Limits - Piecewise Practice Problem

1)

$$f(x) = \begin{cases} \frac{x+2}{x^2-4}, & x < -4 \\ x+3, & -4 \leq x < 3 \\ \frac{x^3}{x-5}, & x \geq 3 \end{cases}$$

a) $\lim_{x \to -\infty} f(x)$

b) $\lim_{x \to -4^-} f(x)$

c) $\lim_{x \to -4^+} f(x)$

d) $\lim_{x \to 4} f(x)$

e) $\lim_{x \to 5^+} f(x)$

f) $\lim_{x \to \infty} f(x)$

g) $\lim_{x \to -4} f(x)$

h) $f(-4)$

Limits Piecewise problems

Use continuity conditions to determine if continuous. If discontinuous, determine if removable/nonremovable

2) $f(x) = \begin{cases} x^2 - 3, & x \geq 4 \\ \frac{x+1}{x^2+1}, & x < 4 \end{cases}$

3) $g(x) = \begin{cases} \frac{x^2-1}{x+1}, & x \neq -1 \\ 3, & x = -1 \end{cases}$

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h) $f(-4)$

a) $\lim_{x \to -\infty} \frac{x+2}{x^2-4} = 0$

b) $\lim_{x \to -4^-} \frac{x+2}{x^2-4} = \frac{-2}{12} = -\frac{1}{6}$

c) $\lim_{x \to -4^+} x+3 = -1$

d) $\lim_{x \to 4} \frac{x^3}{x-5} = \frac{64}{-1} = -64$

e) $\lim_{x \to 5^+} \frac{x^3}{x-5} = \frac{125}{0} \to DNE \nearrow +\infty \searrow -\infty$

$\frac{(5.1)^3}{5.1-5} = \frac{+}{+} = \boxed{+\infty}$

f) $\lim_{x \to \infty} \frac{x^3}{x-5} = \boxed{+\infty}$

g) $\lim_{x \to -4} f(x) = \boxed{DNE}$

↳ $\lim_{x \to -4^-} f(x) = -1/6$

↳ $\lim_{x \to -4^+} f(x) = -1$

$\lim_{x \to -4^-} f(x) \neq \lim_{x \to -4^+} f(x)$

h) $f(-4) = \boxed{-1}$

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Use continuity conditions to determine if continuous. If discontinuous, determine if removable/nonremovable

2) $f(x) = \begin{cases} x^2-3, & x \geq 4 \\ \frac{x+1}{x^2+1}, & x < 4 \end{cases}$

i) $f(4) = 16-3 = 13$

ii) $\lim_{x\to 4^-} \frac{x+1}{x^2+1} = \frac{5}{17}$

$\lim_{x\to 4^+} x^2-3 = 13$

Since $\lim_{x\to 4^-} f(x) \neq \lim_{x\to 4^+} f(x)$

$\lim_{x\to 4} f(x) = DNE$,

Nonremovable discontinuity at $x=4$

3) $g(x) = \begin{cases} \frac{x^2-1}{x+1}, & x \neq -1 \\ 3, & x = -1 \end{cases}$

i) $f(-1) = 3$

ii) $\lim_{x\to -1} \frac{x^2-1}{x+1} = \lim_{x\to -1} \frac{(x+1)(x-1)}{x+1} = -2$

iii) $f(-1) \neq \lim_{x\to -1} f(x)$

Removable discontinuity at $x=-1$