

### Additional Limits Topic FRQ Practice

- 1) Step through Continuity Conditions and find the value of 'a' such that the  $f(x)$  is continuous

$$f(x) = \begin{cases} 3x^3, & x \leq 1 \\ ax + 5, & x > 1 \end{cases}$$

2)

The function  $f$  is defined on all the reals such that  $f(x) = \begin{cases} x^2 + kx - 3 & \text{for } x \leq 1 \\ 3x + b & \text{for } x > 1. \end{cases}$

For which of the following values of  $k$  and  $b$  will the function  $f$  be both continuous and differentiable on its entire domain?

**Particle Motion Practice Problem**

3) Two particles move along the  $x$ -axis. For  $0 \leq t \leq 8$ , the position of particle  $P$  at time  $t$  is given by  $x_P(t) = \ln(t^2 - 2t + 10)$ , while the velocity of particle  $Q$  at time  $t$  is given by  $v_Q(t) = t^2 - 8t + 15$ .

Particle  $Q$  is at position  $x = 5$  at time  $t = 0$ .

- (a) For  $0 \leq t \leq 8$ , when is particle  $P$  moving to the left?
- (b) For  $0 \leq t \leq 8$ , find all times  $t$  during which the two particles travel in the same direction.
- (c) Find the acceleration of particle  $Q$  at time  $t = 2$ . Is the speed of particle  $Q$  increasing, decreasing, or neither at time  $t = 2$ ? Explain your reasoning.

Key

## Additional Limits Topic FRQ Practice

- 1) Step through Continuity Conditions and find the value of 'a' such that the  $f(x)$  is continuous

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Continuity conditions:

- i)  $f(c)$  exists
- ii)  $\lim_{x \rightarrow c} f(x)$  exists  $\left[ \lim_{x \rightarrow c^-} f(x) = \lim_{x \rightarrow c^+} f(x) \right]$
- iii)  $f(c) = \lim_{x \rightarrow c} f(x)$

$$i) f(1) = 3(1)^3 = 3$$

$$ii) \lim_{x \rightarrow 1^-} 3x^3 = \boxed{3} \quad \lim_{x \rightarrow 1^+} ax + 5 = a(1) + 5$$

$$3 = a + 5$$

$$\boxed{-2 = a} \quad \checkmark$$

2)

$$f'(x) = \begin{cases} 2x + k, & x \leq 1 \\ 3, & x > 1 \end{cases}$$

The function  $f$  is defined on all the reals such that  $f(x) = \begin{cases} x^2 + kx - 3 & \text{for } x \leq 1 \\ 3x + b & \text{for } x > 1. \end{cases}$

For which of the following values of  $k$  and  $b$  will the function  $f$  be both continuous and differentiable on its entire domain?

\* set equations equal (at  $x=1$ ) b/c  $f(x)$  is continuous and piecewise function will share same y-value at  $x=1$

at  $x=1$ :

$$x^2 + kx - 3 = 3x + b$$

$$(1)^2 + k(1) - 3 = 3(1) + b$$

$$1 + k - 3 = 3 + b$$

$$k - 2 = 3 + b$$

$$k = 5 + b$$

$$1 = 5 + b$$

$$\boxed{-4 = b}$$

\* set derivatives equal b/c  $f(x)$  is differentiable, and so piecewise function must share the same slopes at  $x=1$

at  $x=1$ :

$$2x + k = 3$$

$$2(1) + k = 3$$

$$2 + k = 3$$

$$\boxed{k = 1}$$

# Particle Motion Practice Problem

Key

- 3) Two particles move along the  $x$ -axis. For  $0 \leq t \leq 8$ , the position of particle  $P$  at time  $t$  is given by  $x_P(t) = \ln(t^2 - 2t + 10)$ , while the velocity of particle  $Q$  at time  $t$  is given by  $v_Q(t) = t^2 - 8t + 15$ . Particle  $Q$  is at position  $x = 5$  at time  $t = 0$ .

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 (b) For  $0 \leq t \leq 8$ , find all times  $t$  during which the two particles travel in the same direction.  
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a) particle P:

$$x_P(t) = \ln(t^2 - 2t + 10)$$

$$x'_P(t) = \frac{2t - 2}{t^2 - 2t + 10}$$

$$* \frac{d}{dx} \ln u = \frac{u'}{u}$$

$$2t - 2 = 0$$

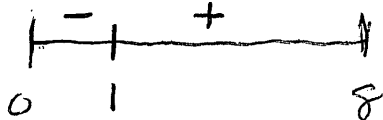
$$t = 1$$

$$t^2 - 2t + 10 = 0$$

none

$$t^2 - 2t + 10 > 0$$

$v_P(t)$



particle P moving left  $0 \leq t < 1$   
 or  $[0, 1)$

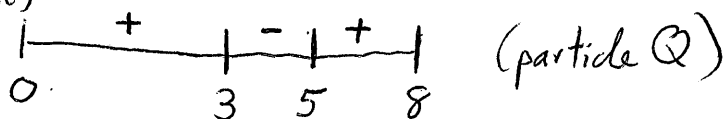
b) particle Q:

$$v_Q(t) = t^2 - 8t + 15$$

$$0 = (t - 3)(t - 5)$$

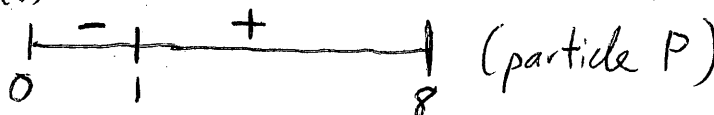
$$t = 3, 5$$

$v_Q(t)$



(particle Q)

$v_P(t)$



(particle P)

Both particles move in same direction in intervals  $(1, 3)$  and  $(5, 8]$   
 b/c velocities of both particles have same signs

c) particle Q:

$$v_Q(t) = t^2 - 8t + 15$$

$$a_Q(t) = 2t - 8$$

$$a_Q(2) = 4 - 8 = -4 < 0$$

$$v_Q(2) = 2^2 - 8(2) + 15 = 3 > 0$$

At  $t = 2$ , since  $a_Q(t)$  and  $v_Q(t)$  have opposite signs, the speed of particle is decreasing.