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Implicit Differentiation In Exercises 63 and 64, use implicit differentiation to find dy/dx.

63. $\underbrace{x}_{f}\underbrace{e^y}_{g} - 10x + 3y = 0$

$\underbrace{1}_{f'} \cdot \underbrace{e^y}_{g} + \underbrace{x}_{f} \cdot \underbrace{e^y\left(\frac{dy}{dx}\right)}_{g'} - 10 + 3\left(\frac{dy}{dx}\right) = 0$

$xe^y\left(\frac{dy}{dx}\right) + 3\left(\frac{dy}{dx}\right) = 10 - e^y$

$\frac{dy}{dx}\left(xe^y + 3\right) = 10 - e^y$

$\boxed{\frac{dy}{dx} = \frac{10 - e^y}{xe^y + 3}}$

64. $e^{xy} + x^2 - y^2 = 10$

$e^{xy}\left[1y + x\left(\frac{dy}{dx}\right)\right] + 2x - 2y\left(\frac{dy}{dx}\right) = 0$

$xe^{xy}\left(\frac{dy}{dx}\right) + ye^{xy} + 2x - 2y\left(\frac{dy}{dx}\right) = 0$

$\frac{dy}{dx}\left(xe^{xy} - 2y\right) = -2x - ye^{xy}$

$\boxed{\frac{dy}{dx} = \frac{-2x - ye^{xy}}{xe^{xy} - 2y}}$

Finding the Equation of a Tangent Line In Exercises 65 and 66, find an equation of the tangent line to the graph of the function at the given point.

65. $xe^y + ye^x = 1, \quad (0, 1)$

$1 \cdot e^y + x \cdot e^y\left(\frac{dy}{dx}\right) + \frac{dy}{dx}e^x + ye^x = 0$

$1e^1 + 0e^1\left(\frac{dy}{dx}\right) + \frac{dy}{dx}e^0 + 1e^0 = 0$

$e + \frac{dy}{dx} + 1 = 0 \quad \frac{dy}{dx} = -e - 1$

$\boxed{y - 1 = (-1 - e)(x - 0)}$

66. $1 + \ln xy = e^{x-y}, \quad (1, 1)$

$1 + \ln x + \ln y = e^{x-y}$

$0 + \frac{1}{x} + \frac{1}{y}\left(\frac{dy}{dx}\right) = e^{x-y}\left[1 - \frac{dy}{dx}\right]$

$1 + 1\left(\frac{dy}{dx}\right) = e^0\left[1 - \frac{dy}{dx}\right]$

$1 + \frac{dy}{dx} = 1 - \frac{dy}{dx}$

$2\frac{dy}{dx} = 0$

$\frac{dy}{dx} = 0$

point $(1, 1)$

slope: $m = 0$

$y - 1 = 0(x - 1)$

$\boxed{y = 1}$

63) $\underbrace{x}_{f}\underbrace{e^y}_{g} - 10x + 3y = 0$

$\frac{d}{dx}e^u = e^u \cdot u'$

$\cancel{ne} \cdot u^u \cdot u'$

$\underbrace{(1)}_{f'} \cdot \underbrace{e^y}_{g} + \underbrace{x \cdot}_{f} \underbrace{e^y\left(\frac{dy}{dx}\right)}_{g'} - 10 + 3\left(\frac{dy}{dx}\right) = 0$

$\frac{dy}{dx}\left(xe^y + 3\right) = 10 - e^y$

$$\boxed{\frac{dy}{dx} = \frac{10 - e^y}{xe^y + 3}}$$

64) $e^{xy} + x^2 - y^2 = 10$

$\frac{d}{dx}e^u = e^u \cdot u'$

$e^{xy} \cdot \left[\underbrace{1}_{f'} \cdot \underbrace{y}_{g} + \underbrace{x}_{f} \cdot \underbrace{\left(\frac{dy}{dx}\right)}_{g'}\right] + 2x - 2y\left(\frac{dy}{dx}\right) = 0$

$ye^{xy} + xe^{xy}\left(\frac{dy}{dx}\right) + 2x - 2y\left(\frac{dy}{dx}\right) = 0$

$\frac{dy}{dx}\left[xe^{xy} - 2y\right] = -2x - ye^{xy}$

$$\boxed{\frac{dy}{dx} = \frac{-2x - ye^{xy}}{xe^{xy} - 2y}}$$

65) $\underbrace{x}_{f}\underbrace{e^y}_{g} + \underbrace{y}_{f}\underbrace{e^x}_{g} = 1 \qquad (0,1)$ Find tangent line equation

$$\underbrace{1}_{f'}\cdot\underbrace{e^y}_{g} + \underbrace{x}_{f}\cdot\underbrace{e^y\left(\frac{dy}{dx}\right)}_{g'} + \underbrace{\frac{dy}{dx}}_{f'}\cdot\underbrace{e^x}_{g} + \underbrace{y}_{f}\cdot\underbrace{e^x}_{g'} = 0$$

$$1e^1 + \cancel{0e^1\left(\frac{dy}{dx}\right)} + \frac{dy}{dx}e^0 + 1e^0 = 0$$

$$e + \frac{dy}{dx} + 1 = 0$$

$$\frac{dy}{dx} = -e-1$$

point: $(0,1)$

slope: $m = -e-1$

$$\boxed{y-1 = (-e-1)(x-0)}$$

66) $1 + \ln(xy) = e^{x-y} \qquad (1,1)$ Find tangent line equation

$$1 + \ln x + \ln y = e^{x-y}$$

$$0 + \frac{1}{x} + \frac{1}{y}\left(\frac{dy}{dx}\right) = e^{x-y}\cdot\left(1 - \frac{dy}{dx}\right)$$

$$\frac{1}{1} + \frac{1}{1}\left(\frac{dy}{dx}\right) = e^{1-1}\cdot\left(1 - 1\left(\frac{dy}{dx}\right)\right)$$

$$1 + 1\left(\frac{dy}{dx}\right) = \cancel{e^0}\left(1 - 1\left(\frac{dy}{dx}\right)\right)$$

$$1 + 1\left(\frac{dy}{dx}\right) = 1 - 1\left(\frac{dy}{dx}\right)$$

$$2\left(\frac{dy}{dx}\right) = 1 - 1$$

$$\frac{dy}{dx} = \frac{0}{2} = 0$$

point: $(1,1)$

slope: $m = 0$

$$\boxed{y-1 = 0(x-1)}$$

$$\boxed{y = 1}$$

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L.4-L.5 Quiz Review Derivatives of Exponential function e^x and a^x

Solution Key

Find $\frac{dy}{dx}$.

1. $f(x) = ln\frac{(3+4x)^5}{\sqrt[4]{1-3x}} = \ln(3+4x)^5 - \ln(1-3x)^{1/4} = 5\ln(3+4x) - \frac{1}{4}\ln(1-3x)$

$$f'(x) = 5\left(\frac{4}{3+4x}\right) - \frac{1}{4}\left(\frac{-3}{1-3x}\right) = \boxed{\frac{20}{3+4x} + \frac{3}{4(1-3x)}}$$

2. $y = \log_3\left(\frac{3x^5}{2x^4-3}\right)^3 = 3\log_3\left(\frac{3x^5}{2x^4-3}\right) = 3\log_3(3x^5) - 3\log_3(2x^4-3)$

Reminder: $* \frac{d}{dx}\log_a u = \frac{1}{\ln a}\left[\frac{u'}{u}\right]$

$$y' = 3\cdot\frac{1}{\ln 3}\left(\frac{15x^4}{3x^5}\right) - 3\left(\frac{1}{\ln 3}\right)\left(\frac{8x^3}{2x^4-3}\right)$$

$$= \boxed{\frac{3}{\ln 3}\left(\frac{5}{x}\right) - \frac{3}{\ln 3}\left(\frac{8x^3}{2x^4-3}\right)}$$

3. $f(x) = xe^{2-x^2}$ * product rule

$$f'(x) = (1)e^{2-x^2} + x\cdot e^{2-x^2}(-2x)$$

$$\boxed{= e^{2-x^2}(1-2x^2)}$$

4. $f(x) = log_5\left(\frac{3-x}{\sqrt{1-x}}\right) = \log_5(3-x) - \log_5(1-x)^{1/2} = \log_5(3-x) - \frac{1}{2}\log_5(1-x)$

$$f'(x) = \frac{1}{\ln 5}\left(\frac{-1}{3-x}\right) - \frac{1}{2}\left(\frac{1}{\ln 5}\right)\left(\frac{-1}{1-x}\right)$$

$$\boxed{= \frac{-1}{\ln 5(3-x)} + \frac{1}{2\ln 5(1-x)}}$$

Reminder: $* \frac{d}{dx}a^u = \ln a \cdot a^u \cdot u'$

5. $f(x) = 8^{x-3x^2}(log(3-2x)^3)$

$$f(x) = 8^{x-3x^2}\cdot 3\log_{10}(3-2x)$$

$$f'(x) = (\ln 8)8^{x-3x^2}\cdot(1-6x)\cdot\log(3-2x)^3 + 8^{x-3x^2}\cdot\frac{3}{\ln 10}\left(\frac{-2}{3-2x}\right)$$

$$\boxed{8^{x-3x^2}\left[(\ln 8)(1-6x)\log(3-2x)^3 - \frac{6}{\ln 10(3-2x)}\right]}$$

5) $f(x) = 8^{x-3x^2} \cdot \log(3-2x)^3$

$$\underbrace{8^{x-3x^2}} \cdot \underbrace{3\log_{10}(3-2x)}$$

$$\overbrace{(\ln 8)8^{x-3x^2} \cdot (1-6x)}^{f'} \cdot \overbrace{3\log_{10}(3-2x)}^{g} + \overbrace{8^{x-3x^2}}^{f} \cdot \overbrace{3 \cdot \frac{1}{\ln 10} \cdot \frac{-2}{3-2x}}^{g'}$$