EXAM I
CALCULUS AB
SECTION I PART A
Time–55 minutes
Number of questions–28

A CALCULATOR MAY NOT BE USED ON THIS PART OF THE EXAMINATION

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the box. Do not spend too much time on any one problem.

In this test:

(1) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(2) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1}x = \arcsin x$).

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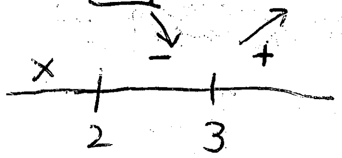
1. If $f'(x) = \ln(x-2)$, then the graph of $y = f(x)$ is decreasing if and only if

[✓ (A)] $2 < x < 3$ (B) $0 < x$ (C) $0 < x < 1$ (D) $x > 1$ (E) $x > 2$



$x-2=1$

$x=3$

$\ln(x-2)=0$

$e^{\ln(x-2)}=e^{0}$

Ans A

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2. For $x \neq 0$, the slope of the tangent to $y = x\cos x$ equals zero whenever

(A) $\tan x = -x$

[✓ (B)] $\tan x = \frac{1}{x}$

(C) $\tan x = x$

(D) $\sin x = x$

(E) $\cos x = x$

*find $y'(x)$ *use product rule: $f'g + fg'$

$y' = 1\cos x + x(-\sin x)$

$0 = \cos x - x\sin x$

$\frac{x\sin x}{\cos x} = \frac{\cos x}{\cos x}$

$x\tan x = 1$

$\boxed{\tan x = \frac{1}{x}}$

Ans B

3. The function F is defined by

$F(x) = G[x + G(x)]$

where the graph of the function G is shown at the right.

The approximate value of $F'(1)$ is

(A) $\frac{7}{3}$

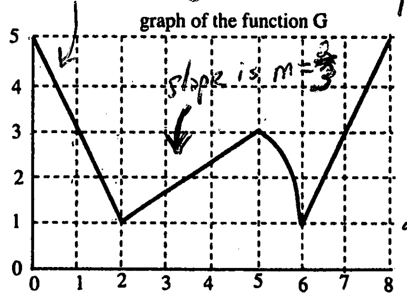
(B) $\frac{2}{3}$

(C) -2

(D) -1

[✓ (E)] $-\frac{2}{3}$

slope of line segment is $m = -\frac{2}{1} = -2$



slope is $m = \frac{2}{3}$

*slope of G graph at $x=1$ is $-\frac{2}{1} = -2$

* Apply chain rule:

$\frac{d}{dx} f(g(x)) = f'(g(x))\cdot g'(x)$

$F(x) = G[x + G(x)]$

$F'(x) = G'[x + G(x)]\cdot(1 + G'(x))$

$F'(x) = G'[1 + G(1)]\cdot(1 + G'(1))$

$F'(1) = G'[1+3]\cdot(1+-2)$

$= G'(4)\cdot(-1)$

$= (\frac{2}{3})(-1)$

$\boxed{F'(1) = \frac{-2}{3}}$

Ans $\boxed{E}$

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4. $\int_2^6 \left(\frac{1}{x} + 2x\right) dx =$

(A) $\ln 4 + 32$

(B) $\ln 3 + 40$

[✓ (C)] $\ln 3 + 32$

(D) $\ln 4 + 40$

(E) $\ln 12 + 32$

$\int \frac{1}{x} + 2x\,dx = \ln|x| + \frac{2x^2}{2}$

$= \ln|x| + x^2\Big]_2^6$

$= \ln 6 + 6^2 - (\ln 2 + 2^2)$

$= \ln 6 + 36 - \ln 2 - 4$

$= \ln 6 - \ln 2 + 32$

*$\ln a - \ln b = \ln(\frac{a}{b})$

$\ln(\frac{6}{2}) + 32$

$= \boxed{\ln 3 + 32}$

Ans $\boxed{C}$

Domain $(0, \infty)$

5. A relative maximum of the function $f(x) = \frac{(\ln x)^2}{x}$ occurs at

(A) 0

(B) 1

(C) 2

(D) e

[✓ (E)] e^2

* find $f'(x)$ using quotient rule, chain rule

* create sign line, apply 1st Derivative Test

$f'(x) = \frac{2(\ln x)(\frac{1}{x})\cdot x - (\ln x)^2(1)}{x^2}$

$f'(x) = \frac{2\ln x - (\ln x)^2}{x^2}$

$\boxed{f'(x) = \frac{\ln x(2-\ln x)}{x^2}}$

$\ln x = 0$

$e^{\ln x} = e^0$

$x = 1$

$2 - \ln x = 0$

$\ln x = 2$

$e^{\ln x} = e^2$

$x = e^2$

$x^2 = 0$

$x = 0$ vertical asymptote

$\times$ | $-$ | $+$ | $-$

0 0.5 1 e e^2 e^3

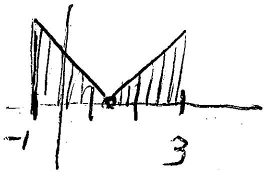
Relative max occurs at $x = e^2$

Ans $\boxed{E}$

$y=|2x-3|$

6. Use a right-hand Riemann sum with 4 equal subdivisions to approximate the integral

$$\int_{-1}^{3} |2x-3|\, dx.$$



(A) 13

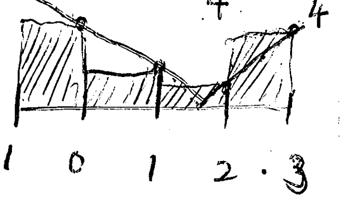
(B) 10

(C) 8.5

[✓ (D) 8]

(E) 6

$\text{width} = \frac{b-a}{n} = \frac{3-(-1)}{4} = \frac{4}{4} = 1$



$\int_{-1}^{3}|2x-3|dx \approx 1\cdot y(0) + 1\cdot y(1) + 1\cdot y(2) + 1\cdot y(3)$

$= 1(3) + 1(1) + 1(1) + 1(3)$

$y(0)=3$
$y(1)=1$
$y(2)=1$
$y(3)=3$

$= \boxed{8}$

Ans $\boxed{D}$

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7. An equation of the line tangent to the graph of $y = x^3 + 3x^2 + 2$ at its point of inflection is ← where $f''(x)=0$

[✓ (A) $y = -3x + 1$]

(B) $y = -3x - 7$

(C) $y = x + 5$

(D) $y = 3x + 1$

(E) $y = 3x + 7$

$y' = 3x^2 + 6x$

$y'' = 6x + 6$

$0 = 6(x+1)$

$x = -1$ ∪

∩ − | +

−1 ↖ POI at $x=-1$

point: $(-1, 4)$ ← $y(-1) = (-1)^3 + 3(-1)^2 + 2$
$= -1 + 3 + 2 = 4$

slope: $m = -3$ ← $y'(-1) = 3(-1)^2 + 6(-1)$
$= 3 - 6 = -3$

$y - y_1 = m(x - x_1)$

$y - 4 = -3(x+1)$

$y - 4 = -3x - 3$

$\boxed{y = -3x + 1}$

Ans $\boxed{A}$

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8. $\int \cos(3-2x)\, dx =$

(A) $\sin(3-2x) + C$

(B) $-\sin(3-2x) + C$

(C) $\frac{1}{2}\sin(3-2x) + C$

[✓ (D) $-\frac{1}{2}\sin(3-2x) + C$]

(E) $-\frac{1}{5}\sin(3-2x) + C$

$u = 3 - 2x$

$\frac{du}{dx} = -2$

$dx = \frac{du}{-2}$

$\int \cos u \cdot \frac{du}{-2}$

$-\frac{1}{2}\int \cos u\, du$

$= -\frac{1}{2}\sin u + c$

$= \boxed{-\frac{1}{2}\sin(3-2x) + c}$

Ans $\boxed{D}$

9. What is $\lim\limits_{x\to\infty} \frac{\sqrt{9x^2+2}}{4x+3}$?

(A) $\frac{3}{2}$ [✓ (B) $\frac{3}{4}$] (C) $\frac{\sqrt{2}}{3}$ (D) 1 (E) The limit does not exist.

* For limits approaching infinity, compare degrees between numerator/denominator

$$\lim_{x\to\infty} \frac{\sqrt{9x^2+2}}{4x+3} \approx \frac{\sqrt{9x}}{4x} = \boxed{\frac{3}{4}}$$

Ans $\boxed{B}$

10. Let the first quadrant region enclosed by the graph of $y = \frac{1}{x}$ and the lines $x = 1$ and $x = 4$ be the base of a solid. If cross sections perpendicular to the x-axis are semicircles, the volume of the solid is

$A = \frac{\pi}{8}[base]^2$

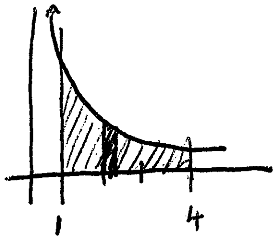
(A) $\frac{3\pi}{64}$

[✓ (B) $\frac{3\pi}{32}$]

(C) $\frac{3\pi}{16}$

(D) $\frac{3\pi}{8}$

(E) $\frac{3\pi}{4}$



$base = \frac{1}{x} - 0 = \frac{1}{x}$

$A = \frac{\pi}{8}\left[\frac{1}{x}\right]^2$

$$V = \frac{\pi}{8}\int_1^4 \frac{1}{x^2}\,dx$$

$$= \int x^{-2}\,dx = \frac{x^{-1}}{-1} = \left.\frac{-1}{x}\right]_1^4 = \frac{-1}{4} - \left(\frac{-1}{1}\right)$$

$$= \frac{\pi}{8}\left[\frac{-1}{4} + \frac{4}{4}\right] = \frac{\pi}{8} \cdot \frac{3}{4} = \boxed{\frac{3\pi}{32}}$$

Ans $\boxed{B}$

11. Let $f(x) = \ln x + e^{-x}$. Which of the following is TRUE at $x = 1$?

[✓ (A) f is increasing]

(B) f is decreasing

(C) f is discontinuous

(D) f has a relative minimum

(E) f has a relative maximum

* We can eliminate choices by evaluating $f'(1)$

$$f'(x) = \frac{1}{x} + e^{-x}(-1)$$

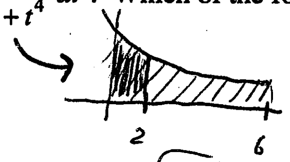
$$f'(x) = \frac{1}{x} - \frac{1}{e^x}$$

$$f'(1) = \frac{1}{1} - \frac{1}{e^1} = \frac{e-1}{e} > 0$$

Since $f'(1) > 0$, graph has positive slope and therefore increasing.

Ans $\boxed{A}$

12. Let F be the function given by $F(x)=\int_0^x \frac{2}{1+t^4}\,dt$. Which of the following statements are true?



I. $F(0)=2$

✓II. $F(2)<F(6)$

✓III. $F''(0)=0$

(A) I only (B) II only (C) III only [✓ (D)] II and III only (E) I, II, III

$F(0)=\int_0^0 \frac{2}{1+t^4}dt=0$

$F(6)>F(2)$ since area is greater for $F(6)$

$F'(x)=\frac{2}{1+x^4}$

$F''(x)=\frac{0(1+x^4)-2(4x^3)}{(1+x^4)}$

$F''(x)=\frac{-8x^3}{1+x^4}$

$F''(0)=\frac{0}{1+0}=0$ ✓

Ans $\boxed{D}$

13. What is the average (mean) value of $2t^3-3t^2+4$ over the interval $-1\le t\le 1$?

(A) 0

(B) $\frac{7}{4}$

[✓ (C)] 3

(D) 4

(E) 6

*Avg. value theorem $=\frac{1}{b-a}\int_a^b f(t)dt$

$=\frac{1}{1-(-1)}\int_{-1}^{1} 2t^3-3t^2+4dt$

$=\frac{1}{2}\cdot\frac{2t^4}{4}-\frac{3t^3}{3}+4t\Big]_{-1}^{1}$

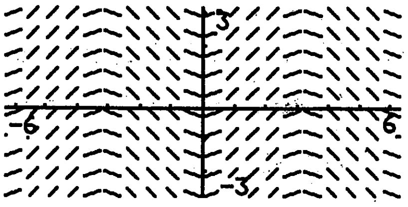
$\frac{1}{2}\left[\frac{1}{2}(1)^4-1^3+4-\left(\frac{1}{2}-(-1)-4\right)\right]$

$\frac{1}{2}\left[\cancel{\frac{1}{2}}-1+4-\cancel{\frac{1}{2}}-1+4\right]$

$\frac{1}{2}[6]=\boxed{3}$

Ans $\boxed{C}$

14. The slope field for a differential equation $\frac{dy}{dx}=f(x,y)$ is given in the figure. The slope field corresponds to which of the following differential equations?



*pattern behavior of slope field resembles sin x graph.

(A) $\frac{dy}{dx}=\tan x\cdot\sec x$

[✓ (B)] $\frac{dy}{dx}=\sin x$

(C) $\frac{dy}{dx}=\cos x$

(D) $\frac{dy}{dx}=-\sin x$

(E) $\frac{dy}{dx}=-\cos x$

Ans $\boxed{B}$

15. What is $\lim_{x\to1}\frac{\sqrt{x}-1}{x-1}$?

(A) 0

[✓ (B)] $\frac{1}{2}$

(C) 1

(D) $\frac{3}{2}$

(E) The limit does not exist.

option 1: L'Hopital's Rule:

$$\lim_{x\to1}\frac{\sqrt{x}-1}{x-1} = \frac{0}{0} \xrightarrow{L'H}$$

$$\lim_{x\to1}\frac{\frac{1}{2}x^{-1/2}}{1} = \frac{1}{2\sqrt{x}} = \boxed{\frac{1}{2}}$$

option 2: *conjugate method

$$\lim_{x\to1}\frac{\sqrt{x}-1}{x-1}\cdot\frac{\sqrt{x}+1}{\sqrt{x}+1} = \frac{\cancel{(x-1)}}{\cancel{(x-1)}(\sqrt{x}+1)}$$

$$\lim_{x\to1}\frac{1}{\sqrt{x}+1} = \frac{1}{\sqrt{1}+1} = \boxed{\frac{1}{2}}$$

Ans $\boxed{B}$

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16. If $y = \cos^2 x - \sin^2 x$, then $y' =$

(A) -1

(B) 0

(C) $-2(\cos x + \sin x)$

(D) $2(\cos x + \sin x)$

[✓ (E)] $-4(\cos x)(\sin x)$

* chain rule

$$\frac{d}{dx}f[g(x)] = f'[g(x)]\cdot g'(x)$$

$$y = [\cos x]^2 - [\sin x]^2$$

$$y' = 2[\cos x]^1(-\sin x) - 2[\sin x]^1\cdot\cos x$$

$$y' = -2\sin x\cos x - 2\sin x\cos x$$

$$y' = -4\sin x\cos x$$

Ans $\boxed{E}$

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17. The area of the first quadrant region bounded above by the graph of $y = 4x^3 + 6x - \frac{1}{x}$ between the values of $x = 1$ and $x = 2$ is

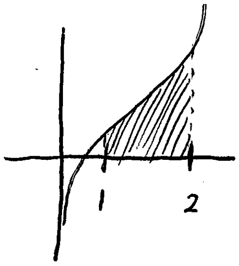
(A) $32 - \ln 2$

(B) $30 - \ln 2$

[✓ (C)] $24 - \ln 2$

(D) $\frac{99}{4}$

(E) 21



$$\text{Area} = \int_1^2 4x^3+6x-\frac{1}{x}\,dx$$

$$= \frac{\cancel{4}x^4}{\cancel{4}} + \frac{6x^2}{2} - \ln|x| \Big]_1^2$$

$$= 2^4+3(2)^2-\ln 2-[1+3-\ln 1]$$

$$= 16+12-\ln 2-4$$

$$= \boxed{24-\ln 2}$$

Ans $\boxed{C}$

18. $\int \frac{x-2}{x-1}\,dx =$

(A) $-\ln|x-1|+C$

(B) $x+\ln|x-1|+C$

$\boxed{\text{(C)}}$ $x-\ln|x-1|+C$

(D) $x+\sqrt{x-1}+C$

(E) $x-\sqrt{x-1}+C$

*option 1: long division:

$$x-1\,\overline{)\,x-2\,}\quad 1+\frac{-1}{x-1}$$

$$\ominus x \overset{\oplus}{-} 1$$

$$-1$$

$$\int 1-\frac{1}{x-1}\,dx$$

$$\boxed{x-\ln|x-1|+c}$$

*option 2: u-sub (change of variable)

$$\int \frac{x-2}{x-1}dx \quad u=x-1 \quad du=dx$$

$$x=u+1$$

$$\int \frac{x-2}{u}$$

$$\int \frac{u+1-2}{u}$$

$$\int \frac{u-1}{u}du$$

$$\int 1-\frac{1}{u}du$$

$$=u-\ln|u|+c$$

$$=x-1-\ln|x-1|+c$$

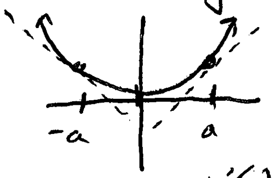
Ans $\boxed{C}$ $=\boxed{x-\ln|x-1|+c}$

19. Suppose that g is a function with the following two properties: $g(-x) = g(x)$ for all x, and $g'(a)$ exists. Which of the following must necessarily be equal to $g'(-a)$?

*even function

(A) $g'(a)$ $\boxed{\text{(B)}}$ $-g'(a)$ (C) $\frac{1}{g'(a)}$ (D) $-\frac{1}{g'(a)}$ (E) none

*example $y=x^2$



$$g'(-a)=-g'(a)$$

$$*\,y'(-2)=-2(2)=-4$$

$$y'(x)=2x \quad y'(2)=2(2)=4$$

Ans $\boxed{B}$

20. An equation for a tangent line to the graph of $y = \text{Arctan}\,\frac{x}{3}$ at the origin is:

$$\frac{d}{dx}\arctan u=\frac{u'}{1+u^2}$$

$\boxed{\text{(A)}}$ $x-3y=0$

(B) $x-y=0$

(C) $x=0$

(D) $y=0$

(E) $3x-y=0$

$$y'=\frac{\frac{1}{3}}{1+\left(\frac{x}{3}\right)^2}$$

$$y'(0)=\frac{\frac{1}{3}}{1+0}=\frac{1}{3}$$

point: $(0,0)$

slope: $m=\frac{1}{3}$

$$y-y_1=m(x-x_1)$$

$$y-0=\frac{1}{3}(x-0)$$

$$y=\frac{1}{3}x$$

$$3y=x$$

$$\boxed{0=x-3y}$$

Ans $\boxed{A}$

21. If $f(x) = \begin{cases} x^2+4 & \text{for } 0 \le x \le 1 \\ 6-x & \text{elsewhere} \end{cases}$ then $\int_0^3 f(x)\,dx$ is a number between

(A) 0 and 5

(B) 5 and 10

[✓ (C) 10 and 15]

(D) 15 and 20

(E) 20 and 25

$\int_0^1 x^2+4\,dx + \int_1^3 6-x\,dx$

$\frac{x^3}{3}+4x\Big]_0^1 + 6x-\frac{x^2}{2}\Big]_1^3 = 6(3)-\frac{9}{2}-\left(6-\frac{1}{2}\right)$

$\frac{1}{3}+4 + 8 \approx \boxed{12.3}$

$18-\frac{9}{2}-6+\frac{1}{2}$

$12-4$

$=8$

Ans $\boxed{C}$

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22. $\frac{d}{dx}\left(\ln e^{3x}\right) =$

$* \ln e^{u} = u$

(A) 1

[✓ (B) 3]

(C) $3x$

(D) $\frac{1}{e^{3x}}$

(E) $\frac{3}{e^{3x}}$

$\frac{d}{dx}(3x) = 3$

Ans $\boxed{B}$

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23. If $g'(x) = 2g(x)$ and $g(-1) = 1$, then $g(x) =$

(A) e^{2x}

(B) e^{-x}

(C) e^{x+1}

(D) e^{2x+2}

(E) e^{2x-2}

$y' = 2y$

$\frac{dy}{dx} = 2y$

$dy = 2y\,dx$

$\frac{dy}{y} = 2\,dx$

$(-1,1)$

$\ln|y| = 2x + C$

$\ln 1 = 2(-1) + C$

$0 = -2 + C$

$2 = C$

$\ln|y| = 2x + 2$

$e^{\ln|y|} = e^{2x+2}$

$|y| = e^{2x+2}$

$\boxed{y = e^{2x+2}}$

Ans $\boxed{D}$

24. The acceleration at time $t > 0$ of a particle moving along the x-axis is $a(t) = 3t + 2$ ft/sec². If at $t = 1$ seconds the velocity is 4 ft/sec and the position is $x = 6$ feet, then at $t = 2$ seconds the position $x(t)$ is

(A) 8 ft (B) 11 ft (C) 12 ft [✓ (D) 13 ft] (E) 15 ft

$V(1) = 4$

$X(1) = 6$

$X(2) = ?$

$v(t) = \int a(t)dt$

$v(t) = \int 3t + 2dt$

$v(t) = \frac{3t^2}{2} + 2t + c$

$4 = \frac{3}{2}(1)^2 + 2(1) + c$

$\frac{1}{2} = c$

$v(t) = \frac{3}{2}t^2 + 2t + \frac{1}{2}$

$x(t) = \int \frac{3}{2}t^2 + 2t + \frac{1}{2} dt$

$= \frac{\cancel{3}}{2}\cdot\frac{t^3}{\cancel{3}} + \frac{2t^2}{2} + \frac{1}{2}t + C$

$6 = \frac{1}{2} + 1 + \frac{1}{2} + c$

$4 = c$

Ans $\boxed{D}$

$x(t) = \frac{1}{2}t^3 + t^2 + \frac{1}{2}t + 4$

$x(2) = 4 + 4 + 1 + 4$

$\boxed{x(2) = 13}$

25. The approximate value of $y = \sqrt{3 + e^x}$ at $x = 0.08$, obtained from the tangent to the graph at $x = 0$, is

$y(0) = \sqrt{3 + e^0} = \sqrt{4} = 2$

(A) 2.01

[✓ (B) 2.02]

(C) 2.03

(D) 2.04

(E) 2.05

*Linear approximation:

a) Find tangent line equation

b) plug in decimal

$y = (3 + e^x)^{1/2}$

$y' = \frac{1}{2}(3 + e^x)^{-1/2}(e^x)$

$y'(0) = \frac{1}{2}(3 + 1)^{-1/2}(1) = \frac{1}{2}\cdot\frac{1}{\sqrt{4}} = \frac{1}{4}$

point: $(0, 2)$

slope: $m = \frac{1}{4}$

$y - 2 = \frac{1}{4}(x - 0)$

$y = \frac{1}{4}x + 2$

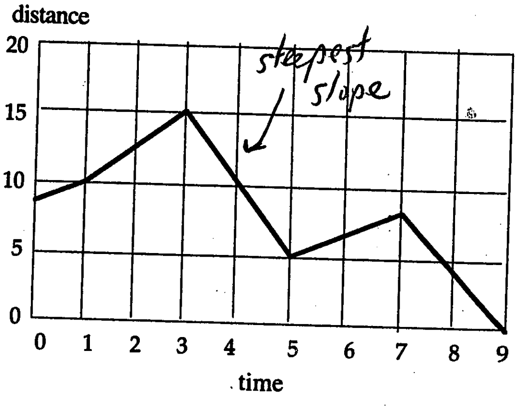
$y(0.08) = \frac{1}{4}(0.08) + 2$

$= 0.02 + 2 = \boxed{2.02}$

Ans $\boxed{B}$

26. A leaf falls from a tree into a swirling wind. The graph at the right shows its vertical distance (feet) above the ground plotted against time (seconds).

According to the graph, in what time interval is the speed of the leaf the greatest?



(A) $1 < t < 3$

[✓ (B) $3 < t < 5$]

(C) $5 < t < 7$

(D) $7 < t < 9$

(E) none of these

Ans $\boxed{B}$

27. Water is flowing into a spherical tank with 6 foot radius at the constant rate of 30π cu ft per hour. When the water is h feet deep, the volume of water in the tank is given by

$$V = \frac{\pi h^2}{3}(18 - h).$$

$\frac{dV}{dt} = 30\pi \ ft^3/hr$

What is the rate at which the depth of the water in the tank is increasing at the moment when the water is 2 feet deep? $h = 2$

(A) 0.5 ft per hr

(B) 1.0 ft per hr

(C) 1.5 ft per hr

(D) 2.0 ft per hr

(E) 2.5 ft per hr

*Related Rates:

$V = \frac{\pi h^2}{3}(18-h)$

$V = 6\pi h^2 - \frac{\pi}{3}h^3$

$\frac{dV}{dt} = 12\pi h\left(\frac{dh}{dt}\right) - 3\left(\frac{\pi}{3}\right)h^2\left(\frac{dh}{dt}\right)$

$30\pi = 12\pi(2)\left(\frac{dh}{dt}\right) - \pi(2)^2\left(\frac{dh}{dt}\right)$

$30\pi = 24\pi\left(\frac{dh}{dt}\right) - 4\pi\left(\frac{dh}{dt}\right)$

$30\pi = 20\pi\left(\frac{dh}{dt}\right)$

$\frac{30\pi}{20\pi} = \frac{dh}{dt}$

$\boxed{\frac{dh}{dt} = \frac{3}{2} = 1.5 \ ft/hr.}$

Ans $\boxed{C}$

---

28. The graph of the function $f(x) = 2x^{5/3} - 5x^{2/3}$ is increasing on which of the following intervals.

I. $1 < x$ II. $0 < x < 1$ III. $x < 0$

(A) I only (B) II only (C) III only (D) I and II only $\boxed{\text{(E)}}$ I and III only

* 1st derivative test:

a) find $f'(x)$

b) set $f'(x) = 0$

c) find critical pts.

d) create sign line

$f(x) = 2x^{5/3} - 5x^{2/3}$

$f'(x) = 2 \cdot \frac{5}{3}x^{2/3} - 5 \cdot \frac{2}{3}x^{-1/3}$

$f'(x) = \frac{10}{3}x^{2/3} - \frac{10}{3x^{1/3}}$

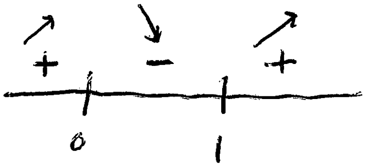
$f'(x) = \frac{10x - 10}{3x^{1/3}}$

$10x - 10 = 0$

$x = 1$

$3x^{1/3} = 0$

$x = 0$



$f(x)$ is increasing $(-\infty, 0) \cup (1, \infty)$

$x < 0$

$x > 1$

Ans $\boxed{E}$

EXAM I
CALCULUS AB
SECTION I PART B
Time–50 minutes
Number of questions–17

A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS ON THIS PART OF THE EXAMINATION

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the box. Do not spend too much time on any one problem.

In this test:

(1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.

(2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(3) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1}x = \arcsin x$).

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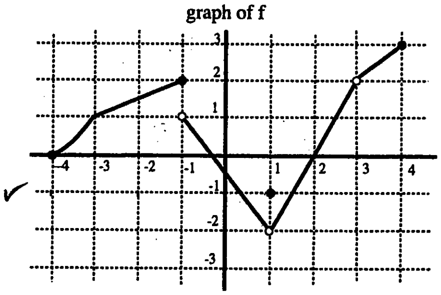
1. The function f is defined on the interval $[-4, 4]$ and its graph is shown to the right. Which of the following statements are true?



I. $\lim_{x\to 1} f(x) = -1$ False: $\lim_{x\to 1} f(x) = -2$

II. $\lim_{h\to 0} \frac{f(2+h)-f(2)}{h} = 2$ ← True: $f'(2)=2$ ✓

III. $\lim_{x\to -1^+} f(x) = f(-3)$ $1 = 1$ ✓ True

(A) I only (B) II only (C) I and II only (D) II and III only (E) I, II, III

Ans
D

---

2. For $f(x) = \sin^2 x$ and $g(x) = 0.5x^2$ on the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, the instantaneous rate of change of f is greater than the instantaneous rate of change of g for which value of x?

(A) −0.8 (B) 0 [✓ (C) 0.9] (D) 1.2 (E) 1.5

$f'(0.9) > g'(0.9)$

Ans
C

3. If $f(x) = 2x^2 - x^3$ and $g(x) = x^2 - 2x$, for what values of a and b is

$$\int_a^b f(x)\,dx > \int_a^b g(x)\,dx \ ?$$

I. $a = -1$ and $b = 0$ II. $a = 0$ and $b = 2$ III. $a = 2$ and $b = 3$

(A) I only

[✓ (B) II only]

(C) I and II only

(D) I and III only

(E) I, II, III

$\int_{-1}^{0} f(x)dx = 0.916$

$\int_{-1}^{0} g(x)dx = 1.33$

$$\boxed{\begin{array}{l}\int_{0}^{2} f(x)dx = 1.33 \\ \int_{0}^{2} g(x)dx = -1.33\end{array}}$$

$\int_{2}^{3} f(x)dx = -3.583$

$\int_{2}^{3} g(x)dx = 1.33$

Ans $\boxed{B}$

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4. If $y^2 - 3x = 7$, then $\dfrac{d^2y}{dx^2} =$

$\dfrac{dy}{dx} = \dfrac{3}{2y}$

(A) $\dfrac{-6}{7y^3}$ (B) $\dfrac{-3}{y^3}$ (C) 3 (D) $\dfrac{3}{2y}$ (E) $\dfrac{-9}{4y^3}$

*Implicit Differentiation

$y^2 - 3x = 7$

$2y\left(\frac{dy}{dx}\right) - 3 = 0$

$\frac{dy}{dx} = \frac{3}{2y} = \frac{3}{2}y^{-1}$

$\frac{d^2y}{dx^2} = \frac{3}{2} \cdot -1y^{-2}\left(\frac{dy}{dx}\right)$

$\frac{d^2y}{dx^2} = \frac{-3}{2y^2} \cdot \left(\frac{3}{2y}\right) = \boxed{\frac{-9}{4y^3}}$

Ans $\boxed{E}$

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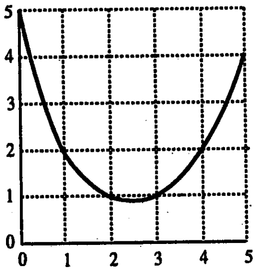
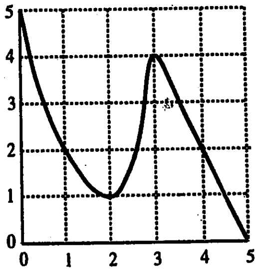
5. The graphs of functions f and g are shown at the right.
If $h(x) = g[f(x)]$, which of the following statements are true about the function h?

graph of f



graph of g



✗ I. $h(0) = 4$.

✓ II. h is increasing at $x = 2$.

✓ III. The graph of h has a horizontal tangent at $x = 4$.

(A) I only (B) II only (C) I and II only (D) II and III only (E) I, II, III

$h(x) = g[f(x)]$

$h(0) = g[f(0)]$

$= g[5]$

$h(0) = 0$

$h'(x) = g'[f(x)] \cdot f'(x)$

$h'(2) = g'[f(2)] \cdot f'(2)$

$= g'[1] \cdot f'(2)$

negative slope negative slope

$h'(2) > 0$ so h is increasing at $x=2$

$h'(x) = g'[f(x)] \cdot f'(x)$

$h'(4) = g'[f(4)] \cdot f'(4)$

$= g'[2] \cdot f'(4)$

$= 0 \cdot 1$

$= 0$ ✓

Ans $\boxed{D}$

$(0,0)$

(x, e^x)

6. The minimum distance from the origin to the curve $y = e^x$ is

(A) 0.72 (B) 0.74 (C) 0.76 [✓ (D)] 0.78 (E) 0.80

$\text{distance} = \sqrt{(x-0)^2+(e^x-0)^2}$

$d = \sqrt{x^2+e^{2x}}$

$d = (x^2+e^{2x})^{1/2}$

* chain rule →

$d'(x) = \frac{1}{2}(x^2+e^{2x})^{-1/2}(2x+2e^{2x})$

$d'(x) = \dfrac{x+e^{2x}}{\sqrt{x^2+e^{2x}}}$

$0 = x+e^{2x}$

$x \approx -0.426$

$d(-0.426) = \sqrt{(-0.426)^2+e^{2(-0.426)}}$

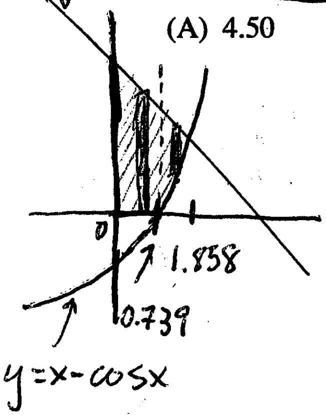
$d \approx 0.7797$

$\approx \boxed{0.78}$

Ans $\boxed{D}$

7. The area of the first quadrant region bounded by the y-axis, the line $y = 4 - x$ and the graph of $y = x - \cos x$ is approximately

(A) 4.50 (B) 4.54 (C) 4.56 (D) 4.58 (E) 5.00



$\text{Area} \approx \int_0^{0.739} 4-x-0\,dx + \int_{0.739}^{1.858} 4-x-(x-\cos x)\,dx$

$= 4.538 \approx \boxed{4.54}$

Ans $\boxed{B}$

set $y''(x) = 0$

8. The graph of $y = x^4 - x^2 - e^{2x}$ changes concavity at $x =$

(A) −0.641 (B) −0.531 (C) −0.421 (D) −0.311 (E) −0.201

$y = x^4 - x^2 - e^{2x}$

$y' = 4x^3 - 2x - 2e^{2x}$

$y'' = 12x^2 - 2 - 4e^{2x}$

$0 = 12x^2 - 2 - 4e^{2x}$

$x \approx -0.5309 = \boxed{-0.531}$

Ans $\boxed{B}$

9. The rate at which ice is melting in a pond is given by $\frac{dV}{dt} = \sqrt{1 + 2^t}$, where V is the volume of ice in cubic feet, and t is the time in minutes. What amount of ice has melted in the first 5 minutes?

(A) 14.49 ft^3 (B) 14.51 ft^3 [✓ (C) 14.53 ft^3] (D) 14.55 ft^3 (E) 14.57 ft^3

$$\text{Amount of Ice} = \int_0^5 \underbrace{\sqrt{1+2^t}}_{ft^3/min}\ \underbrace{dt}_{\cdot\ min} = \boxed{14.53\ ft^3}$$

Ans $\boxed{C}$

10. The region shaded in the figure at the right is rotated about the x-axis. Using the Trapezoid Rule with 5 equal subdivisions, the approximate volume of the resulting solid is

Disc Method

$R(x) = f(x) - 0$

(A) 23

(B) 47

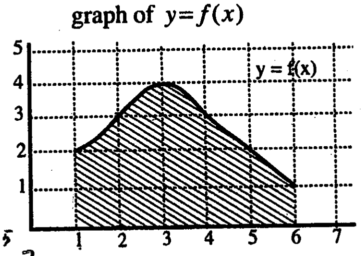
[✓ (C) 127]

(D) 254

(E) 400

$\text{Area(trapezoid)} = \frac{w}{2}[h_1 + h_2]$

$\text{Trapezoid Rule} = \frac{w}{2}[h_1 + 2h_2 + 2h_3 + 2h_4 + 2h_5 + h_6]$



$$V = \pi\int_1^6 [f(x)]^2 dx \approx \frac{1}{2}\left[2^2 + 2(3)^2 + 2(4)^2 + 2(3)^2 + 2(2)^2 + 1^2\right] \approx 81/2 = 40.5$$

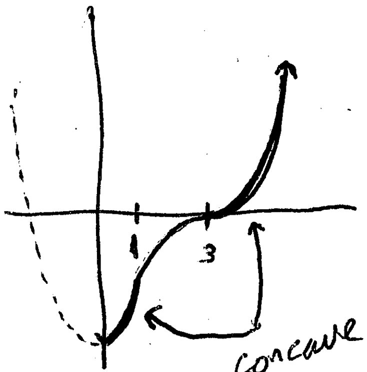
$$\approx \pi \cdot [40.5] \approx 127.23$$

Ans $\boxed{C}$

11. A particle moves along the x-axis so that at time $t \geq 0$, its position is given by $x(t) = (t + 1)(t - 3)^3$. For what values of t is the velocity of the particle increasing?

acceleration >0

(A) all t (B) $0 < t < 1$ C) $0 < t < 3$ (D) $1 < t < 3$ E) $t < 1$ or $t > 3$



concave up

$(0,1) \cup (3,\infty)$

Ans $\boxed{E}$

12. Let $f(x) = \dfrac{\ln e^{2x}}{x-1}$ for $x > 1$. If g is the inverse of f, then $g'(3) =$

(A) 2 (B) 1 (C) 0 (D) –1 (E) –2

$f(3) = 3$ | $g(3) = 3$

$f'(3) = \frac{-1}{2}$ | $g'(3) = \boxed{-2}$

$3 = \dfrac{\ln e^{2x}}{x-1}$

$3 = \dfrac{2x}{x-1}$

$3x - 3 = 2x$

$x = 3$

$f(x) = \dfrac{2x}{x-1}$

$f'(x) = \dfrac{2(x-1)-(2x)(1)}{(x-1)^2} = \dfrac{\cancel{2x}-2-\cancel{2x}}{(x-1)^2}$

$f'(x) = \dfrac{-2}{(x-1)^2}$

$f'(3) = \dfrac{-2}{(3-1)^2} = \dfrac{-2}{4} = \dfrac{-1}{2}$

Ans $\boxed{E}$

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13. $\int \dfrac{e^{x^2} - 2x}{e^{x^2}}\,dx$

(A) $x - e^{x^2} + C$

(B) $x - e^{-x^2} + C$

(C) $x + e^{-x^2} + C$

(D) $-e^{x^2} + C$

(E) $e^{-x^2} + C$

$\int (e^{x^2} - 2x)e^{-x^2}\,dx$

$\int 1 - 2xe^{-x^2}\,dx$

$\int 1dx - \int 2xe^{-x^2}dx$ ← u-sub

~~$u = e^{-x^2}$~~

~~$\frac{du}{dx} = e^{-x^2} \cdot -2x$~~

~~$dx = \frac{du}{\quad}$~~

power rule

$\int \cancel{2x} \cdot e^u \cdot \dfrac{du}{\cancel{-2x}}$

$-\int -e^u\,du$

$1x + e^u + C$

$\boxed{1x + e^{-x^2} + c}$

$u = -x^2$

$\frac{du}{dx} = -2x$

$dx = \dfrac{du}{-2x}$

Ans $\boxed{C}$

14. Suppose f is a function whose derivative is given by $f'(x) = \frac{(x-1)(x-4)^3}{1+x^4}$. Which of the following statements are true?

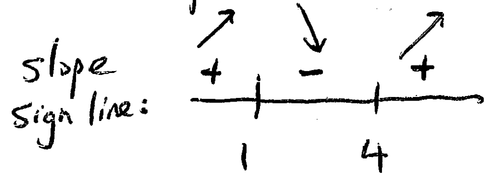
I. The slope of the tangent line to the curve $y = f(x)$ at $x = 2$ is -8. False

II. f is increasing on the interval (1, 4). False, f is decreasing

III. f has a local minimum at $x = 4$. True

(A) I only (B) II only [✓ (C) III only] (D) II and III only (E) I, II, III

*critical points: set $f'(x)=0 \rightarrow x=1, x=4$



$$f'(2) = \frac{(2-1)(2-4)^3}{1+2^4} = \frac{(1)(-2)^3}{17} = \frac{-8}{17} \neq -8$$

Ans
C

15. Let m and b be real numbers and let the function f be defined by

$$f(x) = \begin{cases} 1 + 3bx + 2x^2 & \text{for } x \le 1 \\ mx + b & \text{for } x > 1 . \end{cases}$$

*set piecewise functions equal

*set derivatives equal

If f is both continuous and differentiable at $x = 1$, then

(A) $m = 1, b = 1$

(B) $m = 1, b = -1$

(C) $m = -1, b = 1$

(D) $m = -1, b = -1$

(E) none of the above

set equations equal at x=1

$1+3bx+2x^2 = mx+b$

$1+3b+2 = m+b$

$3+2b = m$

set derivatives equal at x=1

$3b+4x = m+0$

$3b+4(1) = m$

$3+2b = 3b+4$

$\boxed{-1 = b}$

$3b+4 = m$

$3(-1)+4 = m$

$-3+4 = m$

$\boxed{1 = m}$

Ans
B

16. Suppose a car is moving with increasing speed according to the following table.

time (sec)	0	2	4	6	8	10
speed (ft/sec)	30	36	40	48	54	60

The closest approximation of the distance traveled in the first 10 seconds is

(A) 150 ft

(B) 250 ft

(C) 350 ft

[✓ (D)] 450 ft

(E) 550 ft

*Trapezoid Approx: $\frac{w}{2}\left[h_1+2h_2+2h_3+\dots+h_n\right]$

$$\frac{2}{2}\left[30+2(36)+2(40)+2(48)+2(54)+1(60)\right]$$

$$=\underline{446}$$

Ans $\boxed{D}$

17. Consider the function F defined so that $F(x)+5=\int_2^x \sin\left(\frac{\pi t}{4}\right)dt$.

The value of $F(2)+F'(2)$ is

(A) 0

(B) 1

(C) $\frac{\pi}{4}$

(D) 4

[✓ (E)] −4

$$F(x)=\int_2^x \sin\left(\frac{\pi t}{4}\right)dt-5$$

$$F(2)=\int_2^2 \sin\left(\frac{\pi t}{4}\right)dt-5$$

$$=0-5$$

$$=-5$$

SFTC

$$F'(x)=\frac{d}{dx}\int_2^x \sin\left(\frac{\pi t}{4}\right)$$

$$F'(x)=\sin\left(\frac{\pi x}{4}\right)$$

$$F'(2)=\sin\left(\frac{2\pi}{4}\right)=1$$

$$F(2)+F'(2)=-5+1$$

$$=\boxed{-4}$$

Ans $\boxed{E}$