EXAM II
CALCULUS AB
SECTION I PART A
Time–55 minutes
Number of questions–28

A CALCULATOR MAY NOT BE USED ON THIS PART OF THE EXAMINATION

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the box. Do not spend too much time on any one problem.

In this test:

(1) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(2) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1}x = \arcsin x$).

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1. Let $f(x) = 4x^3 - 3x - 1$. An equation of the line tangent to $y = f(x)$ at $x = 2$ is

(A) $y = 25x - 5$

(B) $y = 45x + 65$

(C) $y = 45x - 65$

(D) $y = 65 - 45x$

(E) $y = 65x - 45$

$f(2) = 4(2)^3 - 3(2) - 1 = 32 - 6 - 1 = 25$

$f'(x) = 12x^2 - 3$

$f'(2) = 12(2)^2 - 3 = 45$

point: $(2, 25)$ | $y - 25 = 45(x-2)$

slope: $m = 45$ | $y - 25 = 45x - 90$

$y = 45x - 65$

Ans $\boxed{C}$

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u-sub

2. $\int_0^1 \sin(\pi x)\,dx =$

(A) $\frac{2}{\pi}$ (B) $\frac{1}{\pi}$ (C) 0 (D) $-\frac{2}{\pi}$ (E) $-\frac{1}{\pi}$

$u = \pi x$

$\frac{du}{dx} = \pi$

$dx = \frac{du}{\pi}$

$\int \sin u \cdot \frac{du}{\pi}$

$\frac{1}{\pi}\int \sin u\, du$

$\frac{1}{\pi} \cdot -\cos u$

$-\frac{1}{\pi}\cos(\pi x)\Big]_0^1$

$= -\frac{1}{\pi}\cos(\pi) - \left(-\frac{1}{\pi}\cos 0\right)$

$= -\frac{1}{\pi}(-1) + \frac{1}{\pi}(1) = \boxed{\frac{2}{\pi}}$

Ans $\boxed{A}$

3. $\lim\limits_{h\to 0}\left(\dfrac{\cos(x+h)-\cos x}{h}\right)=$

$\dfrac{\cos x-\cos x}{0}=\dfrac{0}{0} \xrightarrow{\text{L'Hopital's}} \lim\limits_{h\to 0}\dfrac{-\sin(x+h)+0}{1}$

$=-\sin(x+0)$

$=-\sin x$

(A) $\sin x$
(B) $-\sin x$
(C) $\cos x$
(D) $-\cos x$
(E) does not exist

Ans $\boxed{B}$

4. On which of the following intervals, is the graph of the curve $y = x^5 - 5x^4 + 10x + 15$ concave up ?

I. $x < 0$ II. $0 < x < 3$ III. $x > 3$

*concave up when $y''(x) > 0$

(A) I only
(B) II only
(C) III only
(D) I and II only
(E) II and III only

$y' = 5x^4 - 20x^3 + 10$

$y'' = 20x^3 - 60x^2$

$0 = 20x^2(x-3)$

$x = 0, 3$

$\cap \quad \cap \quad \cup$

$- \quad - \quad +$

$0 \quad 3$

Concave up when $x > 3$

Ans $\boxed{C}$

5. The region bounded by the x-axis and the part of the graph of $y = \sin x$ between $x = 0$ and $x = \pi$ is separated into two regions by the line $x = k$. If the area of the region for $0 \le x \le k$ is one-third the area of the region for $k \le x \le \pi$, then $k =$

(A) $\arcsin \frac{1}{3}$
(B) $\arcsin \frac{1}{4}$
(C) $\frac{\pi}{6}$
(D) $\frac{\pi}{3}$
(E) $\frac{\pi}{4}$

$\text{Area} = \int_0^{\pi} \sin x\,dx = -\cos x\Big]_0^{\pi} = -\cos(\pi)-(-\cos 0)$

$= 1 + 1 = 2$

$\int_0^k \sin x\,dx = \frac{1}{2}$

$-\cos x\Big]_0^k = \frac{1}{2}$

$-\cos k - (-\cos 0) = \frac{1}{2}$

$-\cos k + 1 = \frac{1}{2}$

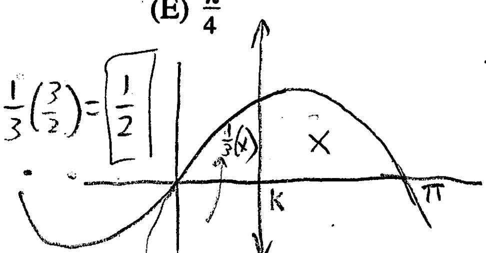
$-\cos k = -\frac{1}{2}$

$\cos k = \frac{1}{2}$

$k = \cos^{-1}\left(\frac{1}{2}\right)$

$\boxed{k = \frac{\pi}{3}}$

$\frac{1}{3}\left(\frac{3}{2}\right) = \boxed{\frac{1}{2}}$



$\frac{x}{3} + \frac{3x}{3} = \frac{4x}{3} = 2$

$4x = 6$

$x = \frac{3}{2}$

Ans $\boxed{D}$

6. A particle starts at time $t = 0$ and moves along a number line so that its position, at time $t \geq 0$, is given by $x(t) = (t-2)^3(t-6)$. The particle is moving to the right for

when $x'(t) = v(t) > 0$

(A) $0 < t < 5$

(B) $2 < t < 6$

(C) $t > 5$

(D) $t \geq 0$

(E) never

*product rule, chain rule

$v(t) = 3(t-2)^2(t-6) + (t-2)^3(1)$

$v(t) = (t-2)^2[3(t-6) + (t-2)]$

$= (t-2)^2[3t-18+t-2]$

$v(t) = (t-2)^2[4t-20]$

$v(t) = (t-2)^2 \cdot 4(t-5)$

$0 = 4(t-2)^2(t-5)$

$t=2,\ t=5$

$-$ $-$ $+$

0 2 5

$v(t) > 0$ when $t > 5$

Ans $\boxed{C}$

7. An antiderivative of $(x^2-1)^2$ is

(A) $\frac{1}{3}(x^2-1)^3 + C$

(B) $\frac{1}{5}x^5 - x + C$

(C) $4x(x^2-1) + C$

(D) $\frac{1}{6x}(x^2-1)^3 + C$

(E) $\frac{1}{5}x^5 - \frac{2}{3}x^3 + x + C$

$\int (x^2-1)^2 dx$ ← Expand, then power rule

$\int (x^2-1)(x^2-1)\,dx$

$\int x^4 - 2x^2 + 1\,dx$

$\boxed{\frac{x^5}{5} - \frac{2x^3}{3} + x + C}$

Ans $\boxed{E}$

8. $\int_{\pi/4}^{\pi/3} \frac{\sec^2 x}{\tan x}\,dx =$ ← u-sub: $u = \tan x$

(A) $\ln\sqrt{3}$ (B) $-\ln\sqrt{3}$ (C) $\ln\sqrt{2}$ (D) $\sqrt{3}-1$ (E) $\ln\frac{\pi}{3} - \ln\frac{\pi}{4}$

$u = \tan x$

$\frac{du}{dx} = \sec^2 x$

$dx = \frac{du}{\sec^2 x}$

$\int \frac{\cancel{\sec^2 x}}{u} \cdot \frac{du}{\cancel{\sec^2 x}}$

$\int \frac{1}{u}\,du$ → $\ln|u| \Big]_1^{\sqrt{3}} = \ln\sqrt{3} - \ln 1$

*convert bounds

If $x = \pi/4$, $u = \tan(\pi/4) = 1$

If $x = \pi/3$, $u = \tan(\pi/3) = \sqrt{3}$

$= \ln\sqrt{3} - 0 = \boxed{\ln\sqrt{3}}$

Ans $\boxed{A}$

9. What is $\lim\limits_{x\to\infty} \dfrac{x^2-6}{2+x-3x^2}$?

* compare degrees b/t numerator and denominator

(A) –3 (B) $-\frac{1}{3}$ (C) $\frac{1}{3}$ (D) 2 (E) The limit does not exist.

$= \boxed{\frac{-1}{3}}$

Ans $\boxed{B}$

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10. $\int_0^2 \sqrt{x^2-4x+4}\,dx$ is:

(A) 1

(B) –1

(C) –2

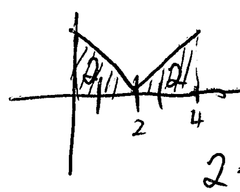
(D) 2

(E) None of the above

$\int \sqrt{(x-2)(x-2)}\,dx = \int \sqrt{(x-2)^2}\,dx$

$\int |x-2|\,dx = \frac{x^2}{2} - 2x \Big]_0^2 = \frac{4}{2} - 4 - (0-0)$

$= |2-4| = \boxed{+2}$



$2+2 = \boxed{4}$ ⟶

Ans $\boxed{D}$

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11. If $g(x) = \dfrac{x-2}{x+2}$, then $g'(2) =$

(A) 1

(B) –1

(C) $\frac{1}{4}$

(D) $-\frac{1}{4}$

(E) 0

* quotient rule

$g'(x) = \dfrac{(1)(x+2) - (x-2)(1)}{(x+2)^2}$

$= \dfrac{x+2-x+2}{(x+2)^2}$

$g'(x) = \dfrac{4}{(x+2)^2}$

$g'(2) = \dfrac{4}{(2+2)^2} = \dfrac{4}{4^2} = \dfrac{4}{16} = \boxed{\frac{1}{4}}$

Ans $\boxed{C}$

12. Which of the following is the solution to the differential equation $\frac{dy}{dx} = y^2$, where $y(-1) = 1$?

plug in $(-1, 1)$

(A) $y = \frac{1}{x}$ for $x \neq 0$

(B) $y = -\frac{1}{x}$ for $x < 0$

(C) $y = -\frac{1}{x}$ for $x > 0$

(D) $y = \frac{1}{x}$ for $x > 0$

(E) $y = \frac{1}{x}$ for $x < 0$

$dy = y^2 dx$

$\frac{dy}{y^2} = dx$

$\int y^{-2} dy = \int dx$

$\frac{y^{-1}}{-1} = x + c$

$-\frac{1}{y} = x + c$

$-\frac{1}{1} = -1 + c$

$-1 = -1 + c$

$0 = c$

$-\frac{1}{y} = x$

$-1 = yx$

$\boxed{\frac{-1}{x} = y}$

Ans $\boxed{B}$

13. The fourth derivative of $f(x) = (2x - 3)^4$ is

(A) $24(2^4)$

(B) $24(2^3)$

(C) $24(2x - 3)$

(D) $24(2^5)$

(E) 0

$f(x) = (2x-3)^4$

$f'(x) = 4(2x-3)^3(2)$

$f''(x) = 2 \cdot 4 \cdot 3(2x-3)^2(2)$

$f'''(x) = 2 \cdot 2 \cdot 4 \cdot 3 \cdot 2(2x-3)^1(2)$

$f^4(x) = 2 \cdot 2 \cdot 2 \cdot 4 \cdot 3 \cdot 2 \cdot (2)$

$= 2^5 \cdot 4 \cdot 3 = 24(2)^4$

Ans $\boxed{A}$

14. If $\int_2^4 f(x)\,dx = 6$, then $\int_2^4 (f(x) + 3)\,dx =$

(A) 3

(B) 6

(C) 9

(D) 12

(E) 15

$= \int_2^4 f(x)dx + \int_2^4 3dx$ power rule

$= 6 + 3x\Big]_2^4 = 3(4) - 3(2)$

$= 12 - 6 = 6$

$6 + 6 = \boxed{12}$

Ans $\boxed{D}$

15. The slope of the tangent line to the curve $2xy + \sin y = 2\pi$ at the point where $y = \pi$ is

(A) -2π

(B) $-\pi$

(C) 0

(D) π

(E) 2π

*implicit differentiation, product rule

$2(y) + 2x\left(\frac{dy}{dx}\right) + \cos y\left(\frac{dy}{dx}\right) = 0$

$\frac{dy}{dx}(2x + \cos y) = -2y$

$\frac{dy}{dx} = \frac{-2y}{2x+\cos y}$

$\left.\frac{dy}{dx}\right|_{(1,\pi)} = \frac{-2\pi}{2+\cos\pi} = \frac{-2\pi}{2-1} = \boxed{-2\pi}$

point: $2x(\pi) + \sin\pi = 2\pi$

$2x\pi + 0 = 2\pi$

$x = 1$

point: $(1, \pi)$

$\boxed{\text{slope: } m = -2\pi}$

Ans $\boxed{A}$

16. If $f(x) = e^{2x}$ and $g(x) = \ln x$, then the derivative of $y = f(g(x))$ at $x = e$ is

(A) e^2

(B) $2e^2$

(C) $2e$

(D) 2

(E) undefined

*chain rule

$y = f[g(x)]$

$y' = f'[g(x)] \cdot g'(x)$

$y'(e) = f'[g(e)] \cdot g'(e)$

$y'(e) = f'(1) \cdot \frac{1}{e}$

$= 2e^2 \cdot \frac{1}{e} = \boxed{2e}$

$f'(x) = e^{2x} \cdot 2$

$g'(x) = \frac{1}{x}$

Ans $\boxed{C}$

17. The area of the region bounded by the lines $x = 1$ and $y = 0$ and the curve $y = xe^{x^2}$ is

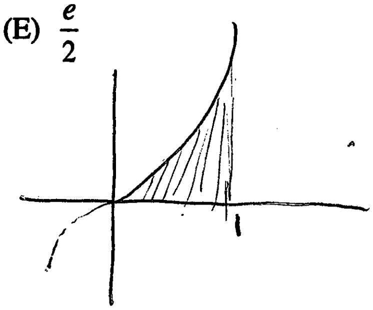
(A) $1 - e$

(B) $e - 1$

(C) $\frac{e-1}{2}$

(D) $\frac{1-e}{2}$

(E) $\frac{e}{2}$



$\text{Area} = \int_0^1 xe^{x^2}\,dx$

$u = x^2$

$\frac{du}{dx} = 2x$

$dx = \frac{du}{2x}$

$\int \cancel{x} \cdot e^u \cdot \frac{du}{2\cancel{x}}$

$\frac{1}{2}\int e^u\,du = \frac{1}{2}e^{x^2}\Big]_0^1 = \frac{1}{2}e^1 - \frac{1}{2}e^0$

$= \frac{1}{2}e - \frac{1}{2}$

$= \boxed{\frac{e-1}{2}}$

Ans $\boxed{C}$

18. If $h(x) = (x^2 - 4)^{3/4} + 1$, then the value of $h'(2)$ is

(A) 3

(B) 2

(C) 1

(D) 0

(E) does not exist

*chain rule

$h'(x) = \frac{3}{4}(x^2-4)^{-1/4}(2x)$

$h'(x) = \frac{3x}{2(x^2-4)^{1/4}}$

$h'(2) = \frac{3(2)}{2[2^2-4]^{1/4}} = \frac{6}{0}$

$h'(2) = DNE$

Ans $\boxed{E}$

19. The derivative of $\sqrt{x} - \frac{1}{x\sqrt[3]{x}}$ is

(A) $\frac{1}{2}x^{-1/2} - x^{-4/3}$

$\boxed{(B)}$ $\frac{1}{2}x^{-1/2} + \frac{4}{3}x^{-7/3}$

(C) $\frac{1}{2}x^{-1/2} - \frac{4}{3}x^{-1/3}$

(D) $-\frac{1}{2}x^{-1/2} + \frac{4}{3}x^{-7/3}$

(E) $-\frac{1}{2}x^{-1/2} - \frac{4}{3}x^{-1/3}$

*power rule

$y = x^{1/2} - \frac{1}{x^{3/3}x^{1/3}} = x^{1/2} - \frac{1}{x^{4/3}}$

$y = x^{1/2} - x^{-4/3}$

$y' = \frac{1}{2}x^{-1/2} - \frac{-4}{3}x^{-7/3}$

$\boxed{y' = \frac{1}{2}x^{-1/2} + \frac{4}{3}x^{-7/3}}$

Ans $\boxed{B}$

20. The function f is continuous at $x = 1$.

If $f(x) = \begin{cases} \frac{\sqrt{x+3} - \sqrt{3x+1}}{x-1} & \text{for } x \neq 1 \\ k & \text{for } x = 1 \end{cases}$

then $k =$

(A) 0 (B) 1 (C) $\frac{1}{2}$ $\boxed{(D)}$ $-\frac{1}{2}$ (E) none of the above

$\lim_{x \to 1} \frac{\sqrt{x+3} - \sqrt{3x+1}}{x-1} \cdot \frac{\sqrt{x+3} + \sqrt{3x+1}}{\sqrt{x+3} + \sqrt{3x+1}}$

$\lim_{x \to 1} \frac{(x+3) - (3x+1)}{(x-1)[\sqrt{x+3} + \sqrt{3x+1}]} \longrightarrow x+3-3x-1$

$\lim_{x \to 1} \frac{2-2x}{(x-1)[\sqrt{x+3} + \sqrt{3x+1}]} = \frac{2\cancel{(1-x)}^{-1}}{\cancel{(x-1)}[\sqrt{x+3} + \sqrt{3x+1}]}$

$= \frac{-1}{\sqrt{4} + \sqrt{4}}$

$= \frac{-2}{2+2} = \frac{-2}{4} = \boxed{\frac{-1}{2}}$

Ans $\boxed{D}$

21. An equation of the normal to the graph of $f(x) = \dfrac{x}{2x-3}$ at $(1, f(1))$ is

(A) $3x + y = 4$

(B) $3x + y = 2$

(C) $x - 3y = -2$

(D) $x - 3y = 4$

(E) $x + 3y = 2$

* perpendicular slope to graph of $f(x)$ at $x=1$

quotient rule

$$f'(x) = \frac{(1)(2x-3) - x(2)}{(2x-3)^2} = \frac{\cancel{2x}-3-\cancel{2x}}{(2x-3)^2}$$

$$f'(1) = \frac{-3}{(2-3)^2} = -3$$

$m_1 = -3$

$\boxed{m_2 = \frac{1}{3}}$

point: $f(1) = \frac{1}{2-3} = -1$ $(1,-1)$

slope: $m = \frac{1}{3}$

$y - y_1 = m(x - x_1)$

$y + 1 = \frac{1}{3}(x-1)$

$3y + 3 = x - 1$

$4 = x - 3y$

$\boxed{x - 3y = 4}$

Ans $\boxed{D}$

22. Let $f(x) = x \ln x$. The minimum value attained by f is

(A) $-\dfrac{1}{e}$

(B) 0

(C) $\dfrac{1}{e}$

(D) -1

(E) There is no minimum.

→ y-value

* 1st derivative test, find derivative using product rule

$f'(x) = 1 \cdot \ln x + x \left(\frac{1}{x}\right)$

$f'(x) = \ln x + 1$

$0 = \ln x + 1$

$e^{\ln x} = e^{-1}$

$x = \frac{1}{e}$

$\searrow$ − $\nearrow$ +

0 $1/e$

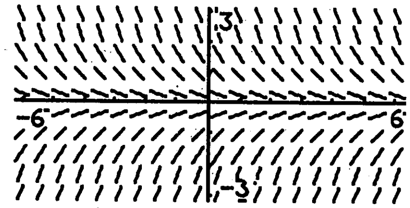
Relative min is $f\left(\frac{1}{e}\right) = \frac{1}{e} \cdot \ln\left(\frac{1}{e}\right)$

$= \frac{1}{e} \cdot \ln(e^{-1})$

$= \boxed{-\frac{1}{e}}$

Ans $\boxed{A}$

23. The slope field for a differential equation $\frac{dy}{dx} = f(x,y)$ is given in the figure. The slope field corresponds to which of the following differential equations?



× (A) $\frac{dy}{dx} = x + y$

× (B) $\frac{dy}{dx} = y^2$

✓ (C) $\frac{dy}{dx} = -y$

× (D) $\frac{dy}{dx} = e^{-x}$

× (E) $\frac{dy}{dx} = 1 - \ln x$

* when $y > 0$, $\frac{dy}{dx} < 0$

* when $y < 0$, $\frac{dy}{dx} > 0$

Ans $\boxed{C}$

24. The average value of $\sec^2 x$ over the interval $0 \le x \le \frac{\pi}{4}$ is

(A) $\frac{\pi}{4}$ (B) $\frac{4}{\pi}$ (C) $\frac{\pi}{8}$ (D) 1 (E) none of the above

Avg. value theorem: $\frac{1}{b-a}\int_a^b f(x)dx$

$= \frac{1}{\frac{\pi}{4}-0}\int_0^{\pi/4} \sec^2 x\,dx$

$= \frac{4}{\pi} \cdot \tan x \Big]_0^{\pi/4}$

$\frac{4}{\pi} \cdot \tan(\pi/4) - \tan(0)$

$\frac{4}{\pi} \cdot [1 - 0] = \boxed{\frac{4}{\pi}}$

Ans $\boxed{B}$

25. Suppose that g is a function that is defined for all real numbers. Which of the following conditions assures that g has an inverse function? Inverse function occurs when $f(x)$ is monotonic

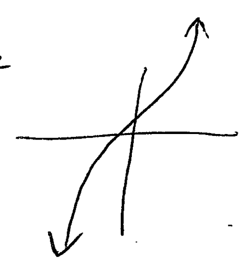
(A) $g'(x) < 1$, for all x → can have pos & neg. slope

(B) $g'(x) > 1$, for all x → all positive slope

(C) $g''(x) > 0$, for all x

(D) $g''(x) < 0$, for all x

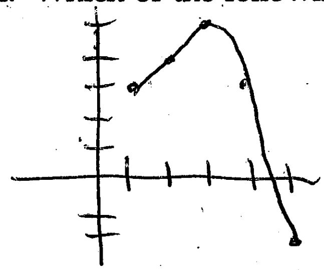
(E) g is continuous.



Ans $\boxed{B}$

26. The function f is continuous and differentiable on the closed interval [1, 5]. The table below gives selected values of f on this interval. Which of the following statements must be TRUE?

x	1	2	3	4	5
$f(x)$	3	4	5	3	−2



X (A) $f'(x)>0$ for $1<x<3$

X (B) $f''(x)<0$ for $3<x<5$

X (C) The maximum value of f on [1, 5] must be 5.

X (D) The minimum value of f on [1, 5] must be −2.

✓(E) There exists a number c, $1 < c < 5$ for which $f(c)=0$.

↳ True, since by IVT, $f(c)=0$ b/c $f(5)<0<f(4)$

Ans $\boxed{E}$

27. If the function G is defined for all real numbers by $G(x) = \int_0^{2x} \cos(t^2)\,dt$, then $G'(\sqrt{\pi}) =$

(A) 2 (B) 1 (C) 0 (D) −1 (E) −2

$G'(x) = \frac{d}{dx}\int_0^{2x} \cos(t^2)dt$

$= \cos(2x)^2 \cdot 2$

$G'(x) = \cos(4x^2)\cdot 2$

$G'(\sqrt{\pi}) = \cos(4\sqrt{\pi}^2)\cdot 2$

$= \cos(4\pi) = +1(2) = \boxed{2}$

$= \cos(2\pi)$

Apply SFTC

$\frac{d}{dx}\int_a^{p(x)} f(t)dt = f(p(x))\cdot p'(x)$ *

Ans $\boxed{A}$

?

28. At time t a particle moving along the x-axis is at position x. The relationship between x and t is given by: $tx = x^2 + 8$. At $x = 2$ the velocity of the particle is

(A) 1

(B) 2

(C) 6

(D) −2

(E) −1

Ans $\boxed{E}$

EXAM II
CALCULUS AB
SECTION I PART B
Time–50 minutes
Number of questions–17

A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS ON THIS PART OF THE EXAMINATION

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the box. Do not spend too much time on any one problem.

In this test:

(1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.

(2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(3) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1}x = \arcsin x$).

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1. Which of the following functions have a derivative at $x = 0$?

I. $y = |x^3 - 3x^2|$

II. $y = \sqrt{x^2 + .01} - |x - 1|$

III. $y = \dfrac{e^x}{\cos x}$

*slope exists at $x=0$ for all three graphs.

(A) None (B) II only (C) III only (D) II and III only (E) I, II, III

Ans $\boxed{E}$

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2. Water is pumped into an empty tank at a rate of $r(t) = 20e^{0.02t}$ gallons per minute. Approximately how many gallons of water have been pumped into the tank in the first five minutes?

(A) 20 gal
(B) 22 gal
(C) 85 gal
(D) 105 gal
(E) 150 gal

$\int_0^5 r(t)\,dt \approx 105$ gallons

Ans $\boxed{D}$

3. Consider the function $f(x) = \dfrac{6x}{a + x^3}$ for which $f'(0) = 3$. The value of a is

*quotient rule

(A) 5
(B) 4
(C) 3
(D) 2
(E) 1

$$f'(x) = \frac{6(a+x^3) - 6x(0+3x^2)}{(a+x^3)^2}$$

$$f'(0) = \frac{6(a+0) - 6(0)(3(0))}{(a+0)^2}$$

$$3 = \frac{6a}{a^2} \quad \Big| \quad 3a^2 = 6a, \quad 3a^2 - 6a = 0 \quad \Big| \quad 3a(a-2) = 0, \quad a = 0, 2$$

Ans $\boxed{D}$

4. Which of the following is true about the function f if $f(x) = \dfrac{(x-1)^2}{2x^2 - 5x + 3}$? $= \dfrac{(x-1)^2}{(2x-3)(x-1)}$

✗ I. f is continuous at $x = 1$.
✗ II. The graph of f has a vertical asymptote at $x = 1$.
✓ III. The graph of f has a horizontal asymptote at $y = \frac{1}{2}$.

*hole at $x = 1$

(A) I only (B) II only (C) III only (D) II and III only (E) I, II, III

$$f(x) = \frac{x^2 - 2x + 1}{2x^2 - \cancel{5}x + 3} \qquad \lim_{x \to \infty} \frac{x^2 - 2x + 1}{2x^2 - 5x + 3} = \boxed{\frac{1}{2}}$$

Ans $\boxed{C}$

5. If $y = u + 2e^u$ and $u = 1 + \ln x$, find $\dfrac{dy}{dx}$ when $x = \dfrac{1}{e}$

(A) e (B) $2e$ (C) $3e$ (D) $\dfrac{2}{e}$ (E) $\dfrac{3}{e}$

$$\frac{dy}{dx} = \frac{du}{dx} + 2e^u\left(\frac{du}{dx}\right)$$

$$u = 1 + \ln x, \qquad \frac{du}{dx} = \frac{1}{x}$$

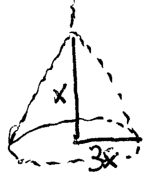
$$\frac{dy}{dx} = \frac{1}{x} + 2e^{1+\ln x}\left(\frac{1}{x}\right)$$

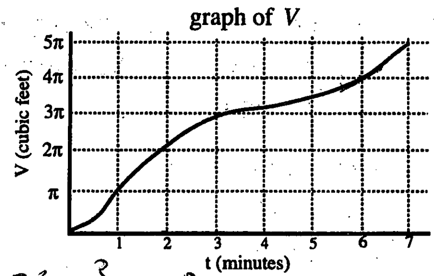
$$\left.\frac{dy}{dx}\right|_{x=\frac{1}{e}} = \frac{1}{1/e} + 2e^{1+\ln(1/e)}\left(\frac{1}{1/e}\right)$$

$$\frac{dy}{dx} = e + 2 \cdot e^1 \cdot e^{\ln e^{-1}} \cdot e$$
$$= e + 2 \cdot e^1 e^{-1} \cdot e^1$$
$$= e + 2e$$
$$= \boxed{3e}$$

Ans $\boxed{C}$

6. ? Sand is being dumped on a pile in such a way that it always forms a cone whose base radius is always 3 times its height. The function V whose graph is sketched in the figure gives the volume of the conical sand pile, $V(t)$, measured in cubic feet, after t minutes. $\left(V(t) = \frac{1}{3}\pi r^2 h\right)$ At what approximate rate is the radius of the base changing after 6 minutes?





$3h = r$ $h = \frac{r}{3}$

$V(6) = 4\pi = \frac{\pi}{9}r^3$ $4\pi \cdot \frac{9}{\pi} = 36 = r^3$ $r = \sqrt[3]{36}$

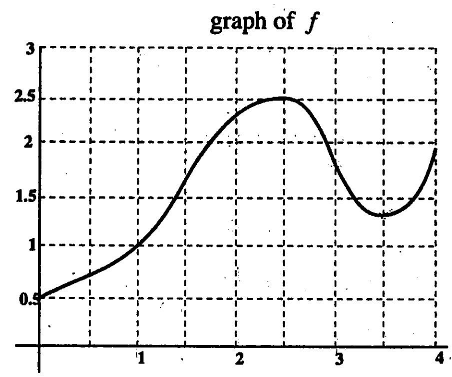
$V = \frac{\pi}{3}(r^2)\left(\frac{r}{3}\right)$ (A) 0.22 ft/min (B) 0.28 ft/min (C) 0.34 ft/min (D) 0.40 ft/min (E) 0.46 ft/min

$V = \frac{\pi}{9}r^3$ $\frac{dV}{dt} = \frac{\pi}{9} \cdot 3r^2\left(\frac{dr}{dt}\right)$ $\pi = \frac{\pi}{9} \cdot 3\left(\sqrt[3]{36}\right)^2 \frac{dr}{dt}$

$\pi = \frac{\pi}{3}\left(\sqrt[3]{36}\right)^2 \frac{dr}{dt}$

Ans $\boxed{A}$

7. A graph of the function f is shown at the right. Which of the following statements are true?



✓ I. $f(1) > f'(3)$ $f'(3) < 0$

✓ II. $\int_1^2 f(x)\,dx > f'(3.5)$ $\rightarrow f'(3.5) = 0$

$f'(2) > f'(2.5)$ ✓

✓ III. $\lim_{h \to 0} \frac{f(2+h) - f(2)}{h} > \frac{f(2.5) - f(2)}{2.5 - 2}$

(A) I only (B) II only (C) I and II only (D) II and III only (E) I, II, III

Ans $\boxed{E}$

? 8. Given: $5x^3 + 40 = \int_a^x f(t)\,dt$. The value of a is

$F(x) - F(a) = 5x^3 + 40$

(A) –2
(B) 2
(C) 1
(D) –1
(E) 0

$15x^2 = f(x)$

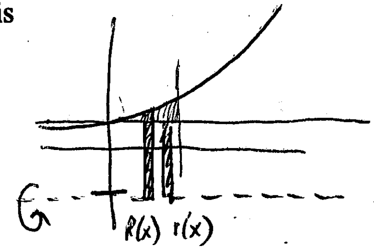
$5x^3 + 40 = \int_a^x 15x^2\,dx$

$\frac{15x^3}{3} - 5a^3 =$

Ans $\boxed{A}$

9. Let R be the region in the first quadrant enclosed by the lines $x = \ln 3$ and $y = 1$ and the graph of $y = e^{x/2}$. The volume of the solid generated when R is revolved about the line $y = -1$ is

(A) 5.128
(B) 7.717
(C) 12.845
(D) 15.482
(E) 17.973



$R(x) = e^{x/2} - (-1) = e^{x/2} + 1$

$r(x) = 1 - (-1) = 2$

* Washer Method

$V = \pi \int_0^{\ln 3} [e^{x/2}+1]^2 - [2]^2 \, dx = 1.632\pi \approx \boxed{5.128}$

Ans $\boxed{A}$

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10. If $\dfrac{dy}{dx} = \dfrac{x\sin(x^2)}{y}$, then y could be

(A) $\sqrt{2 - \cos(x^2)}$
(B) $\sqrt{2} - \cos(x^2)$
(C) $2 - \cos(x^2)$
(D) $\cos(x^2)$
(E) $\sqrt{2 - \cos x}$

$\int y \, dy = \int x \sin(x^2) \, dx$

u-sub: $u = x^2$, $\dfrac{du}{dx} = 2x$, $dx = \dfrac{du}{2x}$

$\int x \sin u \cdot \dfrac{du}{2x}$

$\frac{1}{2}\int \sin u$

$\dfrac{y^2}{2} = -\dfrac{1}{2}\cos(x^2) + C$

$y^2 = -\cos(x^2) + C$

$\boxed{y = \pm\sqrt{C - \cos(x^2)}}$

Ans $\boxed{A}$

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11. If $y = \sin u$, $u = v - \dfrac{1}{v}$, and $v = \ln x$, then value of $\dfrac{dy}{dx}$ at $x = e$ is

(A) 0
(B) 1
(C) $\dfrac{1}{e}$
(D) $\dfrac{2}{e}$
(E) $\cos e$

$\dfrac{dy}{dx} = \cos u \left(\dfrac{du}{dx}\right)$

$u = v - v^{-1}$

$\dfrac{du}{dx} = \dfrac{dv}{dx} + 1v^{-2}\left(\dfrac{dv}{dx}\right)$

$\dfrac{dv}{dx} = \dfrac{1}{x}$

$\dfrac{dv}{dx} = \dfrac{1}{e}$

$\dfrac{du}{dx} = \dfrac{1}{e} + \dfrac{1}{(\ln x)^2}\left(\dfrac{1}{e}\right)$

$= \dfrac{1}{e} + \dfrac{1}{e} = \dfrac{2}{e}$

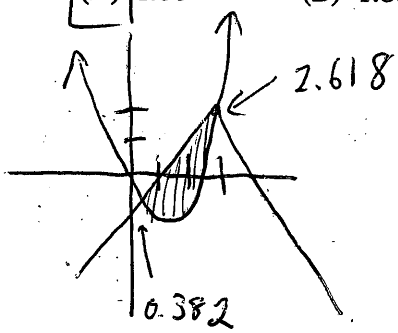
$\dfrac{dy}{dx} = \cos(0) \cdot \dfrac{2}{e}$

$= \boxed{\dfrac{2}{e}}$

Ans $\boxed{D}$

12. The area of the region bounded by the graphs of $y = 2 - |x - 3|$ and $y = x^2 - 2x$ is

$\boxed{\text{(A)}}$ 1.86 (B) 1.88 (C) 1.90 (D) 1.92 (E) 1.94



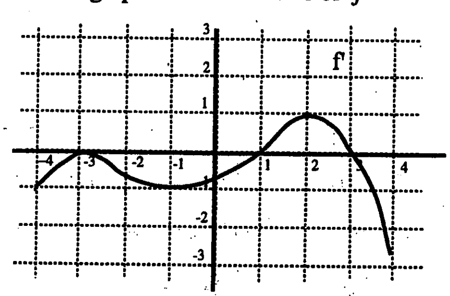
$$\text{Area} = \int_{0.382}^{2.618} x - 1 - (x^2 - 2x) \approx \boxed{1.863}$$

$$y = \begin{cases} 2 - (x-3) = -x + 5, & x > 3 \\ 2 + (x-3) = x - 1, & x < 3 \end{cases}$$

Ans
$\boxed{A}$

13. The figure shows the graph of f', the *derivative* of a function f. The domain of f is the interval $-4 \le x \le 4$. Which of the following are true about the graph of f?

graph of the derivative of f



not at 2

✗ I. At the points where $x = -3$ and $x = 2$ there are horizontal tangents.

✓ II. At the point where $x = 1$ there is a relative minimum point.

✓ III. At the point where $x = -3$ there is an inflection point.

(A) None (B) II only (C) III only (D) II and III only (E) I, II, III

slope sign line for f: $-4 \; \underset{\searrow}{-} \; -3 \; \underset{\searrow}{-} \; 1 \; \underset{\nearrow}{+} \; 3 \; \underset{\searrow}{-} \; 4$

* Rel. min at $x = 1$ since f' changes from $-$ to $+$

Concavity sign line: f: $-4 \; \underset{\cup}{+} \; -3 \; \underset{\cap}{-} \; -1 \; \underset{\cup}{+} \; 2 \; \underset{\cap}{-} \; 4$

* POI at $x = -3$, since f' changes from inc to dec or vice versa

Ans
$\boxed{D}$

14. A differentiable function f has the property that $f(3) = 5$ and $f'(3) = 4$. What is an estimate for $f(2.8)$ using the linear appoximation for f at $x = 3$?

(A) 6.6
(B) 5.8
(C) 5.0
$\boxed{\text{(D)}}$ 4.2
(E) 3.4

point: $(3, 5)$

slope: $m = 4$

$y - 5 = 4(x - 3)$

$y = 4(x-3) + 5$

$y(2.8) \approx 4(2.8 - 3) + 5$

$\approx \boxed{4.2}$

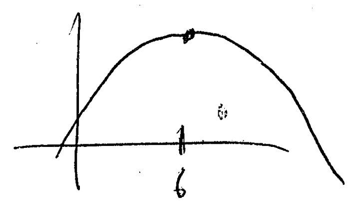
Ans $\boxed{D}$

15. The number of bacteria in a culture is given by $N(t) = 200\ln(t^2 + 36)$, where t is measured in days. On what day is the change in growth a maximum? → Look for POI, where $N''(t) = 0$

(A) 4 (B) 6 (C) 8 (D) 10 (E) 12

$N'(t) = 200 \cdot \frac{1}{t^2+36} \cdot 2t = \frac{400t}{t^2+36}$

$N''(t) = 0$



Ans $\boxed{B}$

16. The acceleration of a particle at time t moving along the x-axis is given by: $a = 4e^{2t}$. At the instant when $t = 0$, the particle is at the point $x = 2$ moving with velocity $v = -2$. The position of the particle at $t = \frac{1}{2}$ is

(A) $e-3$ (B) $e-2$ (C) $e-1$ (D) e (E) $e+1$

$v(t) = \int 4e^{2t}\,dt$ $\quad u = 2t \quad dt = \frac{du}{2} \quad \frac{du}{dt} = 2$

$\int 4 \cdot e^{u} \cdot \frac{du}{2} = 2e^{2t} + c$

$v(t) = 2e^{2t} + C$

$v(0) = 2e^{2(0)} + c$

$-2 = 2e^{0} + c$

$-4 = c$

$v(t) = 2e^{2t} - 4$

$x(t) = \int 2e^{2t} - 4\,dt$

$x(t) = e^{2t} - 4t + C$

$x(0) = e^{0} - 0 + C$

$2 = 1 + C$

$1 = C$

$x(t) = e^{2t} - 4t + 1$

$x(\frac{1}{2}) = e^{1} - 2 + 1 = \boxed{e-1}$

Ans $\boxed{C}$

17. The graph of f', the derivative of a function f, is shown at the right. The graph of f', has a horizontal tangent at $x = 0$.

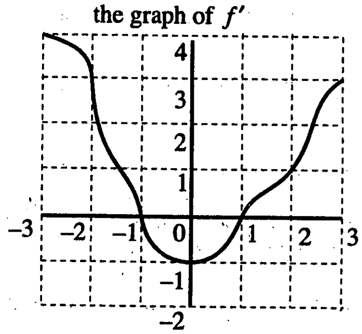
Which of the following statements are true about the function f?

✓ I. f is increasing on the interval $(-2, -1)$.

✓ II. f has an inflection point at $x = 0$.

✗ III. f is concave up on the interval $(-1, 0)$.

the graph of f'



(A) I only (B) II only (C) III only $\boxed{\text{(D)}}$ I and II only (E) II and III only

slope of f: -3 | $+$ ↗ | -1 | $-$ ↘ | 1 | $+$ ↗ | 3

concavity of f: -3 | $-$ ∩ | 0 | $+$ ∪ | 3

Ans $\boxed{D}$