7

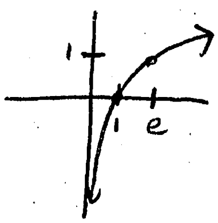
A.P. Calculus AB Test Logarithms/Exponentials

Key A

All questions are worth 5 points each unless otherwise stated. Name Period

ALL WORK MUST BE SHOWN ON ALL PROBLEMS TO RECEIVE FULL CREDIT.

NO CALCULATORS.



1. Let f be the function defined by $f(x) = -\frac{\ln x}{x}$ for all real numbers x. (24 pts)

2 a) (2 points) State the domain of $f(x)$. $(0, \infty)$

8 b) (8 points) Find each relative maximum and relative minimum. Write your answer(s) as ordered pairs and clearly label each answer as either a relative max or min. **Justify each answer.**

$\log_e x = 1$

$e^1 = x$

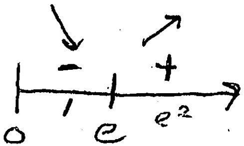
$$f'(x) = \frac{\left(\frac{-1}{x}\right)(x) - (-\ln x)(1)}{x^2} = \frac{-1+\ln x}{x^2}$$

$-1+\ln x = 0$

$\ln x = 1$

$\log_e x = 1$

$x = e^1$



Rel min at $\left(e, \frac{-1}{e}\right)$ b/c $f'(x)$ changes from - to +

c) (2 points) State the range of f.

$\left[-\frac{1}{e}, \infty\right)$

8 d) (8 points) Find each point of inflection on the graph of f. Write your answer(s) as ordered pairs and **justify each answer.**

$$f''(x) = \frac{\frac{1}{x}(x^2) - (-1+\ln x)(2x)}{x^4}$$

$$= \frac{\cancel{x}+2x - 2x\ln x}{x^4}$$

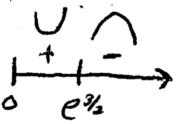
$$f''(x) = \frac{3x-2x\ln x}{x^4} = \frac{x(3-2\ln x)}{x^4}$$

$3-2\ln x = 0$

$-2\ln x = -3$

$\ln x = 3/2$

$e^{3/2} = x$

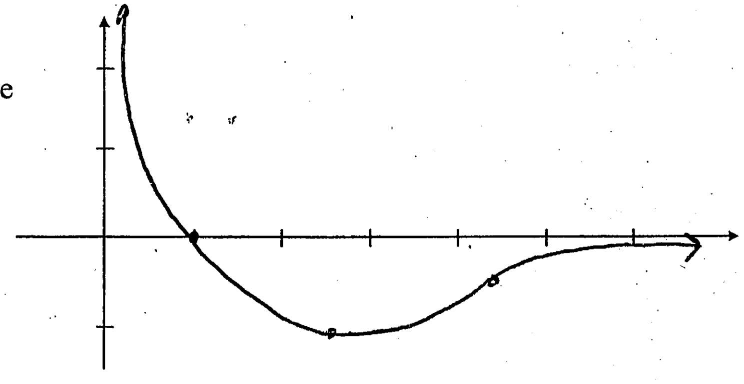


POI at $\left(e^{3/2}, \frac{-3/2}{e^{3/2}}\right) = \left(e^{3/2}, \frac{-3}{2e^{3/2}}\right)$

2 e) (2 points) Find $\lim_{x\to 0^+} f(x)$ and $\lim_{x\to\infty} f(x) \boxed{= 0}$

$\lim_{x\to 0^+} f(x) \boxed{= +\infty}$

5 f) (4 points) Using the results found in parts a – e sketch the graph of f in the xy-plane provided. (Hint: for graphing purposes, let $e \approx 3$ and $e^{\frac{3}{2}} \approx \frac{9}{2}$)



2. The position of a particle moving along the x-axis is given by $x(t) = e^t (t^2 - 3)$ (20 pts)

a) (4 points) Find $v(t)$

$$v(t) = e^t(t^2-3) + e^t(2t)$$

$$= e^t(t^2+2t-3)$$

$$= e^t(t+3)(t-1)$$

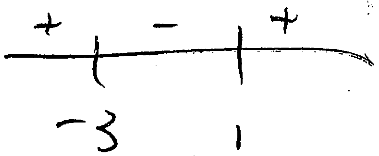
$$t = 1, -3$$

b) (4 points) Find $a(t)$

$$a(t) = e^t(t^2+2t-3) + e^t(2t+2)$$

$$= e^t(t^2+4t-1)$$

c) (3 points) For what values of t is the particle moving to the left?



$$\boxed{(-3, 1)}$$

d) (3 points) For what values of t is the particle located at the origin?

$$x(t) = 0 = e^t(t^2-3)$$

$$t^2 - 3 = 0$$

$$t = \pm\sqrt{3}$$

e) (3 points) At $t = 0$, is the velocity of the particle increasing or decreasing? Justify your answer

$$a(0) = e^0(0^2-3) < 0$$

velocity is decreasing b/c $a(t) < 0$

f) (3 points) At $t = 0$, is the speed of the particle increasing or decreasing? Justify your answer

$a(0) < 0$, $v(0) < 0$

Since $v(t)$ and $a(t)$ have same signs, speed is increasing

Find $\frac{dy}{dx}$ for $y = \ln\left(x\,\sqrt[5]{x - 3x^4}\right)$

$y = \ln x + \ln(x - 3x^4)^{1/5}$

$y = \ln x + \frac{1}{5}\ln(x - 3x^4)$

$y' = \frac{1}{x} + \frac{1}{5} \cdot \frac{1 - 12x^3}{x - 3x^4}$

4. Find $\frac{dy}{dx}$ if $\ln\left(\frac{x}{y}\right) = xy$ (Simplify expression with no complex fraction)

$\ln x - \ln y = xy$

$\frac{1}{x} - \frac{1}{y}\left(\frac{dy}{dx}\right) = 1y + x\left(\frac{dy}{dx}\right)$

$\frac{dy}{dx}\left(-\frac{1}{y} - x\right) = y - \frac{1}{x}$

$\frac{dy}{dx} = \frac{y - \frac{1}{x}}{-\frac{1}{y} - x} \cdot \frac{xy}{xy} = \boxed{\frac{xy^2 - y}{-x - x^2y}}$

5. Find $\frac{dy}{dx}$ if $y = x^2\, 3^{\sqrt[5]{4-x}}$

$y' = 2x \cdot 3^{(4-x)^{1/5}} + x^2 \cdot \ln 3 \cdot 3^{(4-x)^{1/5}} \cdot \frac{1}{5}(4-x)^{-4/5}(-1)$

$\boxed{y' = 2x \cdot 3^{(4-x)^{1/5}} - \frac{x^2 \ln 3\left(3^{(4-x)^{1/5}}\right)}{5(4-x)^{4/5}}}$

6. Find $\frac{dy}{dx}$ if $\sqrt[6]{(4 - 5x^3)^x}$

$y = (4 - 5x^3)^{x/6}$

$\ln y = \ln(4 - 5x^3)^{x/6}$

$\ln y = \left(\frac{x}{6}\right)\left(\ln(4 - 5x^3)\right)$

$\frac{1}{y}\left(\frac{dy}{dx}\right) = \frac{1}{6} \cdot \ln(4 - 5x^3) + \frac{x}{6} \cdot \frac{-15x^2}{4 - 5x^3}$

$\frac{dy}{dx} = y \cdot \frac{\ln(4 - 5x^3)}{6} - \frac{\overset{5}{\cancel{15}}x^3}{2\cancel{6}(4 - 5x^3)}$

$\frac{dy}{dx} = \sqrt[6]{(4 - 5x^3)^x}\left[\frac{\ln(4 - 5x^3)}{6} - \frac{5x^3}{2(4 - 5x^3)}\right]$

7. Find $\frac{d}{dx}\left[f^{-1}(5)\right]$ if $f(x) = x^3 - 3x^2 + 5x + 2$ and $f^{-1}(x)$ is the inverse function of $f(x)$.

$f(1) = 5$	$(f^{-1})(5) = 1$
$f'(1) = 2$	$(f^{-1})'(5) = \frac{1}{2}$

$\boxed{\frac{1}{2}}$