EXAM IV
CALCULUS AB
SECTION I PART A
Time–55 minutes
Number of questions–28

A CALCULATOR MAY NOT BE USED ON THIS PART OF THE EXAMINATION

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the box. Do not spend too much time on any one problem.

In this test:

(1) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(2) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).

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1. What is $\lim\limits_{x\to 0}\left(\dfrac{\frac{1}{x-1}+1}{x}\right)$? $\dfrac{0}{0}$

Method 1: Simplify complex fraction

$\boxed{\text{(A)}}$ -1 (B) 0 (C) 1 (D) 2 (E) the limit does not exist

$$\frac{\frac{1}{x-1}+1}{x} = \frac{\frac{1}{x-1}+\frac{x-1}{x-1}}{x} = \frac{1+x-1}{x-1}\cdot\frac{1}{x} \rightarrow \lim_{x\to 0}\frac{1}{x-1} = \frac{1}{-1} = \boxed{-1}$$

OR | L'Hopital's

$$\lim_{x\to 0}\frac{(x-1)^{-1}+1}{x} \rightarrow \lim_{x\to 0}\frac{-1(x-1)^{-2}+0}{1} = \lim_{x\to 0}\frac{-1}{(x-1)^2} = \frac{-1}{1} = \boxed{-1}$$

Ans $\boxed{A}$

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2. $\displaystyle\int \frac{e^{\sqrt{x}}}{2\sqrt{x}}\,dx$ *u-substitution

(A) $\ln\sqrt{x}+C$ (B) $x+C$ (C) e^x+C (D) $\frac{1}{2}e^{2\sqrt{x}}+C$ $\boxed{\text{(E)}}$ $e^{\sqrt{x}}+C$

$u=\sqrt{x}=x^{1/2}$

$\dfrac{du}{dx}=\dfrac{1}{2}x^{-1/2}=\dfrac{1}{2\sqrt{x}}$

$dx=2\sqrt{x}\,du$

$$\int\frac{e^u}{\cancel{2\sqrt{x}}}\cdot\cancel{2\sqrt{x}}\,du$$

$$\int e^u\,du = e^u + C = \boxed{e^{\sqrt{x}}+C}$$

Ans $\boxed{E}$

3. If $y = \frac{3}{4+x^2}$, then $\frac{dy}{dx} =$

(A) $\frac{3}{2x}$

(B) $\frac{3x}{(1+x^2)^2}$

(C) $\frac{6x}{(4+x^2)^2}$

(D) $\frac{-6x}{(4+x^2)^2}$

(E) $\frac{-3}{(4+x^2)^2}$

* chain rule

$y = 3(4+x^2)^{-1}$

$y' = 3 \cdot -1(4+x^2)^{-2}(2x)$

$\boxed{y' = \frac{-6x}{(4+x^2)^2}}$

Ans $\boxed{D}$

4. If $F(x) = \int_1^x (\cos 6t + 1)\,dt$, then $F'(x) =$

(A) $\sin 6x + x$ (B) $\cos 6x + 1$ (C) $\frac{1}{6}\sin 6x + x$

(D) $-\frac{1}{6}\sin 6x + 1$ (E) $\sin 6x + 1$

$F'(x) = \frac{d}{dx}\int_1^x (\cos 6t + 1)dt = \cos 6x + 1$

SFTC

Ans $\boxed{B}$

5. Consider the curve $x + xy + 2y^2 = 6$. The slope of the line tangent to the curve at the point (2,1) is

(A) $\frac{2}{3}$

(B) $\frac{1}{3}$

(C) $-\frac{1}{3}$

(D) $-\frac{1}{5}$

(E) $-\frac{3}{4}$

*Apply Implicit Differentiation (Find $\frac{dy}{dx}$)

* product rule

$1 + 1(y) + x\left(\frac{dy}{dx}\right) + 4y\left(\frac{dy}{dx}\right) = 0$

* Now plug in (2,1) for (x,y)

$1 + 1(1) + 2\left(\frac{dy}{dx}\right) + 4(1)\left(\frac{dy}{dx}\right) = 0$

$2 + 6\left(\frac{dy}{dx}\right) = 0$

$\frac{dy}{dx} = \frac{-2}{6} = \frac{-1}{3}$

$\boxed{m = \frac{-1}{3}}$

Ans $\boxed{C}$

6. $\lim_{h \to 0} \frac{3\left(\frac{1}{2}+h\right)^5 - 3\left(\frac{1}{2}\right)^5}{h} =$ $\frac{0}{0}$ $\xrightarrow{\text{L'Hopital's}}$ $\lim_{h \to 0} \frac{3 \cdot 5\left(\frac{1}{2}+h\right)^4(1) - 0}{1}$

(A) 0

(B) 1

(C) $\frac{15}{16}$

$= 15\left(\frac{1}{2}\right)^4 = \frac{15}{2^4} = \boxed{\frac{15}{16}}$

(D) the limit does not exist

(E) the limit can not be determined

Ans $\boxed{C}$

7. If $p(x) = (x-1)(x+k)$ and if the line tangent to the graph of p at the point $(4, p(4))$ is parallel to the line $5x - y + 6 = 0$, then $k =$

$5x - y + 6 = 0$

$y = 5x + 6$

slope $m = 5$

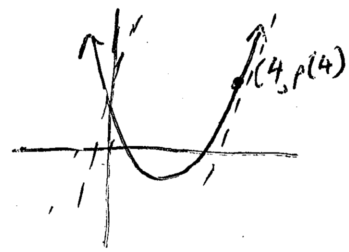
(A) 2

(B) 1

(C) 0

(D) -1

(E) -2



*set $p'(x) = 5$ at $x = 4$

$p'(x) = 1(x+k) + (x-1)(1)$

$= x + k + x - 1 = 2x + k - 1$

$p(x) = x^2 - 1x + kx - k$

$2x + k - 1 = 5$

$2(4) + k - 1 = 5$

$8 + k - 1 = 5$

$\boxed{k = -2}$

Ans $\boxed{E}$

8. If $\cos x = e^y$ and $0 < x < \frac{\pi}{2}$, what is $\frac{dy}{dx}$ in terms of x? *Implicit Differentiation

(A) $-\tan x$

(B) $-\cot x$

(C) $\cot x$

(D) $\tan x$

(E) $\csc x$

$\cos x = e^y$

$-\sin x = e^y\left(\frac{dy}{dx}\right)$

$\frac{dy}{dx} = \frac{-\sin x}{e^y} = \frac{-\sin x}{\cos x} = \boxed{-\tan x}$

substitution, since $\cos x = e^y$

Ans $\boxed{A}$

9. At $t = 0$, a particle starts at the origin with a velocity of 6 feet per second and moves along the x-axis in such a way that at time t its acceleration is $12t^2$ feet per second per second. Through how many feet does the particle move during the first 2 seconds?

Given:
$v(0)=6$
$x(0)=0$

(A) 16 ft
(B) 20 ft
(C) 24 ft
(D) 28 ft
(E) 32 ft

$a(t) = 12t^2$
$v(t) = \int a(t)dt$
$= \int 12t^2 dt$
$v(t) = \frac{12t^3}{3} + c$

$v(t) = 4t^3 + c$
$v(0) = 4(0)^3 + c$
$6 = c$
$v(t) = 4t^3 + 6$
$x(t) = \int v(t)dt$

$x(t) = \int 4t^3 + 6\,dt$
$x(t) = \frac{4t^4}{4} + 6t + c$
$x(t) = t^4 + 6t + c$
$0 = 0 + 0 + c$
$x(t) = t^4 + 6t$
$x(2) = 2^4 + 6(2) = \boxed{28}$ ft

Ans $\boxed{D}$

10. When the area of an expanding square, in square units, is increasing three times as fast as its side is increasing, in linear units, the side is

(A) $\frac{2}{3}$ (B) $\frac{3}{2}$ (C) 3 (D) 2 (E) 1

* Related rates
* Area $= x^2$

$\frac{dA}{dt} = 3\left(\frac{dx}{dt}\right)$

$x = $ ___ ?

$A = x^2$
$\frac{dA}{dt} = 2x\left(\frac{dx}{dt}\right)$
$3\left(\cancel{\frac{dx}{dt}}\right) = 2x\left(\cancel{\frac{dx}{dt}}\right)$

$\boxed{\frac{3}{2} = x}$

Ans $\boxed{B}$

11. The average (mean) value of $\frac{1}{x}$ over the interval $1 \le x \le e$ is

(A) 1 (B) $\frac{1}{e}$ (C) $\frac{1}{e^2} - 1$ (D) $\frac{1+e}{2}$ (E) $\frac{1}{e-1}$

*Avg. value theorem $= \frac{1}{b-a}\int_a^b f(x)dx$

$= \frac{1}{e-1}\int_1^e \frac{1}{x}dx$

$\frac{1}{e-1} \cdot \ln|x| \Big]_1^e$

$\frac{1}{e-1} \cdot (\ln|e| - \ln|1|)$

$= \boxed{\frac{1}{e-1}}$

Ans $\boxed{E}$

12. What is $\lim_{x\to\infty} \dfrac{3x^2+1}{(3-x)(3+x)}$?

(A) –9 $\boxed{\text{(B)}}$ –3 (C) 1 (D) 3 (E) The limit does not exist.

* compare degrees b/t numerator and denominator

$$\lim_{x\to\infty} \frac{3x^2+1}{9+3x-3x-x^2} = \frac{3}{-1} = \boxed{-3}$$

Ans $\boxed{B}$

13. If $\int_{-2}^{2} (x^7 + k)\,dx = 16$, then $k =$

(A) –12

(B) 12

(C) –4

$\boxed{\text{(D)}}$ 4

(E) 0

$$\left.\frac{1}{8}x^8 + kx\right]_{-2}^{2} = \cancel{\tfrac{1}{8}(2)^8} + 2k - \left(\cancel{\tfrac{1}{8}(-2)^8} + -2k\right) = 16$$

$$2k + 2k = 16$$

$$4k = 16$$

$$\boxed{k = 4}$$

Ans $\boxed{D}$

14. Consider the function f defined on $\frac{\pi}{2} \le x \le \frac{3\pi}{2}$ by $f(x) = \dfrac{\tan x}{\sin x}$ for all $x \ne \pi$. If f is continuous at $x = \pi$, then $f(\pi) =$

(A) 2

(B) 1

(C) 0

$\boxed{\text{(D)}}$ –1

(E) –2

* Since there is a hole at $x = \pi$, what ordered pair (point) can we create to fill in hole to create continuous graph?

$$\lim_{x\to\pi} \frac{\tan x}{\sin x} = \frac{0}{0} \rightarrow \tan x \cdot \frac{1}{\sin x} \rightarrow \lim_{x\to\pi} \frac{\cancel{\sin x}}{\cos x} \cdot \frac{1}{\cancel{\sin x}} = \frac{1}{\cos\pi} = -1$$

$$\boxed{f(\pi) = -1}$$

Ans $\boxed{D}$

15. The function $f(x) = x^4 - 18x^2$ has a relative minimum at $x =$

(A) 0 and 3 only

(B) 0 and –3 only

(C) –3 and 3 only

(D) 0 only

(E) –3, 0, 3

* 1st Derivative Test, sign line

$f'(x) = 4x^3 - 36x$

$0 = 4x(x^2 - 9)$

$0 = 4x(x+3)(x-3)$

$x = 0, 3, -3$

$-$ | $+$ | $-$ | $+$ at -3, 0, 3

Rel. min at $x = -3$, $x = 3$ since $f'(x)$ changes sign from $-$ to $+$

Ans $\boxed{C}$

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16. The graph of $y = 3x^5 - 10x^4$ has an inflection point at

(A) (0, 0) and (2, –64)

(B) (0, 0) and (3, –81)

(C) (0, 0) only

(D) (–3, 81) only

(E) (2, –64) only

* find $f''(x)$ create concavity sign line.

$y' = 15x^4 - 40x^3$

$y'' = 60x^3 - 120x^2$

$0 = 60x^2(x - 2)$

$x = 0, 2$

$\cap$ $\cap$ $\cup$

$-$ | $-$ | $+$ at 0, 2

$y(2) = -64$

POI at $x = 2$ since $y''(x)$ change signs.

Ans $\boxed{E}$

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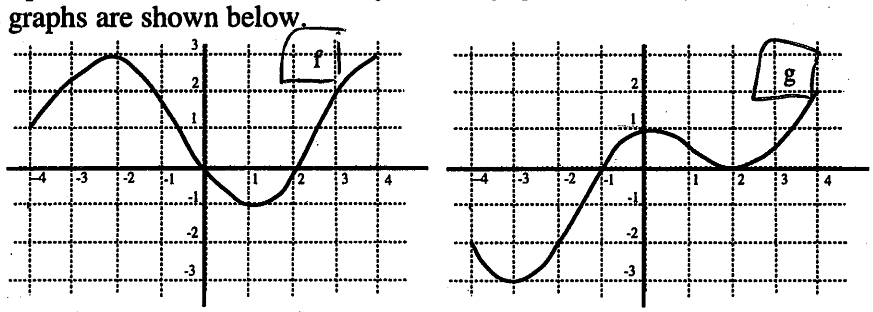
17. The composite function h is defined by $h(x) = f[g(x)]$, where f and g are functions whose graphs are shown below.



The number of points on the graph of h where there are horizontal tangent lines is

(A) 3 (B) 4 (C) 5 (D) 6 (E) 7

$h'(x) = f'[g(x)] \cdot g'(x)$

$0 = f'[g(x)] \cdot g'(x)$

$f'(g(x)) = 0$

$x = 1, 3, 5, -2$

$g'(x) = 0$

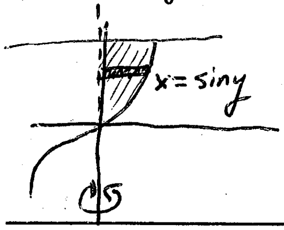
$x = -3, 0, 2$

Ans $\boxed{D}$

18. The region in the first quadrant bounded by the graph of $y = \text{Arcsin } x$, $y = \frac{\pi}{2}$ and the y-axis, is rotated about the y-axis. The volume of the solid generated is given by

(A) $\pi\int_0^{\pi/2} y^2\,dy$ (B) $\pi\int_0^{1} (\text{Arcsin } x)^2\,dx$ (C) $\pi\int_0^{\pi/2} (\text{Arcsin } x)^2\,dx$

(D) $\pi\int_0^{\pi/2} (\sin y)^2\,dy$ (E) $\pi\int_0^{1} (\sin y)^2\,dy$



* Disc Method; About y-axis

$R(y) = \sin y - 0$

$V = \pi\int R(y)^2\,dy$

$V = \pi\int_0^{\pi/2} [\sin y]^2\,dy$

Ans $\boxed{D}$

19. Find the coordinates of the absolute maximum point for the curve $y = xe^{-kx}$ where k is a fixed positive number.

(A) $\left(\frac{1}{k}, \frac{1}{ke}\right)$ (B) $\left(\frac{-1}{k}, \frac{-e}{k}\right)$ (C) $\left(\frac{1}{k}, \frac{1}{e^k}\right)$ (D) $(0, 0)$ (E) there is no maximum

* First Derivative Test
* Apply product, chain rule

$y = xe^{-kx}$

$y' = 1e^{-kx} + xe^{-kx}(-k)$

$0 = \frac{1}{e^{kx}} - \frac{kx}{e^{kx}}$

$0 = \frac{1-kx}{e^{kx}}$

$1 - kx = 0$ $\quad 1 = kx$

$x = \frac{1}{k}$

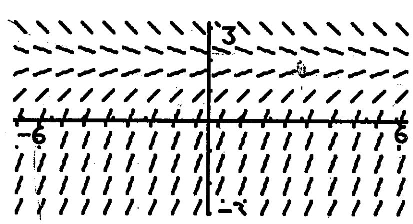
$y\left(\frac{1}{k}\right) = \frac{1}{ke}$

$+ \mid -$ at $\frac{1}{k}$

Abs max at $\left(\frac{1}{k}, \frac{1}{ke}\right)$ since y is always increasing when $x < \frac{1}{k}$ and decreasing when $x > \frac{1}{k}$

Ans $\boxed{A}$

20. The slope field for a differential equation $\frac{dy}{dx} = f(x,y)$ is given in the figure. The slope field corresponds to which of the following differential equations?



(A) $\frac{dy}{dx} = 2 - \ln x$

(B) $\frac{dy}{dx} = 2 - e^{-x}$

(C) $\frac{dy}{dx} = y - 2y^2$

(D) $\frac{dy}{dx} = 2 - y$

(E) $\frac{dy}{dx} = -x^2$

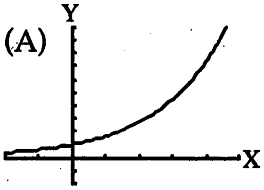
* all slope values are constant for any given y-values
* slope is $\left(\frac{dy}{dx} = 0\right)$ when $y = 2$

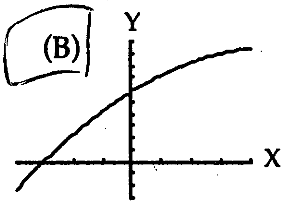
Ans $\boxed{D}$

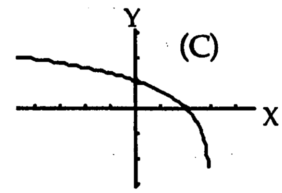
21. If y is a function of x such that $y' > 0$ for all x and $y'' < 0$ for all x, which of the following could be part of the graph of $y = f(x)$?

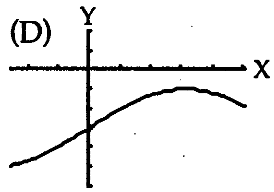
$y' > 0$ (positive slope)

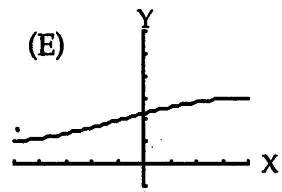
$y'' < 0$ (negative concavity) (concave down)

(A)


(B)


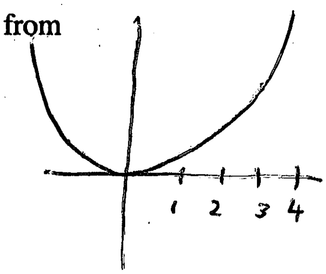
(C)


(D)


(E)


Ans $\boxed{B}$

22. Use the Trapezoid Rule with $n = 3$ to approximate the area under $y = x^2$ from $x = 1$ to $x = 4$.



(A) $\frac{45}{3}$

(B) $\frac{43}{3}$

(C) $\frac{43}{2}$

(D) 43

(E) 21

$* \text{width} = \frac{b-a}{n} = \frac{4-1}{3} = 1$

$\text{Trapezoid Rule} = \frac{w}{2}\left[h_1 + 2h_2 + 2h_3 + h_4\right]$

$= \frac{1}{2}\left[y(1) + 2\cdot y(2) + 2\cdot y(3) + y(4)\right]$

$= \frac{1}{2}\left[1 + 2(4) + 2(9) + 4^2\right]$

$= \frac{1}{2}\left[1+8+18+16\right] = \boxed{\frac{43}{2}}$

Ans $\boxed{C}$

23. If $f(x) = \frac{x^2+1}{e^x}$, then the graph of f is decreasing and concave down on the interval

(A) $(-\infty, 0)$ (B) $(0, 1)$ (C) $(1, 3)$ (D) $(3, 4)$ (E) $(4, \infty)$

quotient rule

* find $f'(x)$, apply 1st Derivative Test
* find $f''(x)$, apply concavity test

$f'(x) = \frac{2x(e^x) - (x^2+1)e^x}{(e^x)^2}$

$0 = \frac{e^x[2x - x^2 - 1]}{e^{2x}} = \frac{2x - x^2 - 1}{e^x}$

$2x - x^2 - 1 = 0$

$x^2 - 2x + 1 = 0$

$(x-1)^2 = 0$

$x = 1$

f': ↓ 1 ↓

$f''(x) = \frac{(2-2x)e^x - (2x-x^2-1)e^x}{(e^x)^2}$

$0 = \frac{e^x[2-2x-2x+x^2+1]}{(e^{2x})}$

$x^2 - 4x + 3 = 0$

$(x-3)(x-1) = 0$

$x = 1, 3$

+ − + (1, 3)

Ans $\boxed{C}$

$f'(x)$ ↓ ↓ ↓

$f''(x)$ ∪ ∩ ∪

1 3

24. The number of bacteria in a culture is growing at a rate of $1500e^{3t/4}$ per unit of time t. At $t = 0$, the number of bacteria present was 2,000. Find the number present at $t = 4$.

$y(0) = 2000$

$y(4) = \_\_?$

(A) $2000e^3$

(B) $6000e^3$

(C) $2000e^6$

(D) $1500e^6$

(E) $1500e^3 + 500$

$y' = 1500e^{3t/4}$

$\frac{dy}{dt} = 1500e^{3t/4}$

$\int dy = \int 1500e^{3t/4}\,dt$ $u = 3t/4$ $\frac{du}{dt} = \frac{3}{4}$ $dt = \frac{4}{3}du$

$y = \frac{4}{3}\int 1500e^{u}\,du$

$y = \frac{4}{3}\cdot 1500e^{3t/4} + c$

$2000 = \frac{4}{3}(1500)e^{0} + c$ $\boxed{C=0}$

$y = 2000e^{3t/4} + 0$

$y(4) = 2000e^{3(4)/4}$

$= \boxed{2000e^{3}}$

Ans $\boxed{A}$

25. A region in the plane is bounded by the graph of $y = \frac{1}{x}$, the x-axis, the line $x = m$ and the line $x = 3m$, $m > 0$. The area of this region

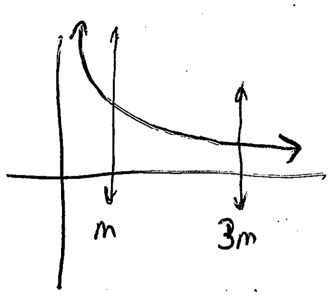
✓ (A) is independent of m

(B) increases as m increases

(C) decreases as m increases

(D) decreases for all $m < \frac{1}{3}$

(E) increases for all $m < \frac{1}{3}$



Ans $\boxed{A}$

26. The formula $x(t) = \ln t + \frac{t^2}{18} + 1$ gives the position of an object moving along the x-axis during the time interval $1 \le t \le 5$. At the instant when the acceleration of the object is zero, the velocity is

(A) 0 (B) $\frac{1}{3}$ (C) $\frac{2}{3}$ (D) 1 (E) undefined

$x'(t) = v(t) = \frac{1}{t} + \frac{2t}{18} + 0 = t^{-1} + \frac{1}{9}t$

$x''(t) = a(t) = -1t^{-2} + \frac{1}{9}$

$0 = \frac{-1}{t^2} + \frac{1}{9}$

$\frac{1}{t^2} = \frac{1}{9}$

$t = 3$

$v(3) = 3^{-1} + \frac{1}{9}(3)$

$v(3) = \frac{1}{3} + \frac{1}{3} = \boxed{\frac{2}{3}}$

Ans $\boxed{C}$

27. $\int 6 \sin x \cos^2 x \, dx =$

(A) $2\sin^3 x + C$

(B) $-2\sin^3 x + C$

(C) $2\cos^3 x + C$

(D) $-2\cos^3 x + C$

(E) $3\sin^2 x \cos^2 x + C$

$6\int \sin x \cdot (\cos x)^2 dx$ *u-substitution

$u = \cos x$

$\frac{du}{dx} = -\sin x$

$dx = \frac{du}{-\sin x}$

$6\int \cancel{\sin x} \cdot u^2 \cdot \frac{du}{\cancel{\sin x}}$

$-6\int u^2 du = -6 \cdot \frac{u^3}{3} + C$

$= \boxed{-2(\cos x)^3 + C}$

Ans $\boxed{D}$

28. If for all $x > 0$, $G(x) = \int_1^x \sin(\ln 2t)\, dt$, then the value of $G''\left(\frac{1}{2}\right)$ is

(A) 0

(B) $\frac{1}{2}$

(C) 1

(D) 2

(E) undefined

$G'(x) = \frac{d}{dx}\int_1^x \sin(\ln 2t) = \sin(\ln 2x)$

$G''(x) = \cos(\ln(2x)) \cdot \frac{2}{2x} = \frac{\cos(\ln(2x))}{x}$

$G''\left(\frac{1}{2}\right) = \frac{\cos(\ln(1))}{1/2} = \frac{1}{1/2} = \boxed{2}$

Ans $\boxed{D}$

EXAM IV
CALCULUS AB
SECTION I PART B
Time–50 minutes
Number of questions–17

A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS ON THIS PART OF THE EXAMINATION

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In this test:

(1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.

(2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(3) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1}x = \arcsin x$).

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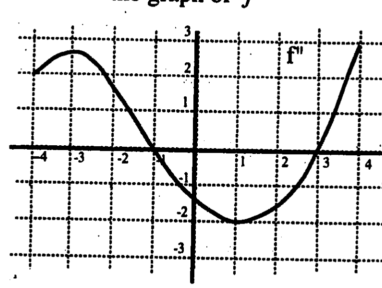
1. The graph of the **second derivative** of a function f is shown at the right. Which of the following is true?

the graph of f''



← $f''(x)=0$

✓ I. The graph of f has an inflection point at $x = -1$.

✓ II. The graph of f is concave down on the interval (–1, 3).

✗ III. The graph of the derivative function f' is increasing at $x = 1$. False since $f(x)$ is concave down at $x=1$

(A) I only (B) II only (C) III only (D) I and II only (E) I, II, III

concavity sign line

∪ ∩ ∪
\+ − +
-4 -1 3 4

Ans $\boxed{D}$

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2. If the function f is continuous for all positive real numbers and if $f(x) = \dfrac{\ln x^2 - x\ln x}{x-2}$ when $x \neq 2$, then $f(2) =$

(A) – 1 (B) –2 (C) $-e$ (D) – ln 2 (E) undefined

$$\lim_{x\to 2}\frac{\ln x^2 - x\ln x}{x-2} = \lim_{x\to 2}\frac{2\ln x - x\ln x}{x-2}$$

$$= \lim_{x\to 2}\frac{-\ln x\cancel{(x-2)}}{\cancel{x-2}} = \boxed{-\ln 2}$$

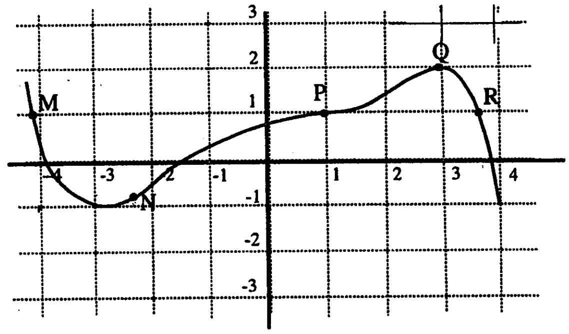
Ans $\boxed{D}$

3. The graph of the function f is shown at the right. At which point on the graph of f are all the following true?

$f(x) > 0$, and $f'(x) < 0$ and $f''(x) < 0$

above x-axis — slope is negative — concave down

the graph of f



(A) M (B) N (C) P (D) Q $\boxed{\text{(E)}}$ R

Ans $\boxed{E}$

4. When using the substitution $u = \sqrt{1+x}$, an antiderivative of $f(x) = 60x\sqrt{1+x}$ is

(A) $20u^3 - 60u + C$

(B) $15u^4 - 30u^2 + C$

(C) $30u^4 - 60u^2 + C$

$\boxed{\text{(D)}}$ $24u^5 - 40u^3 + C$

(E) $12u^6 - 20u^4 + C$

$\int 60x\sqrt{1+x}\,dx$

$u = \sqrt{1+x} = (1+x)^{1/2}$

$\frac{du}{dx} = \frac{1}{2}(1+x)^{-1/2}(1)$ $\quad \frac{du}{dx} = \frac{1}{2\sqrt{1+x}}$

$\int 60(u^2-1)\cdot u \cdot 2\sqrt{1+x}\,du$ $\quad dx = 2\sqrt{1+x}\,du$

$u^2 = 1+x \quad x = u^2-1$

$\int 60(u^2-1)\cdot 2u\cdot u\,du$

$= \int 120u^4 - 120u^2\,du = \frac{120u^5}{5} - \frac{120u^3}{3} + C = \boxed{24u^5 - 40u^3 + C}$

Ans $\boxed{D}$

5. At $x = 0$, which of the following statements is TRUE of the function f defined by $f(x) = \sqrt{x^2 + .0001}$. $= (x^2+0.0001)^{1/2}$

✗ I. f is discontinuous ✓ II. f has a horizontal tangent ✗ III. f' is undefined

(A) I only $\boxed{\text{(B)}}$ II only (C) III only (D) I and III only (E) I, II, III

$f'(x) = \frac{1}{2}(x^2+0.0001)^{-1/2}(2x)$

$f'(x) = \frac{x}{\sqrt{x^2+0.0001}}$

* f has horizontal tangent at $x=0$ ✓

* $f'(0) = \frac{0}{\sqrt{0.0001}}$ ✓

Ans $\boxed{B}$

6. Functions f and g are defined by $f(x) = \frac{1}{x^2}$ and $g(x) = \arctan x$. What is the approximate value of x for which $f'(x) = g'(x)$?

$f(x) = \frac{1}{x^2} = x^{-2}$

(A) −3.36 (B) −2.86 $\boxed{\text{(C)}}$ −2.36 (D) 1.36 (E) 2.36

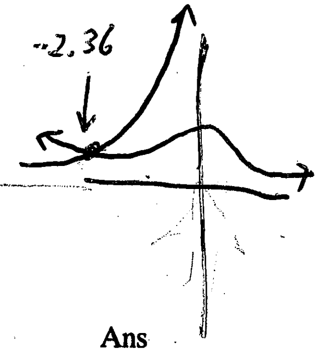
$f'(x) = -2x^{-3} = \frac{-2}{x^3}$

$g(x) = \arctan x$

$g'(x) = \frac{1}{1+x^2}$

$\frac{-2}{x^3} = \frac{1}{1+x^2}$

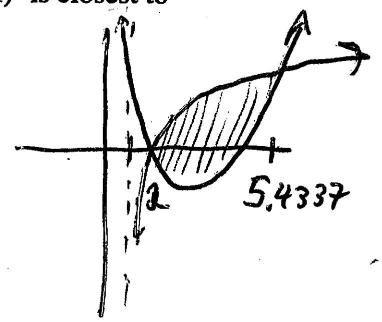
$x \approx -2.36$



Ans $\boxed{C}$

7. The area of the region bounded below by $f(x) = x^2 - 7x + 10$ and above by $g(x) = \ln(x-1)$ is closest to

(A) 7.35
(B) 7.36
(C) 7.38
(D) 7.40
(E) 7.42



$A \approx \int_2^{5.4337} \ln(x-1) - (x^2 - 7x + 10)\,dx$

$\boxed{\text{Area} \approx 7.36}$

Ans $\boxed{B}$

Find Avg. slope

8. The average rate of change of the function $f(x) = \int_0^x \sqrt{1+\cos(t^2)}\,dt$ over the interval [1, 3] is nearest to

(A) 0.85
(B) 0.86
(C) 0.87
(D) 0.88
(E) 0.89

$f(3) = \int_0^3 \sqrt{1+\cos t^2}\,dt = 3.09$

$f(1) = \int_0^1 \sqrt{1+\cos t^2}\,dt = 1.379$

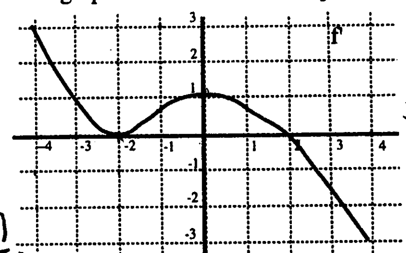
$\text{Avg. RoC} = \frac{f(3) - f(1)}{3-1}$

$= \frac{3.09 - 1.379}{3-1} = \frac{1.715}{2} \approx 0.8576 \approx \boxed{0.86}$

Ans $\boxed{B}$

9. The graph of the *derivative* of f is shown at the right. If the graph of f' has horizontal tangents at $x = -2$ and 0, which of the following is true about the function f?

graph of the derivative of f



$+ \quad + \quad -$
$-4 \quad -2 \quad 2 \quad 4$

× I. f is decreasing at $x = 0$.

✓ II. f has a local maximum at $x = 2$.

✓ III. The graph of f is concave up at $x = -1$.

$\cap \quad \cup \quad \cap$
$- \quad + \quad -$
$-4 \quad -2 \quad 0 \quad 4$

(A) I only (B) II only (C) I and II only $\boxed{\text{(D)}}$ II and III only (E) I, II, III

Ans $\boxed{D}$

10. Let f be the function defined by $f(x) = 3 + \int_2^x \frac{20}{1+t^2}\,dt$. Which of the following is an equation of the line tangent to the graph of f at the point where $x = 2$?

(A) $y = 4(x-2)$
(B) $y - 3 = 7(x-2)$
(C) $y - 2 = 4(x-3)$
$\boxed{\text{(D)}}$ $y - 3 = 4(x-2)$
(E) $y - 2 = 7(x-3)$

$f'(x) = 0 + \frac{d}{dx}\int_2^x \frac{20}{1+t^2}dt = \frac{20}{1+x^2}$

$f'(2) = \frac{20}{1+2^2} = \frac{20}{5} = 4$

$f(2) = 3 + \int_2^2 \frac{20}{1+t^2}dt = 3$

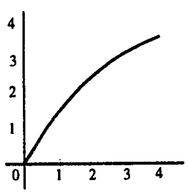
point $(2,3)$ slope: $m = 4$

$y - 3 = 4(x-2)$

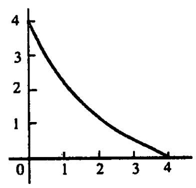
Ans $\boxed{D}$

11. If a left Riemann sum overapproximates the definite integral $\int_0^4 f(x)\,dx$ and a trapezoid sum underapproximates the integral $\int_0^4 f(x)\,dx$, which of the following could be a graph of $y = f(x)$?

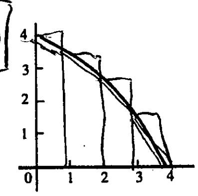
(A)



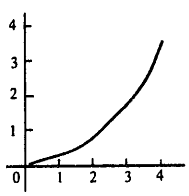
(B)



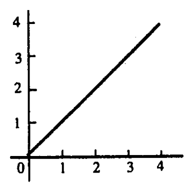
(C)



(D)



(E)

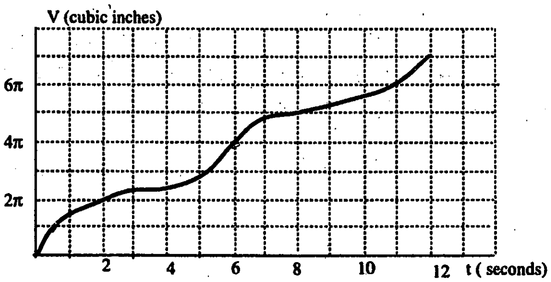


Ans $\boxed{C}$

12. The function V whose graph is sketched below gives the volume of air, $V(t)$, (measured in cubic inches) that a man has blown into a balloon after t seconds.

$$\left(V = \frac{4}{3}\pi r^3\right)$$

The rate at which the radius is changing after 6 seconds is nearest to



$t = 6$

$\frac{dr}{dt} = \_\_\_ ?$

(A) 0.05 in/sec (B) 0.12 in/sec (C) 0.21 in/sec (D) 0.29 in/sec (E) 0.37 in/sec

$$\left.\frac{dV}{dt}\right|_{t=6} \approx \frac{5\pi - 3\pi}{7-5} = \pi \text{ in}^3/\text{sec}$$

$$V(6) = 4\pi$$

$$V = \frac{4}{3}\pi r^3$$

$$4\pi = \frac{4}{3}\pi r^3$$

$$3 = r^3 \quad r = \sqrt[3]{3}$$

$$V = \frac{4}{3}\pi r^3$$

$$\frac{dV}{dt} = \frac{4}{\cancel{3}}\pi \cdot \cancel{3}r^2\left(\frac{dr}{dt}\right)$$

$$\pi = 4\pi(\sqrt[3]{3})^2\left(\frac{dr}{dt}\right)$$

$$\boxed{\frac{dr}{dt} \approx 0.120 \text{ in/sec}}$$

Ans $\boxed{B}$

13. At how many points on the inteval $-2\pi \le x \le 2\pi$ does the tangent to the graph of the curve $y = x\cos x$ have slope $\frac{\pi}{2}$?

* find y' using product rule, set equal to $\pi/2$

(A) 5

(B) 4

(C) 3

(D) 2

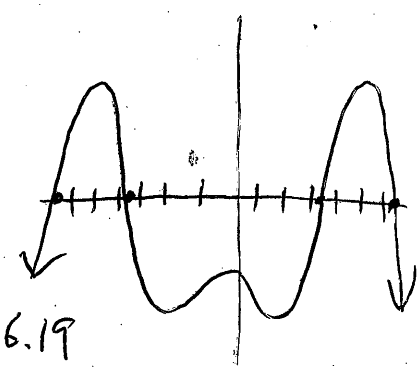
(E) 1

$$y' = 1\cos x + x\cdot(-\sin x)$$

$$\frac{\pi}{2} = \cos x - \sin x$$

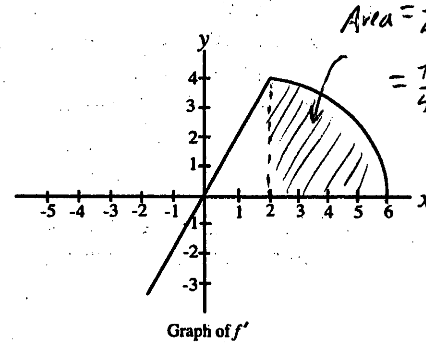
$$\cos x - \sin x - \pi/2 = 0$$

$$x \approx -6.19, -3.808, 3.808, 6.19$$



Ans $\boxed{B}$

14. The graph of $y = f'(x)$, the derivative of a function f, is a line and a quarter-circle shown in the diagram. If $f(2) = 3$, then $f(6) =$



$\text{Area} = \frac{\pi}{4} r^2 = \frac{\pi}{4}(4)^2 = 4\pi$

final pos. = initial pos. + displacement

$x(b) = x(a) + \int_a^b x'(t)\,dt$

(A) 4 (B) 7 $\boxed{\text{(C)}}$ $3+4\pi$ (D) $7+4\pi$ (E) 11

$f(6) = f(2) + \int_2^6 f'(x)\,dx$

$f(6) = 3 + 4\pi$

Ans $\boxed{C}$

15. Let the base of a solid be the first quadrant region bounded above by the graph of $y = \sqrt{x}$ and below by the x-axis on the interval [0, 4]. If every cross section perpendicular to the x-axis is an isosceles right triangle with one leg in the base, then the volume of the solid is

(A) 2

(B) 4

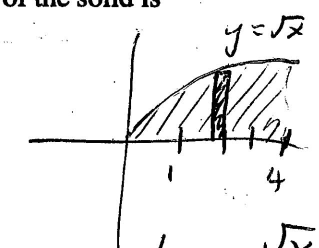
(C) 8

(D) 16

(E) 32

$\text{Area(triangle)} = \frac{1}{2}[\text{base}]^2$

$V = \int_0^4 [\text{Area of cross section}]\,dx$

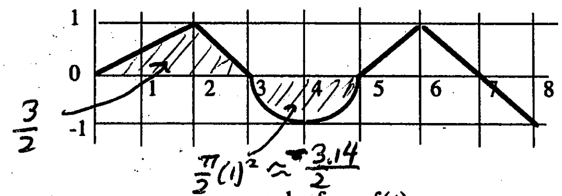


$\text{base} = \sqrt{x} - 0 = \sqrt{x}$

$V = \int_0^4 \frac{1}{2}[\sqrt{x}]^2\,dx = \frac{1}{2}\int x\,dx = \frac{1}{2}\cdot\frac{x^2}{2}\Big]_0^4 = \frac{16}{4} - 0 = \boxed{4}$

Ans $\boxed{B}$

16. Let the function F be defined on the interval $[0, 8]$ by $F(x) = \int_0^x f(t)\,dt$, where the graph of f is shown below. The graph of f consists of four line segments and a semicircle.



graph of $y = f(t)$

In which of the following intervals does F have a zero?

I. $4 < x < 5$ ✓ II. $5 < x < 6$ × III. $6 < x < 7$

(A) I only (B) II only (C) III only (D) I and II only (E) I and III only

Ans $\boxed{D}$

17. The change in N, the number of bacteria in a culture dish at time t, is given by: $\frac{dN}{dt} = 2N$. If $N = 3$ when $t = 0$, the approximate value of t when $N = 1210$ is

(A) 2 (B) 3 (C) 4 (D) 5 (E) 6

(t, N)
$(0, 3)$

$N = 1210$

$\frac{dN}{dt} = 2N$

$\int \frac{dN}{N} = \int 2\,dt$

plug in $(0, 3)$

$\ln|N| = 2t + C$

$\ln|3| = 2(0) + C$

$\ln 3 = C$

$\ln|N| = 2t + \ln 3$

$\ln 1210 = 2t + \ln 3$

$\ln 1210 - \ln 3 = 2t$

$\ln\left|\frac{1210}{3}\right| = 2t$

$5.999 = 2t$

$\boxed{3 \approx t}$

Ans $\boxed{B}$