Review Exercises for Chapter 12

See www.CalcChat.com for worked-out solutions to odd-numbered exercises.

In Exercises 1–4, (a) find the domain of r and (b) determine the values (if any) of t for which the function is continuous.

1. $\mathbf{r}(t) = t\mathbf{i} + \csc t\mathbf{k}$
2. $\mathbf{r}(t) = \sqrt{t}\mathbf{i} + \dfrac{1}{t-4}\mathbf{j} + \mathbf{k}$
3. $\mathbf{r}(t) = \ln t\mathbf{i} + t\mathbf{j} + t\mathbf{k}$
4. $\mathbf{r}(t) = (2t+1)\mathbf{i} + t^2\mathbf{j} + t\mathbf{k}$

In Exercises 5 and 6, evaluate (if possible) the vector-valued function at each given value of t.

5. $\mathbf{r}(t) = (2t+1)\mathbf{i} + t^2\mathbf{j} - \frac{1}{3}t^3\mathbf{k}$
   (a) $\mathbf{r}(0)$ (b) $\mathbf{r}(-2)$ (c) $\mathbf{r}(c-1)$ (d) $\mathbf{r}(1+\Delta t) - \mathbf{r}(1)$
6. $\mathbf{r}(t) = 3\cos t\mathbf{i} + (1 - \sin t)\mathbf{j} - t\mathbf{k}$
   (a) $\mathbf{r}(0)$ (b) $\mathbf{r}\left(\frac{\pi}{2}\right)$ (c) $\mathbf{r}(s-\pi)$ (d) $\mathbf{r}(\pi+\Delta t) - \mathbf{r}(\pi)$

In Exercises 7 and 8, sketch the plane curve represented by the vector-valued function and give the orientation of the curve.

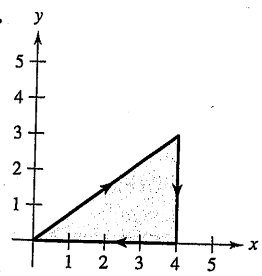
7. $\mathbf{r}(t) = \langle \cos t, 2\sin^2 t\rangle$
8. $\mathbf{r}(t) = \langle t, t/(t-1)\rangle$

In Exercises 9–14, use a computer algebra system to graph the space curve represented by the vector-valued function.

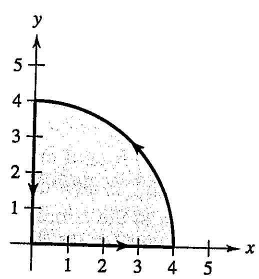
9. $\mathbf{r}(t) = \mathbf{i} + t\mathbf{j} + t^2\mathbf{k}$
10. $\mathbf{r}(t) = 2t\mathbf{i} + t\mathbf{j} + t^2\mathbf{k}$
11. $\mathbf{r}(t) = \langle 1, \sin t, 1\rangle$
12. $\mathbf{r}(t) = \langle 2\cos t, t, 2\sin t\rangle$
13. $\mathbf{r}(t) = \left\langle t, \ln t, \frac{1}{2}t^2\right\rangle$
14. $\mathbf{r}(t) = \left\langle \frac{1}{2}t, \sqrt{t}, \frac{1}{4}t^3\right\rangle$

In Exercises 15 and 16, find vector-valued functions forming the boundaries of the region in the figure.

15.



16.



17. A particle moves on a straight-line path that passes through the points $(-2, -3, 8)$ and $(5, 1, -2)$. Find a vector-valued function for the path. (There are many correct answers.)
18. The outer edge of a spiral staircase is in the shape of a helix of radius 2 meters. The staircase has a height of 2 meters and is three-fourths of one complete revolution from bottom to top. Find a vector-valued function for the helix. (There are many correct answers.)

In Exercises 19 and 20, sketch the space curve represented by the intersection of the surfaces. Use the parameter $x = t$ to find a vector-valued function for the space curve.

19. $z = x^2 + y^2, \quad x + y = 0$
20. $x^2 + z^2 = 4, \quad x - y = 0$

In Exercises 21 and 22, evaluate the limit.

21. $\lim_{t\to 2^-} \left(t^2\mathbf{i} + \sqrt{4-t^2}\mathbf{j} + \mathbf{k}\right)$
22. $\lim_{t\to 0} \left(\dfrac{\sin 2t}{t}\mathbf{i} + e^{-t}\mathbf{j} + e^t\mathbf{k}\right)$

In Exercises 23 and 24, find the following.

(a) $\mathbf{r}'(t)$ (b) $\mathbf{r}''(t)$ (c) $D_t[\mathbf{r}(t)\cdot\mathbf{u}(t)]$
(d) $D_t[\mathbf{u}(t) - 2\mathbf{r}(t)]$ (e) $D_t[\|\mathbf{r}(t)\|],\ t > 0$ (f) $D_t[\mathbf{r}(t)\times\mathbf{u}(t)]$

23. $\mathbf{r}(t) = 3t\mathbf{i} + (t-1)\mathbf{j}, \quad \mathbf{u}(t) = t\mathbf{i} + t^2\mathbf{j} + \frac{2}{3}t^3\mathbf{k}$
24. $\mathbf{r}(t) = \sin t\mathbf{i} + \cos t\mathbf{j} + t\mathbf{k}, \quad \mathbf{u}(t) = \sin t\mathbf{i} + \cos t\mathbf{j} + \dfrac{1}{t}\mathbf{k}$
25. ***Writing*** The x- and y-components of the derivative of the vector-valued function $\mathbf{u}$ are positive at $t = t_0$, and the z-component is negative. Describe the behavior of $\mathbf{u}$ at $t = t_0$.
26. ***Writing*** The x-component of the derivative of the vector-valued function $\mathbf{u}$ is 0 for t in the domain of the function. What does this information imply about the graph of $\mathbf{u}$?

In Exercises 27–30, find the indefinite integral.

27. $\int (\cos t\mathbf{i} + t\cos t\mathbf{j})\,dt$
28. $\int (\ln t\mathbf{i} + t\ln t\mathbf{j} + \mathbf{k})\,dt$
29. $\int \|\cos t\mathbf{i} + \sin t\mathbf{j} + t\mathbf{k}\|\,dt$
30. $\int (t\mathbf{j} + t^2\mathbf{k}) \times (\mathbf{i} + t\mathbf{j} + t\mathbf{k})\,dt$

In Exercises 31 and 32, find r(t) for the given conditions.

31. $\mathbf{r}'(t) = 2t\mathbf{i} + e^t\mathbf{j} + e^{-t}\mathbf{k}, \quad \mathbf{r}(0) = \mathbf{i} + 3\mathbf{j} - 5\mathbf{k}$
32. $\mathbf{r}'(t) = \sec t\mathbf{i} + \tan t\mathbf{j} + t^2\mathbf{k}, \quad \mathbf{r}(0) = 3\mathbf{k}$

In Exercises 33–36, evaluate the definite integral.

33. $\int_{-2}^{2} (3t\mathbf{i} + 2t^2\mathbf{j} - t^3\mathbf{k})\,dt$
34. $\int_0^1 \left(\sqrt{t}\mathbf{j} + t\sin t\mathbf{k}\right)dt$
35. $\int_0^2 (e^{t/2}\mathbf{i} - 3t^2\mathbf{j} - \mathbf{k})\,dt$
36. $\int_{-1}^{1} (t^3\mathbf{i} + \arcsin t\mathbf{j} - t^2\mathbf{k})\,dt$

In Exercises 37 and 38, the position vector r describes the path of an object moving in space. Find the velocity, speed, and acceleration of the object.

37. $\mathbf{r}(t) = \langle \cos^3 t, \sin^3 t, 3t\rangle$
38. $\mathbf{r}(t) = \langle t, -\tan t, e^t\rangle$

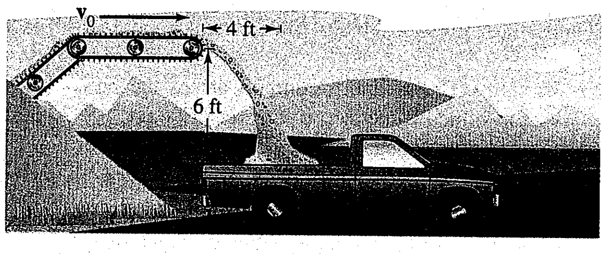
Linear Approximation **In Exercises 39 and 40, find a set of parametric equations for the tangent line to the graph of the vector-valued function at $t = t_0$. Use the equations for the line to approximate $\mathbf{r}(t_0 + 0.1)$.**

39. $\mathbf{r}(t) = \ln(t-3)\mathbf{i} + t^2\mathbf{j} + \frac{1}{2}t\mathbf{k}, \quad t_0 = 4$
40. $\mathbf{r}(t) = 3\cosh t\mathbf{i} + \sinh t\mathbf{j} - 2t\mathbf{k}, \quad t_0 = 0$

Projectile Motion **In Exercises 41–44, use the model for projectile motion, assuming there is no air resistance. [$a(t) = -32$ feet per second per second or $a(t) = -9.8$ meters per second per second.]**

41. A projectile is fired from ground level with an initial velocity of 75 feet per second at an angle of 30° with the horizontal. Find the range of the projectile.

42. The center of a truck bed is 6 feet below and 4 feet horizontally from the end of a horizontal conveyor that is discharging gravel (see figure). Determine the speed ds/dt at which the conveyor belt should be moving so that the gravel falls onto the center of the truck bed.



43. A projectile is fired from ground level at an angle of 20° with the horizontal. The projectile has a range of 80 meters. Find the minimum initial velocity.

44. Use a graphing utility to graph the paths of a projectile if $v_0 = 20$ meters per second, $h = 0$ and (a) $\theta = 30°$, (b) $\theta = 45°$, and (c) $\theta = 60°$. Use the graphs to approximate the maximum height and range of the projectile for each case.

In Exercises 45–52, find the velocity, speed, and acceleration at time t. Then find a · T and a · N at time t.

45. $\mathbf{r}(t) = 5t\mathbf{i}$

46. $\mathbf{r}(t) = (1 + 4t)\mathbf{i} + (2 - 3t)\mathbf{j}$

47. $\mathbf{r}(t) = t\mathbf{i} + \sqrt{t}\mathbf{j}$

48. $\mathbf{r}(t) = 2(t + 1)\mathbf{i} + \dfrac{2}{t + 1}\mathbf{j}$

49. $\mathbf{r}(t) = e^t\mathbf{i} + e^{-t}\mathbf{j}$

50. $\mathbf{r}(t) = t\cos t\,\mathbf{i} + t\sin t\,\mathbf{j}$

51. $\mathbf{r}(t) = t\mathbf{i} + t^2\mathbf{j} + \dfrac{1}{2}t^2\mathbf{k}$

52. $\mathbf{r}(t) = (t - 1)\mathbf{i} + t\mathbf{j} + \dfrac{1}{t}\mathbf{k}$

In Exercises 53 and 54, find a set of parametric equations for the line tangent to the space curve at the given point.

53. $\mathbf{r}(t) = 2\cos t\,\mathbf{i} + 2\sin t\,\mathbf{j} + t\mathbf{k}, \quad t = \dfrac{3\pi}{4}$

54. $\mathbf{r}(t) = t\mathbf{i} + t^2\mathbf{j} + \frac{2}{3}t^3\mathbf{k}, \quad t = 2$

55. ***Satellite Orbit*** Find the speed necessary for a satellite to maintain a circular orbit 600 miles above the surface of Earth.

56. ***Centripetal Force*** An automobile in a circular traffic exchange is traveling at twice the posted speed. By what factor is the centripetal force increased over that which would occur at the posted speed?

In Exercises 57–60, sketch the plane curve and find its length over the given interval.

	Function	*Interval*
57.	$\mathbf{r}(t) = 2t\mathbf{i} - 3t\mathbf{j}$	$[0, 5]$
58.	$\mathbf{r}(t) = t^2\mathbf{i} + 2t\mathbf{k}$	$[0, 3]$
59.	$\mathbf{r}(t) = 10\cos^3 t\,\mathbf{i} + 10\sin^3 t\,\mathbf{j}$	$[0, 2\pi]$
60.	$\mathbf{r}(t) = 10\cos t\,\mathbf{i} + 10\sin t\,\mathbf{j}$	$[0, 2\pi]$

In Exercises 61–64, sketch the space curve and find its length over the given interval.

	Function	*Interval*
61.	$\mathbf{r}(t) = -3t\mathbf{i} + 2t\mathbf{j} + 4t\mathbf{k}$	$[0, 3]$
62.	$\mathbf{r}(t) = t\mathbf{i} + t^2\mathbf{j} + 2t\mathbf{k}$	$[0, 2]$
63.	$\mathbf{r}(t) = \langle 8\cos t, 8\sin t, t\rangle$	$[0, \pi/2]$
64.	$\mathbf{r}(t) = \langle 2(\sin t - t\cos t), 2(\cos t + t\sin t), t\rangle$	$[0, \pi/2]$

In Exercises 65 and 66, use a computer algebra system to find the length of the space curve over the given interval.

65. $\mathbf{r}(t) = \frac{1}{2}t\mathbf{i} + \sin t\,\mathbf{j} + \cos t\,\mathbf{k}, \quad 0 \le t \le \pi$

66. $\mathbf{r}(t) = e^t\sin t\,\mathbf{i} + e^t\cos t\,\mathbf{k}, \quad 0 \le t \le \pi$

In Exercises 67–70, find the curvature K of the curve.

67. $\mathbf{r}(t) = 3t\mathbf{i} + 2t\mathbf{j}$

68. $\mathbf{r}(t) = 2\sqrt{t}\mathbf{i} + 3t\mathbf{j}$

69. $\mathbf{r}(t) = 2t\mathbf{i} + \frac{1}{2}t^2\mathbf{j} + t^2\mathbf{k}$

70. $\mathbf{r}(t) = 2t\mathbf{i} + 5\cos t\mathbf{j} + 5\sin t\mathbf{k}$

In Exercises 71–74, find the curvature and radius of curvature of the plane curve at the given value of x.

71. $y = \frac{1}{2}x^2 + 2, \quad x = 4$

72. $y = e^{-x/2}, \quad x = 0$

73. $y = \ln x, \quad x = 1$

74. $y = \tan x, \quad x = \dfrac{\pi}{4}$

75. ***Writing*** A civil engineer designs a highway as shown in the figure. BC is an arc of the circle. AB and CD are straight lines tangent to the circular arc. Criticize the design.

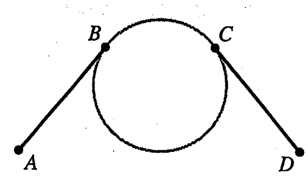


Figure for 75

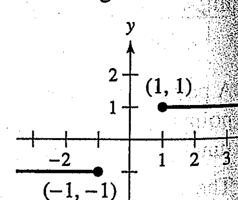


Figure for 76

76. A line segment extends horizontally to the left from the point $(-1, -1)$. Another line segment extends horizontally to the right from the point $(1, 1)$, as shown in the figure. Find a curve of the form

$$y = ax^5 + bx^3 + cx$$

that connects the points $(-1, -1)$ and $(1, 1)$ so that the slope and curvature of the curve are zero at the endpoints.