Calculus AB **Logs and Exponentials Derivatives Unit Quiz Review WS 1**

Key

1. Find $\frac{dy}{dx}$ $\quad y = ln\sqrt{\frac{3-2x}{4x}}$

$y = \ln\left(\frac{3-2x}{4x}\right)^{1/2}$

$y = \frac{1}{2}\ln\left(\frac{3-2x}{4x}\right)$

$y = \frac{1}{2}\ln(3-2x) - \frac{1}{2}\ln(4x)$

$y' = \frac{1}{2}\cdot\frac{-2}{3-2x} - \frac{1}{2}\cdot\frac{4}{4x}$

$\boxed{y' = \frac{-1}{3-2x} - \frac{1}{2x}}$

2. Find $\frac{dy}{dx}$ $\quad y = x^{\sqrt{x+3}}$

$y = x^{(x+3)^{1/2}}$ *log differentiation

$\ln y = \ln x^{(x+3)^{1/2}}$

$\ln y = \underbrace{(x+3)^{1/2}}_{f} \cdot \underbrace{\ln x}_{g}$ ← product Rule

$\frac{1}{y}\left(\frac{dy}{dx}\right) = \underbrace{\frac{1}{2}(x+3)^{-1/2}}_{f'} \cdot \underbrace{\ln x}_{g} + \underbrace{(x+3)^{1/2}}_{f} \cdot \underbrace{\frac{1}{x}}_{g'}$

$\frac{dy}{dx} = y\left[\frac{\ln x}{2(x+3)^{1/2}} + \frac{(x+3)^{1/2}}{x}\right]$

$\boxed{\frac{dy}{dx} = x^{\sqrt{x+3}}\left[\frac{\ln x}{2(x+3)^{1/2}} + \frac{(x+3)^{1/2}}{x}\right]}$

3. Given that $f(g(x)) = x$.
Find $f'(2)$ if $g(2) = 5$, $g(7) = 2$, $g'(2) = 3$ and $g'(7) = -\frac{5}{6}$

* f and g are inverses

$g(\ ) = 2$	$f(2) =$
	$f'(2) =$

→

$g(7) = 2$	$f(2) = 7$
$g'(7) = \frac{-5}{6}$	$f'(2) = \boxed{\frac{-6}{5}}$

4. f(x) = x³ + 2x² – 3 $\quad$ Find $(f^{-1})'(13)$

$13 = x^3 + 2x^2 - 3$

$0 = x^3 + 2x^2 - 16$

*guess and check $\boxed{x = 2}$

$f(2) = 13$	$f^{-1}(13) = 2$
$f'(2) = 20$	$(f^{-1})'(13) = \boxed{\frac{1}{20}}$

$f'(x) = 3x^2 + 4x$

$f'(2) = 3(2)^2 + 4(2) = 3(4) + 8 = 20$

Find $\frac{dy}{dx}$ *for the following*

5. $y = 2\,ln\left(\frac{\sqrt[4]{(3x-2x^4)^3}}{2x^3}\right)$

$y = 2\ln(3x-2x^4)^{3/4} - 2\ln(2x^3)$

$y = 2\cdot\frac{3}{4}\ln(3x-2x^4) - 2\ln(2x^3)$

$y' = \frac{3}{2}\cdot\frac{3-8x^3}{3x-2x^4} - 2\cdot\frac{6x^2}{2x^3}$

$\boxed{y' = \frac{3(3-8x^3)}{2(3x-2x^4)} - \frac{6}{x}}$

6. $y = x^3 e^{5x^2+3x}$ ← *product Rule

$y = \underbrace{x^3}_{f}\,\underbrace{e^{5x^2+3x}}_{g}$

$y' = \underbrace{3x^2}_{f'} \cdot \underbrace{e^{5x^2+3x}}_{g} + \underbrace{x^3}_{f} \cdot \underbrace{e^{5x^2+3x}(10x+3)}_{g'}$

$\boxed{y' = 3x^2 e^{5x^2+3x} + x^3(10x+3)e^{5x^2+3x}}$

Find $\frac{dy}{dx}$ *for the following*

7. $y = 2\log_8\sqrt{x - e^x}$

$$y = 2\log_8(x-e^x)^{1/2}$$

$$y = 2\cdot\tfrac{1}{2}\log_8(x-e^x)$$

$$y = \log_8(x-e^x)$$

$$y' = \left(\frac{1}{\ln 8}\right)\cdot\left(\frac{1-e^x}{x-e^x}\right) = \boxed{\frac{1-e^x}{(\ln 8)(x-e^x)}}$$

8. $f(x) = \underbrace{2^{3x}}_{f}\underbrace{(\log(2-\sqrt{x}))}_{g}$ * product Rule

$$f'(x) = \underbrace{\ln 2\cdot 2^{3x}\cdot 3}_{f'}\cdot\underbrace{\log(2-\sqrt{x})}_{g} + \underbrace{2^{3x}}_{f}\cdot\underbrace{\frac{1}{\ln 10}\cdot\frac{-\frac{1}{2}x^{-1/2}}{2-\sqrt{x}}}_{g'}$$

$$\boxed{f'(x) = (3\ln 2)2^{3x}\cdot\log(2-\sqrt{x}) - \frac{2^{3x}}{(2\ln 10)x^{1/2}(2-\sqrt{x})}}$$

9. $f(x) = \log_2\left(\frac{\sqrt{1-3x}}{(x-5x^2)}\right)$ *expand first

$$y = \log_2(1-3x)^{1/2} - \log_2(x-5x^2)$$

$$y = \tfrac{1}{2}\log_2(1-3x) - \log_2(x-5x^2)$$

$$y' = \frac{1}{2}\cdot\frac{1}{\ln 2}\cdot\frac{-3}{1-3x} - \frac{1}{\ln 2}\cdot\left(\frac{1-10x}{x-5x^2}\right)$$

$$\boxed{y' = \frac{-3}{2\ln 2(1-3x)} - \frac{1-10x}{\ln 2(x-5x^2)}}$$

10. $f(x) = 11^{\sqrt[3]{4x-5x^3-3e^2}}$

$$y = 11^{(4x-5x^3-3e^2)^{1/3}}$$

$$y' = \underbrace{(\ln 11)}\cdot\underbrace{11^{\sqrt[3]{4x-5x^3-3e^2}}}\cdot\underbrace{\tfrac{1}{3}(4x-5x^3-3e^2)^{-2/3}\cdot(4-15x^2)}$$

$$y' = (\ln a)\cdot a^u\cdot u'$$

chain Rule
out: $(\ )^{1/3}$
in: $4x-5x^3-3e^2$

$$\boxed{y' = \frac{(\ln 11)\,11^{\sqrt[3]{4x-5x^3-3e^2}}(4-15x^2)}{3(4x-5x^3-3e^2)^{2/3}}}$$

11. Find dy/dx $\ln\left(\frac{y}{\sqrt{x}}\right) + xy - y + 2x^5 = 3$

$$\ln y - \ln\sqrt{x} + xy - y + 2x^5 = 3$$

$$\ln y - \tfrac{1}{2}\ln x + xy - y + 2x^5 = 3$$

$$\frac{1}{y}\left(\frac{dy}{dx}\right) - \frac{1}{2}\left(\frac{1}{x}\right) + \underbrace{1}_{f'}\cdot\underbrace{y}_{g} + \underbrace{x}_{f}\cdot\underbrace{dy/dx}_{g'} - 1\left(\frac{dy}{dx}\right) + 10x^4 = 0$$

$$\frac{dy}{dx}\left(\frac{1}{y} + x - 1\right) = \frac{1}{2x} - y - 10x^4$$

$$\boxed{\frac{dy}{dx} = \frac{\frac{1}{2x} - y - 10x^4}{\frac{1}{y} + x - 1}}$$

12) Find the tangent line equation for the function $f(x) = \underbrace{e^{-x}}_{f}\underbrace{(\ln x)}_{g}$ at (1, 0) (product Rule)

$$f'(x) = \underbrace{e^{-x}(-1)}_{f'}\cdot\underbrace{\ln x}_{g} + \underbrace{e^{-x}}_{f}\cdot\underbrace{\frac{1}{x}}_{g'}$$

$$f'(1) = e^{-1}(-1)(\ln 1) + e^{-1}\left(\frac{1}{1}\right)$$

$$f'(1) = \frac{1}{e}(-1)(0) + \frac{1}{e} = \frac{1}{e}$$

point: $(1, 0)$
slope: $m = 1/e$

$$y - y_1 = m(x - x_1)$$

$$\boxed{y - 0 = \frac{1}{e}(x-1)}$$