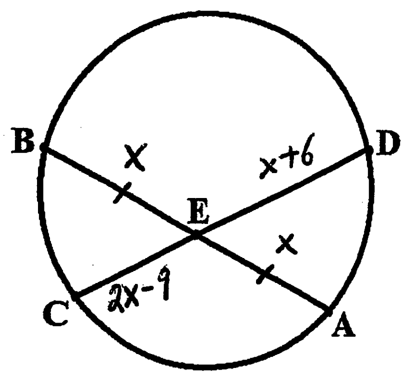
$x \cdot x = (2x-9)(x+6)$

$x^2 = 2x^2 + 12x - 9x - 54$

$0 = x^2 + 3x - 54$

$0 = (x-6)(x+9)$

$\boxed{x = 6}$ $\cancel{x = -9}$



$x = 6$

$CD = CE + ED$

$CD = 2x - 9 + x + 6$

$= 3x - 3$

$CD = 3(6) - 3 = \boxed{15}$

(a) 3cm (b) 6cm (c) 12cm (d) 15cm (e) 24cm

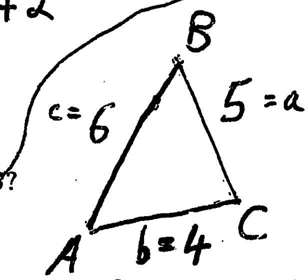
$*a^3+b^3=(a+b)(a^2-ab+b^2)$

36. If $x + \frac{1}{x} = 4$ what is the value of $x^3 + \frac{1}{x^3}$?

(a) 52 (b) 56 (c) 60 (d) 64 (e) 68

$$x^3+\frac{1}{x^3}=\left(x+\frac{1}{x}\right)\left(x^2-1+\frac{1}{x^2}\right)$$
$$=(4)(14-1)$$
$$=4\cdot 13=\boxed{52}$$

$$\left(x+\frac{1}{x}\right)^2=x^2+2+\frac{1}{x^2}$$
$$16=x^2+\frac{1}{x^2}+2$$
$$14=x^2+\frac{1}{x^2}$$



37. Given $\triangle ABC$ has sides 4,5, and 6 and $\angle C$ is the biggest angle, what is $\cos A + \cos B$?

(a) $\frac{1}{8}$ (b) $\frac{2}{3}$ (c) $\frac{21}{16}$ (d) $\frac{6}{5}$ (e) $\frac{3}{2}$

$$\cos A=\frac{b^2+c^2-a^2}{2bc}$$
$$\cos B=\frac{a^2+c^2-b^2}{2ac}$$

$$\cos A=\frac{16+36-25}{2(24)}$$
$$\cos B=\frac{25+36-16}{2(30)}$$

$$\cos A=\frac{27}{48} \quad \cos B=\frac{45}{60}$$
$$\cos A+\cos B$$

38. Find the real solutions of the equation.

$$x + 2\sqrt{x} - 3 = 0$$

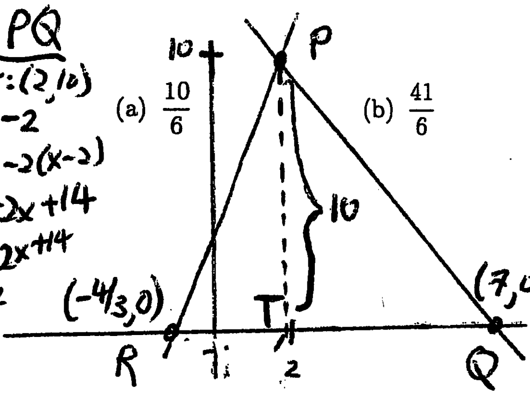
(a) $x = 1$ or $x = -3$
(b) $x = 1$ or $x = -1$
(c) $x = 0$ or $x = 1$
(d) $x = 0$
(e) $x = 1$

$$(\sqrt{x})^2+2\sqrt{x}-3=0$$
$$(\sqrt{x}+3)(\sqrt{x}-1)=0$$

$$\sqrt{x}+3=0 \quad\Big|\quad \sqrt{x}-1=0$$
$$\cancel{\sqrt{x}=-3} \quad\Big|\quad \sqrt{x}=1$$
$$\boxed{x=1}$$

39. A line with slope -2 and a line with slope 3 intersect at point $(2, 10)$. The line with slope -2 intersects the x-axis at point Q and the line with slope 3 intersects the x-axis at point R. What is the area of $\triangle PQR$?

(a) $\frac{10}{6}$ (b) $\frac{41}{6}$ (c) $\frac{21}{3}$ (d) $\frac{41}{3}$ (e) $\frac{125}{3}$



line PQ
point: $(2,10)$
$m=-2$
$y-10=-2(x-2)$
$y=-2x+14$
$0=-2x+14$
$x=7$

line PR
point: $(2,10)$
$m=3$
$y-10=3(x-2)$
$y=3x+4$
$0=3x+4$
$x=-4/3$

$RQ = 4/3 + 7 = 25/3$

$\text{Area} = \frac{1}{2}(RQ)(PT) = \frac{1}{2}\left(\frac{25}{3}\right)(10) = \frac{250}{6} = \boxed{125/3}$

40. Let $f(x)$ be a function on the real numbers with the property that $f(a + b) = f(a) \cdot f(b)$ for all real numbers a and b. If $f(4) = 6$, find $f(10)$.

(a) $6\sqrt{2}$ (b) $6\sqrt{3}$ (c) $36\sqrt{2}$ (d) $36\sqrt{6}$ (e) 108

$f(4) = f(2+2) = f(2) \cdot f(2) = 6$

$f(2)^2 = 6$

$f(2) = \sqrt{6}$

$f(10) = f(8+2) = f(8) \cdot f(2)$

$f(8) = f(4+4) = f(4) \cdot f(4)$

Since $f(4)=6$, $f(4) \cdot f(4) = 36$

$f(10) = f(8) \cdot f(2)$

$= 36 \cdot \sqrt{6}$

$\boxed{f(10) = 36\sqrt{6}}$

ROUND 1

P. What is the base 10 representation of the base 3 number 1101111?

$1 \times 3^6 = 729$
$1 \times 3^5 = 243$
$0 \times 3^4 = 0$
$1 \times 3^3 = 27$
$1 \times 3^2 = 9$
$1 \times 3^1 = 3$
$1 \times 3^0 = 1$

$\boxed{1012}$

1. Rewrite the expression $\log_{216} 6 - \log_2 256$ as a rational number.

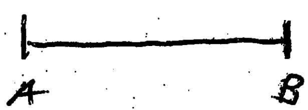
$\log_{216} 6 = x$
$216^x = 6$
$216^{1/3} = 6 \quad x = 1/3$

$\log_2 256 = y$
$2^y = 256$
$y = 8$

$\frac{1}{3} - 8 = \frac{1}{3} - \frac{24}{3} = \boxed{\frac{-23}{3}}$

2. Joe travels from Town A to Town B at a speed of 30 miles per hour. Jacob travels in the opposite direction (from Town B to Town A) at a speed of 45 miles per hour. The distance between Town A and Town B is 450 miles. If they start at the same time, how long does it take for Joe to complete his journey after the two people pass each other ?

Joe 30mph → ← 45mph Jacob



$d = rt$
$450 = (30+45)t$
$6 = t$
$t = 6$ hrs for Joe and Jacob to meet

Joe takes $\frac{450}{30} = 15$ hrs total.
After 6 hrs, Joe still has $15 - 6 = 9$ hrs. to complete journey.

3. What is the coefficient of x^2y^2 when $(x+3y)^4$ is expanded into standard form?

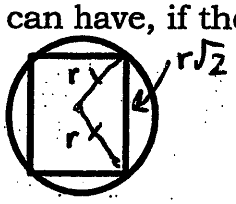
$(a+b)^0$ 1
$(a+b)^1$ 1 1
$(a+b)^2$ 1 2 1
$(a+b)^3$ 1 3 3 1
$(a+b)^4$ 1 4 6 4 1

$6a^2b^2$

$6(x)^2(3y)^4 = 6 \cdot 3^4 x^2 y^2 =$

4. The area of a circle is $n \cdot \pi$, where n is an integer. What is the smallest value that n can have, if the area of the square is greater than 1000?

$A_{circle} = \pi r^2$
$n \cdot \pi = \pi r^2$
$n = r^2$



$A_{square} = (r\sqrt{2})^2 = 2r^2$
$1000 = 2r^2$
$500 = r^2$

$r^2 > 500$
$r^2 \geq 501$
$r^2 = 501$

$\boxed{n = 501}$

5. What is the units digit of 10! + 4! ?

$10 \cdot 9 \cdot 8 \cdot 7 \cdot \dots \cdot 1 + 4 \cdot 3 \cdot 2 \cdot 1$

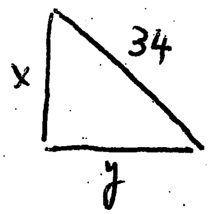
___0 + 24 = 24

$\boxed{\text{units digit} = 4}$

ROUND 2

80

P. What is the perimeter of a right triangle with hypotenuse of 34 and an area of 240?



(-2, 0)

$\left(\frac{29}{10}, \frac{21}{10}\right)$

1. Find all ordered pair solutions to the system of equations below.

$$\begin{cases} 3x-7y+6=0 \\ x^2-y^2=4 \end{cases}$$

199 2. The solution to the linear equation below can be written as a ratio in lowest terms $x=\frac{a}{b}$. Find $a+b$.

$$\frac{5}{28}(x-4)-\frac{2}{7}x+1=\frac{3}{14}x-(5-2x)+\frac{1}{2}$$

$\frac{5x}{28}-\frac{5}{7}-\frac{2}{7}x+1-\frac{3}{14}x+5-2x-\frac{1}{2}$

$\frac{-65x}{28}=\frac{-134}{28}$ $x=\frac{134}{65}$

$134+65=\boxed{199}$

$-y$ 3. Simplify the complex fraction in lowest terms.

$$\frac{\frac{1}{y+1}+\frac{1}{y-1}}{\frac{1}{y+1}-\frac{1}{y-1}}=\frac{\frac{y-1+y+1}{y^2-1}}{\frac{y-1-y-1}{y^2-1}}=\frac{y-1+y+1}{y-2-y}=\frac{2y}{-2}=\boxed{-y}$$

360°

510 4. Let $\boldsymbol{A}$ be the sum of the exterior angles of an octagon and $\boldsymbol{B}$ be an interior angle of a regular dodecagon. What is $\boldsymbol{A+B}$?

$A=360$

$B=\frac{180(n-2)}{n}$ $n=12$, $B=150$

$A+B=150+360=\boxed{510}$

5. Find the remainder when $2x^6-3x^4+40x^3+10x^2+x-21$ is divided by $x+3$.

$$\begin{array}{r|rrrrrrr} -3 & 2 & 0 & -3 & 40 & 10 & 1 & -21 \\ & \downarrow & -6 & 18 & -45 & 15 & -75 & 222 \\ \hline & 2 & -6 & 15 & -5 & 25 & -74 & \boxed{201} \end{array}$$