EXAM III
CALCULUS AB
SECTION I PART A
Time–55 minutes
Number of questions–28

A CALCULATOR MAY NOT BE USED ON THIS PART OF THE EXAMINATION

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the box. Do not spend too much time on any one problem.

In this test:

(1) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(2) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1}x = \arcsin x$).

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slope undefined

1. Which of the following is a function with a vertical tangent at $x = 0$?

(A) $f(x) = x^3$ (B) $f(x) = \sqrt[3]{x}$ (C) $f(x) = \frac{1}{x}$ (D) $f(x) = \sin x$ (E) $f(x) = \tan x$

$f(x) = \sqrt[3]{x} = x^{1/3}$ $f(0) = 0$ but $f'(0) = $ undefined

$f'(x) = \frac{1}{3}x^{-2/3}$

$f'(x) = \frac{1}{3x^{2/3}}$

$f'(0) = $ undefined

Ans $\boxed{B}$

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2. $\displaystyle\int_0^5 \frac{dx}{\sqrt{1+3x}} =$

(A) 4 (B) $\frac{8}{3}$ $\boxed{(C)}$ 2 (D) $\frac{16}{9}$ (E) 1

* u-sub

$u = 1 + 3x$

$\frac{du}{dx} = 3$

$dx = \frac{du}{3}$

$\int \frac{1}{u^{1/2}} \cdot \frac{du}{3}$

$\frac{1}{3}\int u^{-1/2}\,du$

* convert bounds:

If $x = 0$, $u = 1 + 3(0) = 1$

If $x = 5$, $u = 1 + 3(5) = 16$

$\frac{1}{3} \cdot \frac{u^{1/2}}{1/2} = \frac{2}{3}u^{1/2}\Big]_1^{16}$

$= \frac{2}{3}(16)^{1/2} - \frac{2}{3}(1)^{1/2}$

$= \frac{2}{3}(4) - \frac{2}{3}(1)$

$= \frac{2}{3}(4-1) = \frac{2}{3} \cdot 3 = \boxed{2}$

Ans $\boxed{C}$

3. Which function is NOT continuous everywhere?

✓ (A) $y = |x|$

✓ (B) $y = x^{2/3}$

✓ (C) $y = \sqrt{x^2 + 1}$

✓ (D) $y = \dfrac{x}{x^2 + 1}$

× [✓ (E)] $y = \dfrac{4x}{(x + 1)^2}$, $x \neq -1$

Ans
E

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4. The area of the first quadrant region bounded by the curve $y = e^{-x}$, the x-axis, the y-axis and the line $x = 2$ is equal to

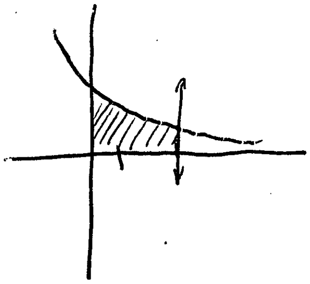
(A) 1

(B) 2

(C) $\ln e^x$

(D) $\dfrac{1}{e^2} - 1$

[✓ (E)] $1 - \dfrac{1}{e^2}$



$$\text{Area} = \int_0^2 e^{-x}\,dx \qquad u = -x \qquad \frac{du}{dx} = -1 \qquad dx = -du$$

$$-\int e^u\,du = -e^{-x}\Big]_0^2 = -e^{-2} - (-e^0)$$

$$= -\frac{1}{e^2} + 1$$

$$= \boxed{1 - \frac{1}{e^2}}$$

Ans
E

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5. If $g(x) = x + \cos x$, then $\lim_{h \to 0} \underbrace{\dfrac{g(x + h) - g(x)}{h}}_{\text{limit definition of derivative}} =$

(A) $\sin x + \cos x$

(B) $\sin x - \cos x$

[✓ (C)] $1 - \sin x$

(D) $1 - \cos x$

(E) $x^2 - \sin x$

$$\boxed{g'(x) = 1 - \sin x}$$

Ans
C

6. $\int_0^4 \frac{2x}{x^2+9}\,dx =$

(A) 25

(B) 16

[✓ (C)] $\ln\frac{25}{9}$

(D) $\ln 4$

(E) $\ln\frac{5}{3}$

*u-sub $\quad$ $u=x^2+9$ $\quad$ $\frac{du}{dx}=2x$ $\quad$ $dx=\frac{du}{2x}$

$\int \frac{\cancel{2x}}{u}\cdot\frac{du}{\cancel{2x}}$

$=\int \frac{1}{u}\,du = \ln|u|+c$

$\ln|x^2+9|\Big]_0^4$

$=\ln|25|-\ln|9|$

$=\ln\left(\frac{25}{9}\right)$

Ans $\boxed{C}$

$\frac{f(x+h)-f(x)}{h}$

7. A function g is defined for all real numbers and has the following property: $g(a+b)-g(a) = 4ab+2b^2$. Find $g'(x)$.

(A) 4

(B) -4

(C) $2x^2$

(D) $4x$

(E) does not exist

* $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

$g'(a)=\lim_{h\to 0}\frac{g(a+b)-g(a)}{b}$

$g'(a)=\lim_{h\to 0}\frac{4ab+2b^2}{b}=\lim_{h\to 0}\frac{b(4a+2b)}{b}$

$g'(a)=4a+0$

$\boxed{g'(x)=4x}$

Ans $\boxed{D}$

8. Given the function defined by $f(x)=x^5-5x^4+3$, find all values of x for which the graph of f is concave up? *$f''(x)>0$

(A) $x>0$

[✓ (B)] $x>3$

(C) $0<x<3$

(D) $x<0$ or $x>3$

(E) $x<0$ or $x>5$

$f'(x)=5x^4-20x^3$

$f''(x)=20x^3-60x^2$

$0=20x^2(x-3)$

$x=0,3$

∩ ∪

− +

0 3

Ans $\boxed{B}$

9. If $f(x) = 2 + |x|$, find the average value of the function f on the interval $-1 \le x \le 3$.

(A) $\frac{7}{4}$ (B) $\frac{9}{4}$ (C) $\frac{11}{4}$ (D) $\frac{13}{4}$ (E) $\frac{15}{4}$

$$\text{Avg. value} = \frac{1}{b-a}\int_a^b f(x)\,dx$$

$$|x| = \begin{cases} x, & x > 0 \\ -x, & x < 0 \end{cases}$$

$$\frac{1}{3-(-1)}\int_{-1}^{3} 2+|x|\,dx = \frac{1}{4}\left[\int_{-1}^{0} 2+-x\,dx + \int_{0}^{3} 2+x\,dx\right]$$

$$\frac{1}{4}\left[2x-\frac{x^2}{2}\right]_{-1}^{0} = +2+\frac{1}{2}$$

$$2x+\frac{x^2}{2}\Big]_0^3 = 6+\frac{9}{2}$$

$$\frac{1}{4}[8+5] = \boxed{\frac{13}{4}}$$

Ans $\boxed{D}$

10. A particle starts at (5, 0) when $t = 0$ and moves along the x-axis in such a way that at time $t > 0$ its velocity is given by $v(t) = \frac{1}{1+t}$. Determine the position of the particle at $t = 3$.

(A) $\frac{97}{16}$

(B) $\frac{95}{16}$

(C) $\frac{79}{16}$

(D) $1 + \ln 4$

(E) $5 + \ln 4$

* final position = initial position + displacement

$$x(b) = x(a) + \int_a^b v(t)\,dt$$

$$x(3) = x(0) + \int_0^3 \frac{1}{1+t}\,dt$$

$$x(3) = 5 + \ln|1+t|\Big]_0^3$$

$$= 5 + \ln 4 - \ln 1 = \boxed{5+\ln 4}$$

Ans $\boxed{E}$

11. If $g(x) = \sqrt[3]{x-1}$ and f is the inverse function of g, then $f'(x) =$

[✓ (A) $3x^2$]

(B) $3(x-1)^2$

(C) $-\frac{1}{3}(x-1)^{-4/3}$

(D) $\frac{1}{3}(x-1)^{2/3}$

(E) does not exist

$$y = \sqrt[3]{x-1}$$

$$x = \sqrt[3]{y-1}$$

$$x^3 = y-1$$

$$y = x^3+1$$

$$f(x) = x^3+1$$

$$\boxed{f'(x) = 3x^2}$$

Ans $\boxed{A}$

12. Suppose $F(x) = \int_0^{\cos x} \sqrt{1+t^3}\,dt$ for all real x, then $F'\left(\frac{\pi}{2}\right) =$

[✓ (A) −1]

(B) 0

(C) $\frac{1}{2}$

(D) 1

(E) $\frac{\sqrt{3}}{2}$

$$*\text{SFTC}: \frac{d}{dx}\int_a^{p(x)} f(t)\,dt = f(p(x))\cdot p'(x)$$

$$F(x) = \int_0^{\cos x} \sqrt{1+t^3}\,dt$$

$$F'(x) = \frac{d}{dx}\int_0^{\cos x} \sqrt{1+t^3}\,dt$$

$$F'(x) = \sqrt{1+\cos^3 x}\cdot(-\sin x)$$

$$F'(\pi/2) = \sqrt{1+[\cos \pi/2]^3}\cdot(-\sin(\pi/2))$$

$$= \sqrt{1+0}\cdot(-1)$$

$$= \boxed{-1}$$

Ans $\boxed{A}$

13. If the line $3x - y + 2 = 0$ is tangent in the first quadrant to the curve $y = x^3 + k$, then $k =$

(A) 5

(B) −5

[✓ (C) 4]

(D) 1

(E) −1

$y = 3x + 2 \rightarrow$ slope: $m = 3$ point: $(1, 5)$

$y = x^3 + k$

$y' = 3x^2$

$3 = 3x^2$

$1 = x^2$ $x = 1$

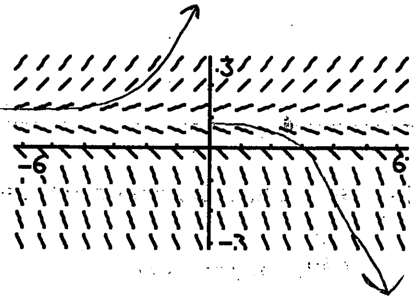
$y = x^3 + k$

$5 = (1)^3 + k$

$\boxed{4 = k}$

Ans $\boxed{C}$

14. The slope field for a differential equation $\frac{dy}{dx} = f(x,y)$ is given in the figure. Which of the following statements are true?



✗ I. A solution curve that contains the point (0, 2) also contains the point (−2, 0).

✓ II. As y approaches 1, the rate of change of y approaches zero.

✓ III. All solution curves for the differential equation have the same slope for a given value of y

(A) I only (B) II only (C) I and II only [✓ (D) II and III only] (E) I, II, III

Ans $\boxed{D}$

15. $\frac{d}{dx}$[Arctan $3x$] =

$* \frac{d}{dx} \text{arctan}\, u = \frac{u'}{1+u^2}$

(A) $\frac{1}{1+9x^2}$

[✓ (B)] $\frac{3}{1+9x^2}$

(C) $\frac{3}{\sqrt{4x^2-1}}$

(D) $\frac{3}{1+3x}$

(E) none of the above

$\frac{d}{dx}\text{arctan}(3x) = \frac{3}{1+(3x)^2} = \boxed{\frac{3}{1+9x^2}}$

Ans $\boxed{B}$

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16. $\lim_{x\to 1} \frac{x^2+2x-3}{x^2-1} =$ $\frac{0}{0} \longrightarrow \lim_{x\to 1} \frac{(x+3)\cancel{(x-1)}}{(x+1)\cancel{(x-1)}} = \frac{4}{2} = \boxed{2}$ ✓

(A) −2

(B) −1

(C) 10

(D) 1

[✓ (E)] 2

or $\frac{0}{0} \xrightarrow{L'H} \lim_{x\to 1} \frac{2x+2}{2x} = \frac{4}{2} = \boxed{2}$ ✓

L'Hopital's Rule

Ans $\boxed{E}$

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17. If f and g are continuous functions such that $g'(x) = f(x)$ for all x, then $\int_2^3 f(x)\,dx =$

(A) $g'(2) - g'(3)$

(B) $g'(3) - g'(2)$

[✓ (C)] $g(3) - g(2)$

(D) $f(3) - f(2)$

(E) $f'(3) - f'(2)$

$\int_2^3 g'(x)dx = g(3) - g(2)$

Ans $\boxed{C}$

* related rates: $\frac{d}{dt}$

18. Let $y = 2e^{\cos x}$. Both x and y vary with time in such a way that y increases at the constant rate of 5 units per second. The rate at which x is changing when $x = \frac{\pi}{2}$ is ; find $\frac{dx}{dt} =$ ____

$\frac{dy}{dt} = 5$ units/sec

(A) 10 units/sec

(B) −10 units/sec

[✓ (C)] −2.5 units/sec

(D) 2.5 units/sec

(E) −0.4 units/sec

$$y = 2e^{\cos x}$$

$$\frac{dy}{dx} = 2e^{\cos x}\cdot(-\sin x)\left(\frac{dx}{dt}\right)$$

$$5 = 2e^{\cos(\pi/2)}\cdot\left(-\sin\left(\pi/2\right)\right)\left(\frac{dx}{dt}\right)$$

$$5 = 2e^{0}(-1)\left(\frac{dx}{dt}\right)$$

$$5 = -2\left(\frac{dx}{dt}\right)$$

$$\frac{-5}{2} = \frac{dx}{dt}$$

$$\boxed{dx/dt = -2.5}$$

Ans $\boxed{C}$

19. $\int_1^2 \frac{dx}{x^3} =$

* power rule, move variable to numerator

[✓ (A)] $\frac{3}{8}$

(B) $-\frac{3}{8}$

(C) $\frac{15}{64}$

(D) $\frac{3}{4}$

(E) $\frac{15}{16}$

$$\int x^{-3}dx = \frac{x^{-2}}{-2}\Big]_1^2 = \frac{-1}{2x^2}\Big]_1^2 = \frac{-1}{8} - \left(\frac{-1}{2}\right)$$

$$= \frac{-1}{8} + \frac{4}{8} = \boxed{\frac{3}{8}}$$

Ans $\boxed{A}$

20. The maximum distance, measured horizontally, between the graphs of $f(x) = x$ and $g(x) = x^2$ for $0 \le x \le 1$, is

(A) 1 (B) $\frac{3}{4}$ (C) $\frac{1}{2}$ [✓ (D)] $\frac{1}{4}$ (E) $\frac{1}{8}$

$$\text{distance} = \sqrt{(x^2 - x)^2} = x^2 - x$$

$$d'(x) = 2x - 1$$

$$0 = 2x - 1$$

$$x = 1/2$$

$$d(x) = x^2 - x$$

$$d(1/2) = \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)$$

$$\frac{1}{4} - \frac{1}{2} = -\frac{1}{4}$$

Ans $\boxed{D}$

$$\left|-\frac{1}{4}\right| = \boxed{\frac{1}{4}}$$

21. Let f be the function defined by $f(x) = \begin{cases} x+1 & \text{for } x < 0 \\ 1+\sin \pi x & \text{for } x \geq 0. \end{cases}$ Then $\int_{-1}^{1} f(x)\,dx =$

(A) $\frac{3}{2}$

(B) $\frac{3}{2} - \frac{2}{\pi}$

(C) $\frac{1}{2} - \frac{2}{\pi}$

[✓ (D)] $\frac{3}{2} + \frac{2}{\pi}$

(E) $\frac{1}{2} + \frac{2}{\pi}$

$$\int_{-1}^{1} f(x)dx = \int_{-1}^{0} x+1dx + \int_{0}^{1} 1+\sin(\pi x)dx$$

$u = \pi x$

$\frac{du}{dx} = \pi$

$dx = \frac{du}{\pi}$

$\int 1+\sin u \cdot \frac{du}{\pi}$

$$\frac{x^2}{2}+x\Big]_{-1}^{0} \qquad x + -\frac{1}{\pi}\cos(\pi x)\Big]_{0}^{1}$$

$$= 0+0-\left(\frac{1}{2}-1\right) + 1 - \frac{1}{\pi}\cos\pi - \left(0-\frac{1}{\pi}\cos 0\right)$$

$$= -\frac{1}{2} + 1 + 1 + \frac{1}{\pi} + \frac{1}{\pi} = \boxed{\frac{3}{2}+\frac{2}{\pi}}$$

Ans $\boxed{D}$

22. Let f be a differentiable function with $f(3) = 4$ and $f'(3) = 8$, and let g be the function defined by $g(x) = x\sqrt{f(x)}$. Which of the following is an equation of the line tangent to the graph of g at the point where $x = 3$?

(A) $y-4 = 8(x-3)$

(B) $y-3 = 8(x-6)$

[✓ (C)] $y-6 = 8(x-3)$

(D) $y-6 = 12(x-3)$

(E) $y-6 = -14(x-3)$

$g(3) = 3\sqrt{f(3)} = 3\cdot\sqrt{4} = 6$ point: $(3,6)$

$g'(x) = 1\cdot\sqrt{f(x)} + x\cdot\frac{1}{2}(f(x))^{-1/2}f'(x)$ ← product rule chain rule

$g'(3) = 1\cdot\sqrt{f(3)} + 3\left(\frac{1}{2}\right)[f(3)]^{-1/2}f'(3)$

$= 1\sqrt{4} + \frac{3}{2}\left(\frac{1}{2}\right)(8) = 2+6 = 8$

point: $(3,6)$ slope: $m=8$

$y-6 = 8(x-3)$ ← $g'(3) = 8$

Ans $\boxed{C}$

23. Let f be the function defined by $f(x) = x^{2/3}(5-2x)$. f is increasing on the interval

(A) $x < -\frac{5}{2}$ (B) $x > 0$ (C) $x < 1$ (D) $0 < x < \frac{5}{8}$ [✓ (E)] $0 < x < 1$

Apply
*First Derivative Test

$f(x) = x^{2/3}(5-2x)$

$f'(x) = \frac{2}{3}x^{-1/3}(5-2x) + x^{2/3}(-2)$

$= \frac{2}{3}\cdot\frac{1}{x^{1/3}}(5-2x) + -2x^{2/3}$

$= \frac{2}{3x^{1/3}}\left[(5-2x) - 3x\right]$

$f'(x) = \frac{2}{x^{1/3}}\left[5-5x\right]$

$0 = \frac{2(5-5x)}{x^{1/3}}$ $x = 0, 1$

↘ ↗ ↘
$-$ $+$ $-$
0 1

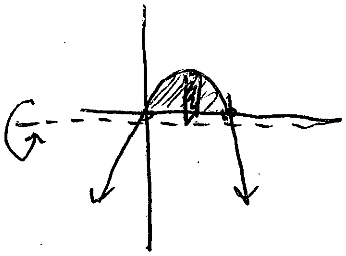
$f(x)$ is increasing in interval $0<x<1$ since $f'(x) > 0$

Ans $\boxed{E}$

24. Let R be the region in the first quadrant bounded by the x-axis and the curve $y = 2x - x^2$. The volume produced when R is revolved about the x-axis is

$y = x(2-x)$

[✓ (A) $\frac{16\pi}{15}$] (B) $\frac{8\pi}{3}$ (C) $\frac{4\pi}{3}$ (D) 16π (E) 8π



*Disc Method

$$V = \pi\int_0^2 [R(x)]^2 dx$$

$$R(x) = 2x - x^2 - 0$$

$$V = \pi\int_0^2 [2x-x^2]^2 dx = \int 4x^2 - 4x^3 + x^4 dx$$

$$\pi \cdot \frac{4x^3}{3} - \frac{4x^4}{4} + \frac{x^5}{5}\Big]_0^2 = \left(\frac{32}{3} - 16 + \frac{32}{5}\right)\pi = \boxed{\frac{16}{15}\pi}$$

Ans $\boxed{A}$

25. What are all values of k for which the graph of $y = 2x^3 + 3x^2 + k$ will have three distinct x-intercepts?

(A) all $k < 0$

(B) all $k > -1$

(C) all k

[✓ (D) $-1 < k < 0$]

(E) $0 < k < 1$

$y' = 6x^2 + 6x$

$0 = 6x(x+1)$

$x = 0, -1$

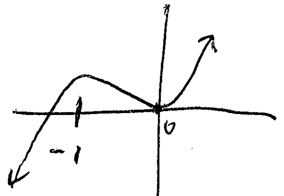
↗ ↘ ↗

\+ − +

−1 0

$y(-1) = -2 + 3 = 1$

$y(0) = 0$



D: $-1 < k < 0$ will ensure the graph will have 3 distinct x-intercepts.

Ans $\boxed{D}$

26. Use the Trapezoid Rule with $n = 4$ to approximate the integral $\int_1^5 f(x)\,dx$ for the function f whose graph is shown at the right.

(A) 7

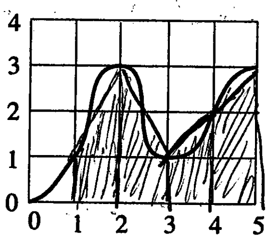
[✓ (B) 8]

(C) 9

(D) 10

(E) 11

graph of $y = f(x)$



$$A \approx \frac{w}{2}\left[h_1 + 2h_2 + 2h_3 + 2h_4 + h_5\right]$$

$$\int_1^5 f(x)dx \approx \frac{1}{2}\left[1 + 2(3) + 2(1) + 2(2) + 3\right]$$

$$= \frac{16}{2} = \boxed{8}$$

Ans $\boxed{B}$

27. A point moves so that x, its distance from the origin at time t, $t \geq 0$ is given by: $x(t) = \cos^3 t$. The first time interval in which the point is moving to the right is $x'(t) > 0$

(A) $0 < t < \frac{\pi}{2}$

(B) $\frac{\pi}{2} < t < \pi$

[✓ (C)] $\pi < t < \frac{3\pi}{2}$

(D) $\frac{3\pi}{2} < t < 2\pi$

(E) none of these

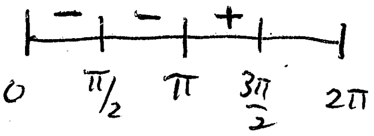
$x(t) = (\cos t)^3$

$x'(t) = 3[\cos t]^2 \cdot -\sin t$

$0 = 3\cos^2 t \cdot -\sin t$

$\cos t = 0$ | $-\sin t = 0$

$t = \frac{\pi}{2}, \frac{3\pi}{2}$ | $t = 0, \pi, 2\pi$



$x'(t) > 0$ in interval $\pi < t < \frac{3\pi}{2}$

Ans $\boxed{C}$

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28. Which of the following is the solution to the differential equation $\frac{dy}{dx} = \frac{x}{y}$, where $y(-2) = -1$?

(A) $y = \sqrt{x^2 - 3}$ for $-\sqrt{3} < x < \sqrt{3}$

(B) $y = -\sqrt{x^2 - 3}$ for $x > \sqrt{3}$

(C) $y = \sqrt{x^2 - 3}$ for $x > \sqrt{3}$

(D) $y = \sqrt{x^2 - 3}$ for $x < -\sqrt{3}$

[✓ (E)] $y = -\sqrt{x^2 - 3}$ for $x < -\sqrt{3}$

$\frac{dy}{dx} = \frac{x}{y}$

$y\,dy = x\,dx$

$\frac{y^2}{2} = \frac{x^2}{2} + C$

$\frac{1}{2} = \frac{4}{2} + C$

$-\frac{3}{2} = C$

* plug in $(-2, -1)$

$\frac{y^2}{2} = \frac{x^2}{2} - \frac{3}{2}$

$y^2 = x^2 - 3$

$y = \pm\sqrt{x^2 - 3}$

$y = \sqrt{x^2 - 3}$ | $\boxed{y = -\sqrt{x^2 - 3}}$

Ans $\boxed{E}$

EXAM III
CALCULUS AB
SECTION I PART B
Time–50 minutes
Number of questions–17

A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS ON THIS PART OF THE EXAMINATION

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the box. Do not spend too much time on any one problem.

In this test:

(1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.

(2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(3) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1}x = \arcsin x$).

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1. The derivative of the function g is $g'(x) = \cos(\sin x)$. At the point where $x = 0$ the graph of g

I. is increasing, II. is concave down, III. attains a relative maximum point.

[✓ (A)] I only (B) II only (C) III only (D) I and III only (E) I, II, III

$g'(0) = \cos[\sin 0]$

$g'(0) = \cos 0 = 1 > 0$ so graph of $g(x)$ is increasing

Ans $\boxed{A}$

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2. The approximate *average* rate of change of the function $f(x) = \int_0^x \sin(t^2)\,dt$ over the interval [1, 3] is

(A) 0.19 [✓ (B)] 0.23 (C) 0.27 (D) 0.31 (E) 0.35

Avg. Rate of change is $\dfrac{f(3)-f(1)}{3-1} \approx \dfrac{0.7736-0.310}{3-1} = \dfrac{0.46356}{2}$

$f(1) = 0.310$

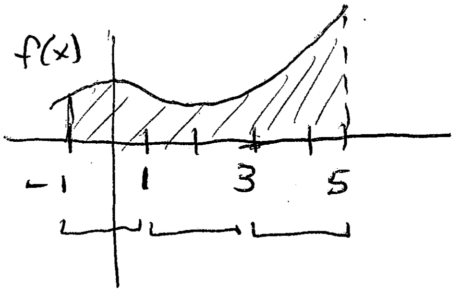
$f(3) = 0.7736$

$\approx \boxed{0.2317}$

Ans $\boxed{B}$

3. When $\int_{-1}^{5} \sqrt{x^3 - x + 1}\, dx$ is approximated by using the mid-points of 3 rectangles of equal width, then the approximation is nearest to

(A) 22.6 [✓ (B)] 22.9 (C) 23.2 (D) 23.5 (E) 23.8



$W = \frac{b-a}{n} = \frac{5-(-1)}{3} = \frac{6}{3} = 2$

$\int_{-1}^{5} f(x)dx \approx 2 \cdot f(0) + 2 \cdot f(2) + 2 \cdot f(4)$

$= 2(1) + 2(2.645) + 2(7.810)$

$= \boxed{22.91}$

Ans $\boxed{B}$

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4. Find the total area of the regions between the graph of the curve $y = x^3 - 5x^2 + 4x$ and the x-axis.

(A) 11.74
(B) 11.77
(C) 11.80
(D) 11.83
(E) 11.86

Ans $\boxed{D}$

---

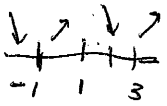
5. The graph of $y = \frac{\sin x}{x}$ has

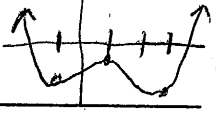
I. a vertical asymptote at $x = 0$
II. a horizontal asymptote at $y = 0$
III. an infinite number of zeros

(A) I only
(B) II only
(C) III only
(D) I and III only
(E) II and III only

Ans $\boxed{E}$

∫f(t)dt





6. The graph of the function f is shown at the right. The graphs of the five functions:

$y = f(x+1)$, → shift one unit left (IV)

$y = f(x)+1$, → shift one unit up (II)

$y = f(-x)$, → reflection (x-axis) III

$y = f'(x)$ and V (use sign-line)

$y = \int_1^x f(t)\,dt$ I.

are shown in the *wrong* order.

The correct order is

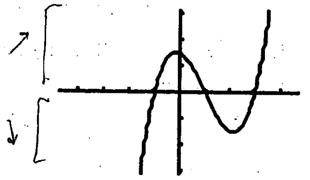
(A) II, IV, III, V, I

(B) IV, II, III, I, V

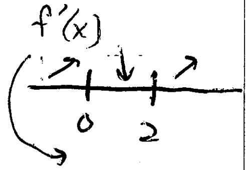
[✓ (C)] IV, II, III, V, I

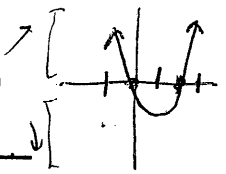
(D) IV, III, II, V, I

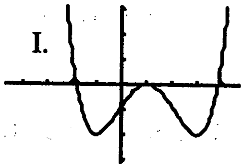
(E) II, IV, III, I, V



the graph of f





I.


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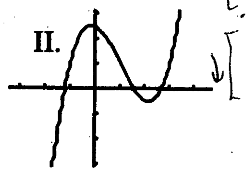
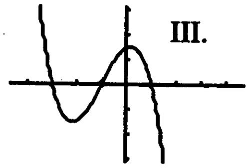
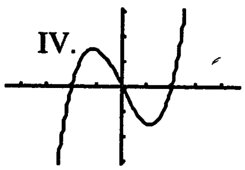
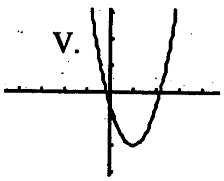


III.


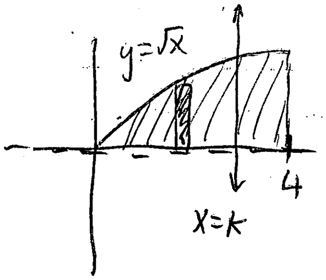
IV.


V.


Ans $\boxed{C}$

7. The region in the first quadrant bounded above by the graph of $y = \sqrt{x}$ and below by the x-axis on the interval $[0, 4]$ is revolved about the x-axis. If a plane perpendicular to the x-axis at the point where $x = k$ divides the solid into parts of equal volume, then $k =$

(A) 2.77 (B) 2.80 [✓ (C)] 2.83 (D) 2.86 (E) 2.89



$$\text{Total Volume} = \pi\int_0^4 [\sqrt{x}]^2 dx = \int x\,dx = \frac{x^2}{2}\Big]_0^4 = \frac{4^2}{2} = \frac{16}{2} = 8\pi$$

$$\pi\int_0^k (\sqrt{x})^2 dx = 4\pi$$

$$\int x\,dx$$

$$\frac{x^2}{2}\Big]_0^k = \pi\frac{k^2}{2} - 0 = 4\pi$$

$$\frac{k^2}{2} = 4$$

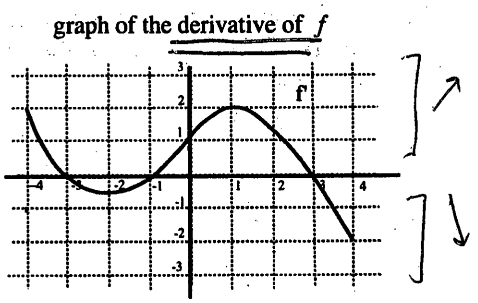
$$k^2 = 8$$

$$k = \sqrt{8}$$

Ans $\boxed{C}$

$\boxed{k \approx 2.828}$

8. The graph of the **derivative** of a function f is shown to the right. If the graph of f' has horizontal tangents at $x = -2$ and 1, which of the following is true about the function f?



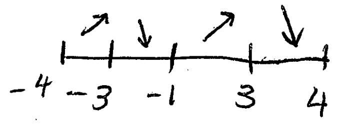
X I. f is increasing on the interval (–2, 1).

✓ II. f is continuous at $x = 0$.

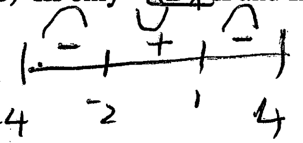
✓ III. The graph of f has an inflection point at $x = -2$.

(A) I only (B) II only (C) III only [✓ (D)] II and III only (E) I, II, III

slope sign line:



concavity sign line



POI at x=-2

Ans $\boxed{\text{D}}$

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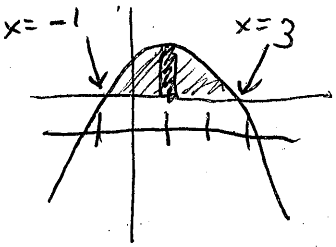
9. The area of the region completely bounded by the curve $y = -x^2 + 2x + 4$ and the line $y = 1$ is



(A) 8.7

(B) 9.7

[✓ (C)] 10.7

(D) 11.7

(E) 12.7

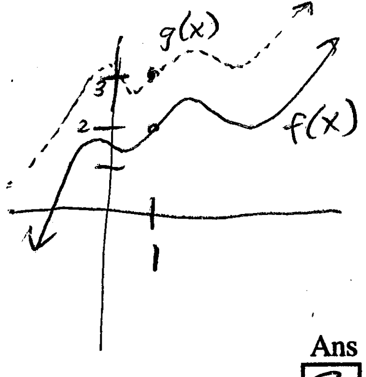
$$Area = \int_{-1}^{3} (-x^2+2x+4) - 1\,dx$$

$$= 10.6\overline{6} \approx \boxed{10.7}$$

Ans $\boxed{\text{C}}$

same slope everywhere

10. If functions f and g are defined so that $\boxed{f'(x) = g'(x)}$ for all real numbers x with $f(1) = 2$ and $g(1) = 3$, then the graph of f and the graph of g



(A) intersect exactly once;

(B) intersect no more than once;

[✓ (C)] do not intersect;

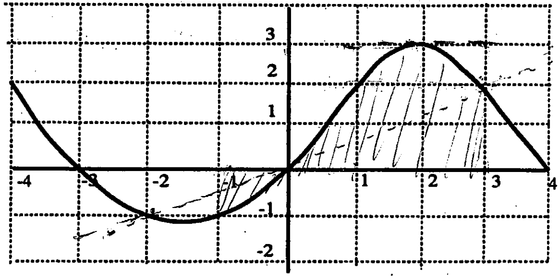
(D) could intersect more than once;

(E) have a common tangent at each point of tangency.

Ans $\boxed{\text{C}}$

11. The graph of a function f whose domain is the interval $[-4, 4]$ is shown in the figure. If the graph of f has horizontal tangents at $x = -1.5$ and 2, which of the following statements are true?

the graph of the function f



Avg. ROC

$$\frac{f(3)-f(-2)}{3-(-2)}$$

$$\frac{2-(-1)}{5} = \frac{3}{5}$$

× I. The average rate of change of f over the interval from $x = -2$ to $x = 3$ is $\frac{1}{5}$.

✓ II. The slope of the tangent line at the point where $x = 2$ is 0.

III. The left-sum approximation of $\int_{-1}^{3} f(t)\,dt$ with 4 equal subdivisions is 4.

A) I only (B) I and II only (C) II and III only (D) I and III only (E) I, II, III

$$\int_{-1}^{3} f(t)dt \approx 1(-1) + 1(0) + 1(2) + 1(3) = -1+2+3 = \boxed{4}$$

Ans $\boxed{C}$

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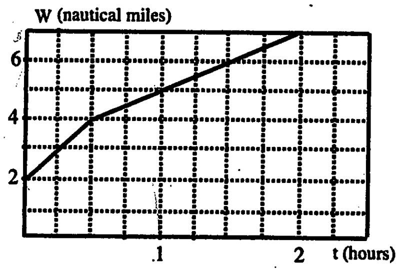
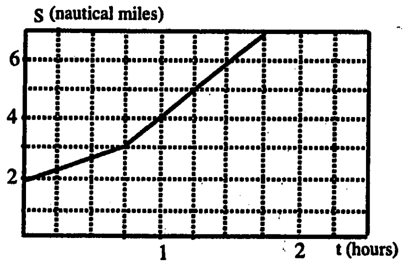
12. One ship traveling west is $W(t)$ nautical miles west of a lighthouse and a second ship traveling south is $S(t)$ nautical miles south of the lighthouse at time t (hours). The graphs of W and S are shown below. At what approximate rate is the distance between the ships increasing at $t = 1$? (nautical miles per hour = knots)





(A) 1 knot (B) 4 knots (C) 7 knots (D) 10 knots (E) 13 knots

Ans $\boxed{B}$

13. Two particles move along the x-axis and their positions at time $0 \le t \le 2\pi$ are given by $x_1 = \cos 2t$ and $x_2 = e^{(t-3)/2} - 0.75$. For how many values of t do the two particles have the same velocity?

(A) 0

(B) 1

(C) 2

(D) 3

(E) 4

Ans

$\boxed{E}$

14. The line $x - 2y + 9 = 0$ is tangent to the graph of $y = f(x)$ at $(3, 6)$ and is also parallel to the line through $(1, f(1))$ and $(5, f(5))$. If f is differentiable on the closed interval $[1, 5]$ and $f(1) = 2$, find $f(5)$.

(A) 2

(B) 3

(C) 4

(D) 5

(E) none of these

Ans

$\boxed{C}$

15. If $\frac{d}{dx}[f(x)] = g(x)$ and $\frac{d}{dx}[g(x)] = f(3x)$, then $\frac{d^2}{dx^2}\left[f(x^2)\right]$ is

[✓ (A)] $4x^2f(3x^2) + 2g(x^2)$

(B) $f(3x^2)$

(C) $f(x^4)$

(D) $2xf(3x^2) + 2g(x^2)$

(E) $2xf(3x^2)$

$\frac{d}{dx} f(x^2) = g(x^2) \cdot 2x$

$\frac{d}{dx}\left[g(x^2) \cdot 2x\right] = f(3x^2) \cdot 2x \cdot 2x + g(x^2) \cdot 2$

$= 4x^2 f(3x^2) + 2g(x^2)$

Ans

A

---

16. The point (1, 9) lies on the graph of an equation $y = f(x)$ for which $\frac{dy}{dx} = 4x\sqrt{y}$ where $x \geq 0$ and $y \geq 0$. When $x = 0$ the value of y is

(A) 6

(B) 4

(C) 2

(D) $\sqrt{2}$

(E) 0

Ans

B

17.

x	-2	1	4	7
$g(x)$	3	1	5	-2

Let g be a continuous, differentiable function defined for all real numbers. Selected values of g are given in the table above. The graph of g must have

(A) two points of inflection and at least one relative maximum

(B) two zeros and at least one relative minimum

(C) one zero and two points of inflection

(D) one point of inflection and at least two relative maxima

(E) at least one point of inflection and at least one relative minimum

Ans

E