Additional Limits Topic FRQ Practice

1) Step through Continuity Conditions and find the value of '**a**' such that the f(x) is continuous

$$f(x) = \begin{cases} 3x^3, & x \le 1 \\ ax + 5, & x > 1 \end{cases}$$

2)

The function f is defined on all the reals such that $f(x) = \begin{cases} x^2 + kx - 3 & \text{for } x \le 1 \\ 3x + b & \text{for } x > 1. \end{cases}$

For which of the following values of k and b will the function f be both continuous and differentiable on its entire domain?

Particle Motion Practice Problem

3) Two particles move along the x-axis. For $0 \le t \le 8$, the position of particle P at time t is given by $x_P(t) = \ln\left(t^2 - 2t + 10\right)$, while the velocity of particle Q at time t is given by $v_Q(t) = t^2 - 8t + 15$. Particle Q is at position $x = 5$ at time $t = 0$.

(a) For $0 \le t \le 8$, when is particle P moving to the left?

(b) For $0 \le t \le 8$, find all times t during which the two particles travel in the same direction.

(c) Find the acceleration of particle Q at time $t = 2$. Is the speed of particle Q increasing, decreasing, or neither at time $t = 2$? Explain your reasoning.

Key

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Continuity conditions:

i) $f(c)$ exists

ii) $\lim_{x\to c} f(x)$ exists $\left[\lim_{x\to c^-} f(x) = \lim_{x\to c^+} f(x)\right]$

iii) $f(c) = \lim_{x\to c} f(x)$

i) $f(1) = 3(1)^3 = 3$

ii) $\lim_{x\to 1^-} 3x^3 = \boxed{3}$ $\quad \lim_{x\to 1^+} ax+5 = a(1)+5$

$3 = a + 5$

$\boxed{-2 = a}$ ✓

2)

$f'(x) = \begin{cases} 2x+k & , x \le 1 \\ 3 & , x > 1 \end{cases}$

The function f is defined on all the reals such that $f(x) = \begin{cases} x^2 + kx - 3 & \text{for } x \le 1 \\ 3x + b & \text{for } x > 1. \end{cases}$

For which of the following values of k and b will the function f be both continuous and differentiable on its entire domain?

* set equations equal (at $x=1$) b/c f(x) is continuous and piecewise function will share same y-value at $x=1$

at $x=1$:

$x^2+kx-3 = 3x+b$

$(1)^2+k(1)-3 = 3(1)+b$

$1+k-3 = 3+b$

$k-2 = 3+b$

$k = 5+b$

$1 = 5+b$

$\boxed{-4 = b}$

* set derivatives equal b/c f(x) is differentiable, and so piecewise function must share the same slopes at $x=1$

at $x=1$:

$2x+k = 3$

$2(1)+k = 3$

$2+k = 3$

$\boxed{k=1}$

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a) particle P:

$* \ \frac{d}{dx}\ln u = \frac{u'}{u}$

$x_1(t) = \ln(t^2 - 2t + 10)$

$x_1'(t) = \dfrac{2t-2}{t^2-2t+10}$

$2t - 2 = 0$	$t^2 - 2t + 10 = 0$
$t = 1$	$\boxed{\text{none}}$
	$t^2 - 2t + 10 > 0$

$v_1(t)$: $-$ on $(0, 1)$, $+$ on $(1, 8)$

0   1   8

particle P moving left $0 \le t < 1$

or $[0, 1)$

b) particle Q:

$v_2(t) = t^2 - 8t + 15$

$0 = (t-3)(t-5)$

$t = 3, 5$

$v_2(t)$: $+$ on $(0, 3)$, $-$ on $(3, 5)$, $+$ on $(5, 8)$ (particle Q)

0   3   5   8

$v_1(t)$: $-$ on $(0, 1)$, $+$ on $(1, 8)$ (particle P)

0   1   8

Both particles move in same direction in intervals $(1, 3)$ and $(5, 8]$ b/c velocities of both particles have same signs

c) particle Q:

$v_2(t) = t^2 - 8t + 15$

$a_2(t) = 2t - 8$

$a_2(2) = 4 - 8 = -4 < 0$

$v_2(2) = 2^2 - 8(2) + 15 = 3 > 0$

At $t = 2$, since $a_2(t)$ and $v_2(t)$ have opposite signs, the speed of particle is decreasing.