A.P. Calculus AB **Test Logarithms/Exponentials**

All questions are worth 6 points each unless otherwise stated. Name Key B Period

ALL WORK MUST BE SHOWN ON FREE RESPONSE QUESTIONS TO RECEIVE CREDIT.

NO CALCULATORS.

1. Let f be the function defined by $f(x) = \frac{\ln x}{x^2}$ for all real numbers x. (24 pts)

a) (2 points) State the domain of $f(x)$. $(0, \infty)$

b) (8 points) Find each relative maximum and relative minimum. Write your answer(s) as ordered pairs and clearly label each answer as either a relative max or min. **Justify each answer.**

$$f'(x) = \frac{(\frac{1}{x})(x^2) - (\ln x)(2x)}{x^4} = \frac{x - 2x\ln x}{x^4} = \frac{1-2\ln x}{x^3}$$

$1 = 2\ln x$

$\ln x = 1/2$

$e^{1/2} = x$

$0 \quad + \quad e^{1/2} \quad -$

Rel max at $(e^{1/2}, \frac{1}{2e})$ b/c $f'(x)$ changes from + to −

c) (2 points) State the range of f.

$(-\infty, \frac{1}{2e})$

d) 6 points) Find each point of inflection on the graph of f. Write your answer(s) as ordered pairs and justify each answer.

$$f''(x) = \frac{-2(\frac{1}{x})[x^3] - (1-2\ln x)(3x^2)}{x^6} = \frac{-2x^2 - 3x^2 + 6x^2\ln x}{x^6}$$

$$\boxed{f''(x) = \frac{-5+6\ln x}{x^4}}$$

$-5 + 6\ln x = 0$

$\ln x = 5/6$

$e^{5/6} = x$

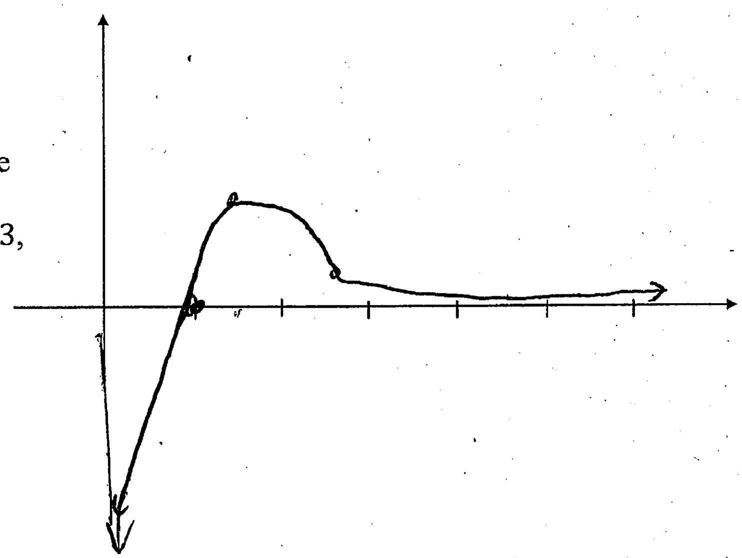
$0 \quad - \quad e^{5/6} \quad +$

POI at $(e^{5/6}, \frac{5/6}{(e^{5/6})^2})$ b/c $f''(x)$ change signs

e) (2 points) Find $\lim_{x \to 0^+} f(x)$ and $\lim_{x \to \infty} f(x)$

$\lim_{x \to 0^+} \frac{\ln x}{x^2} = -\infty \qquad \lim_{x \to \infty} f(x) = 0$

f) (4 points) Using the results found in parts a – e sketch the graph of f in the xy-plane provided.
(Hint: for graphing purposes, let $e \approx 3$, $e^{\frac{1}{2}} \approx 1.5$, $e^{\frac{5}{6}} \approx 2.5$)



2. The position of a particle moving along the x-axis is given by $x(t) = e^t (t^2 - 8)$ (20 pts)

a) (4 points) Find $v(t)$

$v(t) = e^t(t^2-8) + e^t(2t)$

$= \boxed{e^t(t^2+2t-8) = e^t(t+4)(t-2)}$

b) (4 points) Find $a(t)$

$a(t) = e^t(t^2+2t-8) + e^t(2t+2)$

$\boxed{a(t) = e^t(t^2+4t-6)}$

c) (3 points) For what value of t is the particle moving to the right?

$v(t) = 0$ at $t = 2, -4$

+   −   +

−4   2

$(-\infty, -4) \cup (2, \infty)$

d) (3 points) For what values of t is the particle located at the origin?

$x(t) = 0 = e^t(t^2-8)$

$\boxed{t = \pm\sqrt{8}}$

e) (3 points) At $t = 3$, is the velocity of the particle increasing or decreasing? Justify your answer

$a(3) = e^3(3^2+12-6) > 0$

velocity increasing b/c $a(t) > 0$

f) (3 points) At $t = 3$, is the speed of the particle increasing or decreasing? Justify your answer

$v(3) > 0$, $a(3) > 0$

Since $v(t)$ and $a(t)$ have same signs, speed is increasing.

4. Find $\frac{dy}{dx}$ for $y = \ln\left(\frac{4-x}{\sqrt[3]{x-5x^4}}\right)^4$ $4\ln(4-x) - 4\ln(x-5x^4)^{1/3}$

$y = 4\ln(4-x) - \frac{4}{3}\ln(x-5x^4)$

$y' = 4\left(\frac{-1}{4-x}\right) - \frac{4}{3}\left(\frac{1-20x^3}{x-5x^4}\right)$

5. Find $\frac{dy}{dx}$ if $\ln\left(\frac{y}{x}\right) = xy$ (Simplify expression with no complex fraction)

$\ln y - \ln x = xy$

$\frac{1}{y}\frac{dy}{dx} - \frac{1}{x} = 1y + x\left(\frac{dy}{dx}\right)$

$\frac{dy}{dx}\left(\frac{1}{y}\right) - x\left(\frac{dy}{dx}\right) = y + \frac{1}{x}$

$\frac{dy}{dx} = \frac{y + \frac{1}{x}}{\frac{1}{y} - x} \cdot \frac{xy}{xy}$

$\boxed{\frac{dy}{dx} = \frac{xy^2 + y}{x - x^2y}}$

6. Find $\frac{dy}{dx}$ if $y = x^3 4^{\sqrt[5]{2-3x}}$

$y = x^3 \cdot 4^{(2-3x)^{1/5}}$

$y' = 3x^2 \cdot 4^{(2-3x)^{1/5}} + x^3 \cdot \ln 4 \cdot 4^{(2-3x)^{1/5}} \cdot \frac{1}{5}(2-3x)^{-4/5}(-3)$

7. Find $\frac{dy}{dx}$ if $\sqrt[5]{(7-2x^4)^x} = y$

$y = (7-2x^4)^{x/5}$

$\ln y = \frac{x}{5}\ln(7-2x^4)$

$\frac{1}{y}\frac{dy}{dx} = \frac{1}{5}\ln(7-2x^4) + \frac{x}{5} \cdot \frac{-8x^3}{7-2x^4}$

$\frac{dy}{dx} = \sqrt[5]{(7-2x^4)^x}\left[\frac{\ln(7-2x^4)}{5} - \frac{8x^4}{5(7-2x^4)}\right]$

8. Find $\frac{d}{dx}\left[f^{-1}(8)\right]$ if $f(x) = \boxed{x^3 - 3x^2 + 5x + 2}$ and $f^{-1}(x)$ is the inverse function of $f(x)$.

$f(2) = 8$

$(f^{-1})(8) = 2$

$f'(2) = 5$

$(f^{-1})'(8) = \boxed{\frac{1}{5}}$

$f'(x) = 3x^2 - 6x + 5$

$f'(2) = 12 - 12 + 5$

$= 5$

$8 = x^3 - 3x^2 + 5x + 2$
$-8 \qquad\qquad\qquad -8$

$0 = x^3 - 3x^2 + 5x - 6$

$8 - 12 + 10 - 6$ $\quad 0, 1, 5$

$-4 + 10 - 6 = 0$