EXAM V
CALCULUS AB
SECTION I PART A
Time–55 minutes
Number of questions–28

A CALCULATOR MAY NOT BE USED ON THIS PART OF THE EXAMINATION

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the box. Do not spend too much time on any one problem.

In this test:

(1) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(2) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1}x = \arcsin x$).

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1. If $y = \cos^2(2x)$, then $\frac{dy}{dx} =$

*chain rule

$y = [\cos(2x)]^2$

$y' = 2[\cos(2x)]^1 \cdot -\sin(2x) \cdot 2$

$\boxed{y' = -4\cos(2x)\sin(2x)}$

(A) $2\cos 2x \sin 2x$
[✓ (B)] $-4\sin 2x \cos 2x$
(C) $2\cos 2x$
(D) $-2\cos 2x$
(E) $4\cos 2x$

Ans
B

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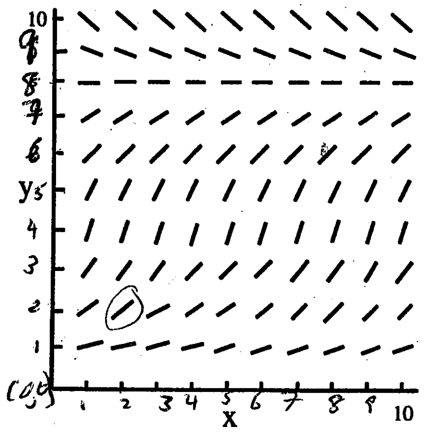
2. A slope field for a differential equation $\frac{dy}{dx} = f(x,y)$ is given in the figure at the right. Which of the following statements are true?



✓ I. The value of $\frac{dy}{dx}$ at the point (2, 2) is approximately 1.

✓ II. As y approaches 8 the rate of change of y approaches zero.

✗ III. All solution curves for the differential equation have the same slope for a given value of x.

(A) I only (B) II only [✓ (C)] I and II only (D) II and III only (E) I, II, III

Ans
C

3. The slope of the line tangent to the graph of $y = \ln\sqrt{x}$ at $(e^2, 1)$ is

(A) $\frac{e^2}{2}$ (B) $\frac{2}{e^2}$ [✓ (C)] $\frac{1}{2e^2}$ (D) $\frac{1}{2e}$ (E) $\frac{1}{e}$

$y = \ln(x)^{1/2}$ | $y' = \frac{1}{2}\left(\frac{1}{x}\right)$

$y = \frac{1}{2}\ln x$ | $y'(e^2) = \frac{1}{2}\left(\frac{1}{e^2}\right) = \frac{1}{2e^2}$

Ans $\boxed{C}$

4. Which of the following functions is both continuous and differentiable at all x in the interval $-2 \le x \le 2$?

(A) $f(x) = |x^2 - 1|$

(B) $f(x) = \sqrt{x^2 - 1}$

[✓ (C)] $f(x) = \sqrt{x^2 + 1}$

(D) $f(x) = \frac{1}{x^2 - 1}$ VA: $x = 1, -1$

(E) none of these

Ans $\boxed{C}$

5. Find the point on the graph of $y = \sqrt{x}$ between (1, 1) and (9, 3) at which the tangent to the graph has the same slope as the line through (1, 1) and (9, 3).

*MVT: $f'(c) = \frac{f(b) - f(a)}{b - a}$

(A) (1, 1)

(B) $(2, \sqrt{2})$

(C) $(3, \sqrt{3})$

[✓ (D)] (4, 2)

(E) none of the above

*Apply MVT: set $y' = \frac{3-1}{9-1} = \frac{2}{8} = \frac{1}{4}$

$y' = \frac{1}{2}x^{-1/2} = \frac{1}{4}$

$\frac{1}{2\sqrt{x}} = \frac{1}{4}$

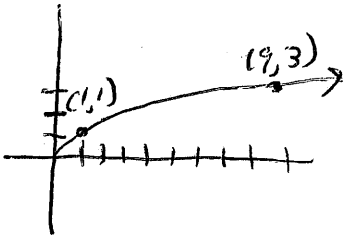
$2\sqrt{x} = 4$

$\sqrt{x} = 2$

$x = 2^2 = 4$

$y(4) = \sqrt{4} = 2$

point: (4, 2)



Ans $\boxed{D}$

6. Consider the function $f(x) = \frac{x^4}{2} - \frac{x^5}{10}$. The *derivative* of f attains its maximum value at $x =$

$\rightarrow$ maximum occurs where $f''(x) = 0$

[✓ (A) 3]

(B) 4

(C) 5

(D) 0

(E) there is no maximum

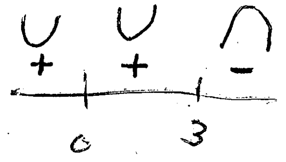
$$f'(x) = \frac{1}{2} \cdot 4x^3 - \frac{1}{10} \cdot 5x^4$$

$$f''(x) = 2 \cdot 3x^2 - \frac{1}{2} \cdot 4x^3$$

$$0 = 6x^2 - 2x^3$$

$$0 = 2x^2(3 - x)$$

$$x = 0, 3$$



max value of $f'(x)$ occurs at $x = 3$ since $f''(x)$ changes from + to −

Ans $\boxed{A}$

7. The acceleration, $a(t)$, of a body moving in a straight line is given in terms of time t by $a(t) = 4 - 6t$. If the velocity of the body is 20 at $t = 0$ and if $s(t)$ is the distance of the body from the origin at time t, what is $s(3) - s(1)$?

(A) −10

(B) 0

(C) 10

(D) 20

[✓ (E) 30]

$$a(t) = 4 - 6t$$

$$v(t) = \int 4 - 6t \, dt$$

$$v(t) = 4t - \frac{6t^2}{2} + c$$

$$20 = 0 - 0 + c$$

$$v(t) = 4t - 3t^2 + 20$$

$$v(0) = 20$$

$$\int_1^3 v(t)dt = s(3) - s(1)$$

$$\int_1^3 4t - 3t^2 + 20 dt = \frac{4t^2}{2} - \frac{3t^3}{3} + 20t \Big]_1^3$$

$$2(3)^2 - 3^3 + 20(3) - (2 - 1 + 20)$$

$$18 - 27 + 60 - 21$$

$$= \boxed{30}$$

Ans $\boxed{E}$

8. $\lim\limits_{x\to1}\left(\frac{\sqrt{x+3}-2}{1-x}\right) = \frac{2-2}{1-1} = \frac{0}{0} \xrightarrow{L'H} \lim\limits_{x \to 1} \frac{\frac{1}{2}(x+3)^{-1/2} - 0}{-1} = \frac{-1}{2(1+3)^{1/2}} = \boxed{\frac{-1}{4}}$

*L'Hopital's

(A) 0.5

(B) 0.25

(C) 0

[✓ (D) −0.25]

(E) −0.5

$= \boxed{-0.25}$

Ans $\boxed{D}$

9. Let f be defined by $f(x) = \begin{cases} \dfrac{x^2-2x+1}{x-1} & \text{for } x \neq 1 \\ k & \text{for } x = 1. \end{cases}$

Determine the value of k for which f is continuous for all real x.

[✓ (A)] 0

(B) 1

(C) 2

(D) 3

(E) none of the above

$$\lim_{x\to 1}\frac{x^2-2x+1}{x-1} = \lim_{x\to 1}\frac{(x-1)\cancel{(x-1)}}{\cancel{(x-1)}} = 0$$

$k=0$

Ans

A

10. The average value of $f(x)=e^{2x}+1$ on the interval $0 \le x \le \frac{1}{2}$ is

[✓ (A)] e (B) $\frac{e}{2}$ (C) $\frac{e}{4}$ (D) $2e-1$ (E) $\frac{e^{2x}+1}{2}$

Avg. value theorem

$$\frac{1}{b-a}\int_a^b f(x)dx$$

$$\frac{1}{\frac{1}{2}-0}\int_0^{1/2} e^{2x}+1dx$$

$$2\left(\frac{1}{2}e^{2x}+x\right)\Big]_0^{1/2}$$

$$e^{2(1/2)}+2(1/2)-(e^0+0)$$

$$e^1+1-1=\boxed{e}$$

Ans

A

11. A particle moves along the x-axis in such a way that its velocity at time $t>0$ is given by $v=\frac{e^t}{t}$. At what value of t does v attain its minimum? set $v'(t)=0$

(A) 0

[✓ (B)] 1

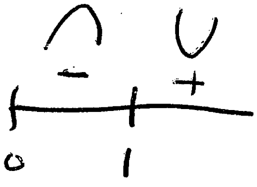
(C) e

(D) -1

(E) There is no minimum value of v.

$$v'(t)=\frac{e^t(t)-e^t(1)}{t^2}$$

$$0=\frac{e^t(t-1)}{t^2} \qquad \cancel{e^t=0} \;\Big|\; \begin{matrix} t-1=0 \\ t=1 \end{matrix}$$



Ans

B

12. $\int \frac{4x}{1+x^2}\,dx =$

(A) $4\text{Arctan}\,x + C$ (B) $\frac{4}{x}\text{Arctan}\,x + C$ (C) $\frac{1}{2}\ln(1+x^2) + C$

[✓ (D)] $2\ln(1+x^2) + C$ (E) $2x^2 + 4\ln|x| + C$

*u-sub $\frac{du}{dx} = 2x$

$u = 1+x^2$ $dx = \frac{du}{2x}$

$\int \frac{4x}{u}\cdot\frac{du}{2x}$

$2\int \frac{1}{u}du = 2\ln|u| + c$

$= \boxed{2\ln|1+x^2| + c}$

Ans $\boxed{D}$

13. Let $f(x) = x^4 + ax^2 + b$. The graph of f has a relative maximum at $(0, 1)$ and an inflection point when $x = 1$. The values of a and b are

$f'(x) = 0$ at $x = 0$

(A) $a = 1$, $b = -6$

(B) $a = 1$, $b = 6$

(C) $a = -6$, $b = 5$

[✓ (D)] $a = -6$, $b = 1$

(E) $a = 6$, $b = 1$

$f'(x) = 4x^3 + 2ax$

$x=1$ $f''(x) = 12x^3 + 2a$

$\to 0 = 12(1)^3 + 2a$

$-12 = 2a$

$\boxed{a = -6}$

$f'(x) = 4x^3 + 2(-6)x$

$0 = 4(0)^3 + 0$

$f(x) = x^4 + -6x^2 + b$

$f(0) = 0^4 - 6(0)^2 + b$

$\boxed{1 = b}$

Ans $\boxed{D}$

14. $\int_1^2 \frac{x^2 - x}{x^3}\,dx =$

[✓ (A)] $\ln 2 - \frac{1}{2}$ (B) $\ln 2 + \frac{1}{2}$ (C) $\frac{1}{2}$ (D) 0 (E) $\frac{1}{4}$

$\int (x^2 - x)x^{-3}dx$

$\int_1^2 x^{-1} - x^{-2}dx$

$\int \frac{1}{x} - x^{-2}dx$

$\ln|x| - \frac{x^{-1}}{-1}$

$\left.\ln|x| + \frac{1}{x}\right]_1^2 = \ln 2 + \frac{1}{2} - (\ln(1) + 1)$

$\ln 2 + \frac{1}{2} - 1$

$\ln 2 - \frac{1}{2}$

Ans $\boxed{A}$

$S=6x^2$

$V=x^3$

$\frac{dx}{dt}=0.2 \text{ in/s}$

$S=150 \text{ in}^2$

$\frac{dV}{dt}=$ ___?

15. The edge of a cube is increasing at the uniform rate of 0.2 inches per second. At the instant when the total surface area becomes 150 square inches, what is the rate of increase, in cubic inches per second, of the volume of the cube?

(A) 5 in^3/sec

(B) 10 in^3/sec

[✓ (C) 15 in^3/sec]

(D) 20 in^3/sec

(E) 25 in^3/sec

$S=6x^2$

$150=6x^2$

$25=x^2$

$5=x$

$V=x^3$

$\frac{dV}{dt}=3x^2\left(\frac{dx}{dt}\right)$

$\frac{dV}{dt}=3(5)^2\left(\frac{1}{5}\right)$

$\frac{dV}{dt}=3\cdot 5=\boxed{15 \text{ in}^3/\text{s}}$

Ans $\boxed{C}$

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16. $\int_0^{\sqrt{3}} \frac{x\,dx}{\sqrt{1+x^2}} =$

(A) $\frac{1}{2}$

[✓ (B) 1]

(C) 2

(D) ln 2

(E) Arctan $2-\frac{\pi}{4}$

u-sub

$u=1+x^2$

$\frac{du}{dx}=2x$

$dx=\frac{du}{2x}$

$\int \frac{x}{u^{1/2}}\cdot\frac{du}{2x} = \frac{1}{2}\int u^{-1/2}\,du$

$\frac{1}{2}\cdot\frac{u^{1/2}}{1/2} = \sqrt{1+x^2}\Big]_0^{\sqrt{3}} = \sqrt{1+3}-\sqrt{1+0}$

$=\sqrt{4}-1$

$=2-1=\boxed{1}$

Ans $\boxed{B}$

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17. Which of the following is true about the graph of $f(x)=\ln|x^2-4|$ in the interval $(-2, 2)$?

✗ (A) f is increasing.

✗ (B) f attains a relative minimum at (0, 0).

✗ (C) f has a range of all real numbers.

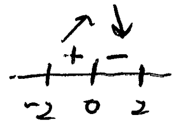
✓ [✓ (D)] f is concave down.

✗ (E) f has an asymptote at $x=0$.

$\ln|(x-2)(x+2)|$

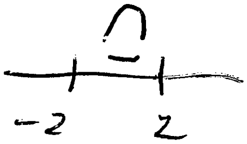
$\ln|x-2|+\ln|x+2|$

$f'(x)=\frac{2x}{x^2-4}$



$f''(x)=\frac{(2)(x^2-4)-(2x)(2x)}{(x^2-4)^2}$

$f''(x)=\frac{-2(x^2+4)}{(x^2-4)^2}=\frac{-2(x^2+4)}{(x^2-4)^2}$



Ans $\boxed{D}$

$=\frac{2x^2-8-4x^2}{(x^2-4)^2}=\frac{-2x^2-8}{(x^2-4)^2}$

18. If $g(x) = \text{Arcsin } 2x$, then $g'(x) =$

(A) $2\text{Arccos } 2x$ (B) $2\csc 2x \cot 2x$ (C) $\frac{2}{1+4x^2}$

(D) $\frac{2}{\sqrt{4x^2-1}}$ [✓ (E) $\frac{2}{\sqrt{1-4x^2}}$]

$\frac{d}{dx}\text{arcsin}\, u = \frac{u'}{\sqrt{1-u^2}}$ $g'(x) = \frac{2}{\sqrt{1-(2x)^2}} = \frac{2}{\sqrt{1-4x^2}}$

Ans $\boxed{E}$

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19. $\int x(x^2-1)^4 \, dx =$

$u = x^2 - 1$ $dx = \frac{du}{2x}$

$\frac{du}{dx} = 2x$

(A) $\frac{1}{10}(x^2)(x^2-1)^5 + C$

[✓ (B) $\frac{1}{10}(x^2-1)^5 + C$]

$\int \cancel{x}\cdot u^4 \cdot \frac{du}{2\cancel{x}}$

(C) $\frac{1}{5}(x^3-x)^5 + C$

(D) $\frac{1}{5}(x^2-1)^5 + C$

$\frac{1}{2}\int u^4 du = \frac{1}{2}\cdot\frac{u^5}{5} + C$

(E) $\frac{1}{5}(x^2-x)^5 + C$

$= \boxed{\frac{1}{10}(x^2-1)^5 + C}$

Ans $\boxed{B}$

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20. If $y = e^{kx}$, then $\frac{d^5y}{dx^5} =$

(A) k^5e^x

$y' = e^{kx}\cdot k$

[✓ (B) k^5e^{kx}]

$y'' = e^{kx}\cdot k^2$

(C) $5!\,e^{kx}$

⋮

(D) $5!\,e^x$

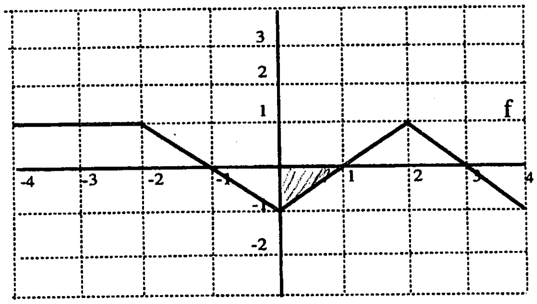
(E) $5e^{kx}$

$y^5(x) = e^{kx}\cdot k^5$

Ans $\boxed{B}$

21. The graph of f is shown at the right. Which of the following statements are true?

the graph of f



$f(2) = 1$

$f'(1) = 1$

X I. $f(2) > f'(1)$

✓ II. $\int_0^1 f(x)\,dx > f'(3.5)$ $\quad -\frac{1}{2} > -1$ ✓

X III. $\int_{-1}^1 f(x)\,dx > \int_{-1}^2 f(x)\,dx$

$-1 > -\frac{1}{2}$

(A) I only [✓ (B) II only] (C) I and II only (D) II and III only (E) I, II, III

Ans $\boxed{B}$

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22. If $g(x) = \sqrt{x}(x-1)^{2/3}$, then the $\boxed{\text{domain of } g' \text{ is}}$

(A) $\{x \mid 0 < x\}$

(B) $\{x \mid x \neq 0 \text{ and } x \neq 1\}$

[✓ (C) $\{x \mid 0 < x < 1 \text{ or } x > 1\}$]

(D) $\{x \mid 0 < x < 1\}$

(E) $\{x \mid \text{all real numbers}\}$

D: $\sqrt{x}$ is $[0, \infty)$

$g'(x) = \frac{1}{2}x^{-1/2}(x-1)^{2/3} + \sqrt{x} \cdot \frac{2}{3}(x-1)^{-1/3}$

$2x-2+3$

$= \frac{(x-1)^{2/3}}{2\sqrt{x}} + \frac{2\sqrt{x}}{3(x-1)^{1/3}} = \frac{2(x-1)+3}{6\sqrt{x}(x-1)^{1/3}}$

$g'(x) = \frac{2x+1}{6\sqrt{x}(x-1)^{1/3}}$ $\quad x > 0, x \neq 1, 0$

Ans $\boxed{C}$

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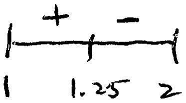
23. A particle moves along the x-axis so that its distance from the origin at time t is given by $10t - 4t^2$. What is the *total* distance covered by the point between $t = 1$ and $t = 2$?



(A) 1.0

(B) 1.5

(C) 2.0

[✓ (D) 2.5]

(E) 3.0

$x(t) = 10t - 4t^2$

$0 = 2t(5-2t)$

$t = 0, 2.5$

$x'(t) = 10 - 8t$

$0 = 2(5-4t)$

$t = 1.25$

$\cancel{\int_1^{5/4} 10t - 4t^2\,dt} + \cancel{\int_{5/4}^{2} 10t - 4t^2\,dt}$

$\cancel{\left[\frac{10t^2}{2} - \frac{4t^3}{3}\right]_1^{5/4}}$

$x(1) = 6$

$x(1.25) = 6.25$ > 0.25

$x(2) = 4$ > 2.25

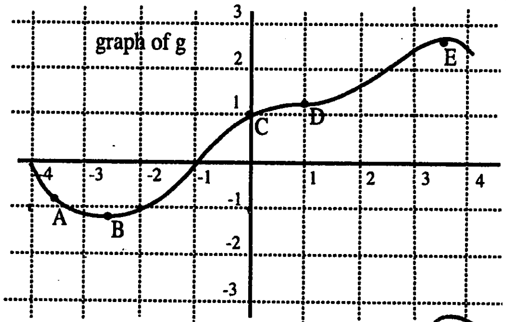
$= \boxed{2.5}$

Ans $\boxed{D}$

24. At which point on the graph of $y = g(x)$ below is $g'(x) = 0$ and $g''(x) = 0$?

$= 0$ concavity is (POI)

* slope is zero



(A) A (B) B (C) C [✓ (D)] D (E) E

Ans $\boxed{D}$

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25. If y is a differentiable function of x, then the slope of the tangent to the curve $xy - 2y + 4y^2 = 6$ at the point where $y = 1$ is → $x(1) - 2 + 4 = 6$ $x = 4$ point $(4,1)$

(A) $\frac{1}{12}$ [✓ (B)] $-\frac{1}{10}$ (C) $-\frac{1}{6}$ (D) $\frac{1}{4}$ (E) $-\frac{5}{6}$

* implicit differentiation
* product rule

$$1(y) + x\left(\frac{dy}{dx}\right) - 2\left(\frac{dy}{dx}\right) + 8y\left(\frac{dy}{dx}\right) = 0$$

$$1(1) + (4)\left(\frac{dy}{dx}\right) + 6\left(\frac{dy}{dx}\right) = 0$$

$$1 + 7\left(\frac{dy}{dx}\right) = 0$$

$$1 = -10\left(\frac{dy}{dx}\right)$$

$$\boxed{\left.\frac{dy}{dx}\right|_{(4,1)} = \frac{-1}{10}}$$

Ans $\boxed{B}$

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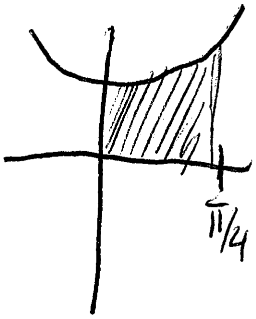
26. The area of the region bounded above by $y = 1 + \sec^2 x$, below by $y = 0$, on the left by $x = 0$ and on the right by $x = \frac{\pi}{4}$ is approximately

(A) 1 (B) 1.25 (C) 1.5 [✓ (D)] 1.75 (E) 2



$$\text{Area} = \int_0^{\pi/4} 1 + \sec^2 x \, dx \approx \boxed{1.75}$$

Ans $\boxed{D}$

27) A solution of the equation $\frac{dy}{dx} + 2xy = 0$ that contains the point $(0, e)$ is

[✓ (A)] $y = e^{1-x^2}$

(B) $y = e^{1+x^2}$

(C) $y = e^{1-x}$

(D) $y = e^{1+x}$

(E) $y = e^{x^2}$

$$\frac{dy}{dx} = -2xy$$

$$dy = -2xy\,dx$$

$$\int \frac{dy}{y} = \int -2x\,dx$$

$$\ln|y| = -\frac{2x^2}{2} + c$$

$$\ln|y| = -x^2 + c$$

plug in $(0, e)$

$$\ln|e| = -0^2 + c$$

$$1 = c$$

$$\ln|y| = -x^2 + 1$$

$$e^{\ln|y|} = e^{-x^2+1}$$

$$|y| = e^{-x^2+1}$$

$$y = e^{-x^2+1}$$

check $(0, e)$

Ans $\boxed{A}$

28. Which of the following are true about the function $F(x) = \int_1^x \ln(2t - 1)\,dt$?

✓ I. $F(1) = 0$ ✓ II. $F'(1) = 0$ ✗ III. $F''(1) = 1$

[✓ (A)] I and II only

(B) I and III only

(C) II and III only

(D) I, II, III

(E) none

$$F'(x) = \frac{d}{dx}\int_0^x \ln(2t-1)\,dt = \ln(2x-1)$$

$$F'(1) = \ln(2(1)-1) = 0$$

$$F''(x) = \frac{2}{2x-1}$$

$$F''(1) = \frac{2}{2-1} = 2$$

Ans $\boxed{A}$

27. A solution of the equation $\frac{dy}{dx} + 2xy = 0$ that contains the point $(0, e)$ is

(A) $y = e^{1-x^2}$

(B) $y = e^{1+x^2}$

(C) $y = e^{1-x}$

(D) $y = e^{1+x}$

(E) $y = e^{x^2}$

Ans ☐

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28. Which of the following are true about the function $F(x) = \int_1^x \ln(2t - 1)\,dt$?

I. $F(1) = 0$ II. $F'(1) = 0$ $F''(1) = 1$

(A) I and II only

(B) I and III only

(C) II and III only

(D) I, II, III

(E) none

Ans ☐

EXAM V
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SECTION I PART B
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Number of questions–17

A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS ON THIS PART OF THE EXAMINATION

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the box. Do not spend too much time on any one problem.

In this test:

(1) The <u>exact</u> numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.

(2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(3) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1}x = \arcsin x$).

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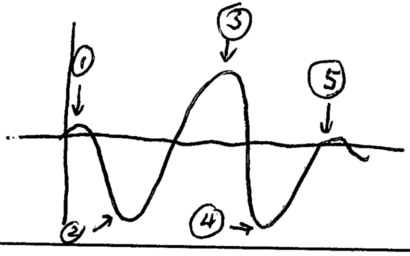
1. How many points of inflection does the graph of $y = \cos x + \frac{1}{3}\cos 3x - \frac{1}{5}\cos 5x$ have on the interval $0 \le x \le \pi$?

$y' = -\sin x - \frac{1}{3}\sin 3x \cdot 3 + \frac{1}{5}\sin 5x \cdot 5$

$y' = -\sin x - \sin 3x + \sin 5x$

(A) 1
(B) 2
(C) 3
(D) 4
[✓ (E)] 5



Ans $\boxed{E}$

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2. Oil is leaking from a tanker at the rate of $R(t) = 500e^{-0.2t}$ gallons per hour, where t is measured in hours. The amount of oil that has leaked out after 10 hours is closest to

(A) 2140 gals
(B) 2150 gals
[✓ (C)] 2160 gals
(D) 2170 gals
(E) 2180 gals

$\text{Amount} = \int_0^{10} R(t)dt \approx 2161.662$

Ans $\boxed{C}$

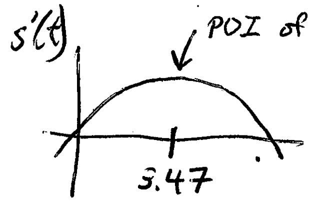
3. The sale of lumber S (in millions of square feet) for the years 1980 to 1990 is modeled by the function

$$S(t) = 0.46\cos(0.45t + 3.15) + 3.4$$

where t is the time in years with $t = 0$ corresponding to the beginning of 1980. Determine the year when lumber sales were increasing at the greatest rate. where $s''(t)$ changes from + to − (POI)

(A) 1982
[✓ (B) 1983]
(C) 1984
(D) 1985
(E) 1986

$$s'(t) = 0.46(-\sin(0.45t+3.15))\cdot 0.45 + 0$$



Ans $\boxed{B}$

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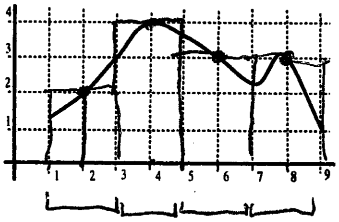
4. The graph of f over the interval [1, 9] is shown in the figure. Using the data in the figure, find a midpoint approximation with 4 equal subdivisions for $\int_1^9 f(x)\,dx$.

width $= \frac{b-a}{n} = \frac{9-1}{4} = 2$

the graph of f



(A) 20 (B) 21 (C) 22 (D) 23 [✓ (E) 24]

$$\int_1^9 f(x)dx \approx 2\cdot f(2) + 2\cdot f(4) + 2\cdot f(6) + 2\cdot f(8)$$
$$2\cdot 2 + 2\cdot 4 + 2\cdot 3 + 2\cdot 3 = \boxed{24}$$

Ans $\boxed{E}$

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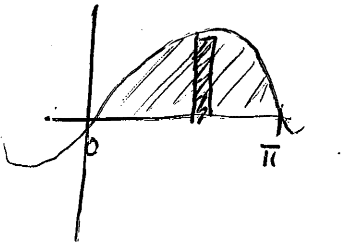
5. Let the base of a solid be the first quadrant region enclosed by the x-axis and one arch of the graph of $y = \sin x$. If all cross sections perpendicular to the x-axis are squares, then the volume of the solid is approximately

(A) 0.52
(B) 0.79
(C) 1.05
[✓ (D) 1.57]
(E) 2.00



Area $= [\text{base}]^2$

$$\text{Volume} = \int_0^{\pi} [\sin x]^2\,dx \approx \boxed{1.571}$$

Ans $\boxed{D}$

6. If $f(x) = 2x + \sin x$ and the function g is the inverse of f, then $g'(2) =$

(A) 0.32
(B) 0.34
[✓ (C)] 0.36
(D) 0.38
(E) 0.40

$2 = 2x + \sin x$

$0 = 2x + \sin x - 2$

$x \approx 0.684$

$f(\overset{0.684}{\downarrow}) = 2 \quad | \quad g(2) = 0.684$

$f'(\underset{0.684}{\uparrow}) \approx 2.775 \quad | \quad g'(2) = \dfrac{1}{2.775}$

$\approx \boxed{0.36}$

$f'(x) = 2 + \cos x$

$f'(0.684) = 2 + \cos(0.684)$

$= 2.775$

Ans $\boxed{C}$

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7. Administrators at Massachusetts General Hospital believe that the hospital's expenditures $E(B)$, measured in dollars, are a function of how many beds B are in use with

$$E(B) = 14000 + (B + 1)^2.$$

On the other hand, the number of beds B is a function of time t, measured in days, and it is estimated that

$$B(t) = 20\sin\left(\frac{t}{10}\right) + 50.$$

At what rate are the expenditures decreasing when $t = 100$?

(A) 120 dollars/day
(B) 125 dollars/day
(C) 130 dollars/day
[✓ (D)] 135 dollars/day
(E) 140 dollars/day

$B = 20\sin\left(\frac{100}{10}\right) + 50$

$B \approx 39.119$

$E'(B) = 0 + 2(B+1)' \cdot B'(t)$ * chain rule

$E'(39.119) = 2(39.119 + 1)(-1.678) \approx -134.25$ dollars/day

$B'(t) = 20\cos\left(\frac{t}{10}\right)\left(\frac{1}{10}\right)$

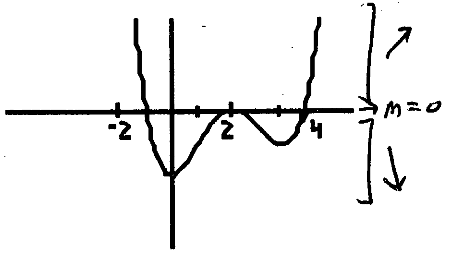
$B'(100) = 20\cos(10)\left(\frac{1}{10}\right)$

$= -1.678$

Ans $\boxed{D}$

8. Let f be a function that has domain $[-2, 5]$. The graph of f' is shown at the right. Which of the following statements are TRUE?

The graph of f'



✓ I. f has a relative maximum at $x = -1$.

✗ II. f has an absolute minimum at $x = 0$.

✓ III. The graph of f is concave down for $-2 < x < 0$.

✓ IV. The graph of f has inflection points at $x = 0$ and $x = 2$ and $x = 3$.

(A) I, II, IV [✓ (B) I, III, IV] (C) II, III, IV (D) I, II, III (E) I, II, III, IV

f' ↗ ↘ ↘ ↗ ; + − − + ; −2 −1 2 4 5

f'' ∩ ∪ ∩ ∪ ; − + − + ; −2 0 2 3 5

Ans $\boxed{B}$

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9. On which interval is the graph of $f(x) = 4x^{3/2} - 3x^2$ both concave down and increasing?

$f''(x) < 0$ $f'(x) > 0$

(A) $(0, 1)$

(B) $\left(0, \frac{1}{2}\right)$

(C) $\left(0, \frac{1}{4}\right)$

(D) $\left(\frac{1}{4}, \frac{1}{2}\right)$

[✓ (E) $\left(\frac{1}{4}, 1\right)$]

$f'(x) = 4 \cdot \frac{3}{2} x^{1/2} - 6x \longrightarrow 0 = 6\sqrt{x} - 6x$ | $x = 0, 1$

$0 = 6\sqrt{x}(\sqrt{x} - 1)$

↘ ↗ ; − + ; 0 1

$f''(x) = 4 \cdot \frac{3}{2} \cdot \frac{1}{2} \cdot x^{-1/2} - 6$

$0 = \frac{3}{\sqrt{x}} - 6$

$0 = \frac{3 - 6\sqrt{x}}{\sqrt{x}}$

$3 - 6\sqrt{x} = 0$ | $\sqrt{x} = \frac{1}{2}$

$3 = 6\sqrt{x}$ | $x = 1/4$

∪ ∩ ; + − ; 0 1/4

Ans $\boxed{E}$

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10. The average rate of change of the function $f(x) = x^2 - \frac{1}{e^x}$ over the interval $[0, 3]$ equals the instantanous rate of change of f at $x =$

*MVT $f'(x) = \frac{f(b) - f(a)}{b - a}$

(A) 0.313 [✓ (B) 1.553] (C) 2.573 (D) 3.317 (E) 9.950

$f(3) = 3^2 - \frac{1}{e^3} = 8.95$

$f(0) = 0 - \frac{1}{e^0} = -1$

set $f'(x) = \frac{f(3) - f(0)}{3 - 0}$

$f(x) = x^2 - e^{-x}$

$f'(x) = 2x + e^{-x}$

$2x + \frac{1}{e^x} = \frac{8.95 - (-1)}{3}$

$2x + \frac{1}{e^x} = 3.3167$

$2x + \frac{1}{e^x} - 3.3167 = 0$

$\boxed{x \approx 1.5524}$

Ans $\boxed{B}$

11. If $\sin 3x - 1 = \int_a^x f(t)\,dt$, then the value of a is

$3\cos 3x = f(x)$

(A) 0

(B) 1

(C) -1

(D) $\frac{\pi}{3}$

[✓ (E) $\frac{\pi}{6}$]

$\int_a^x f(t)dt = \sin 3x - 1 - [\sin 3a - 1]$

$\cancel{\sin 3x - 1} = \cancel{\sin 3x - 1} - \sin 3a + 1$

$0 = -\sin 3a + 1$

$\sin 3a = +1$

$3a = \sin^{-1}(1)$

$3a = \pi/2$

$\boxed{a = \pi/6}$

Ans $\boxed{E}$

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12. If $xy^2 = 20$ and x is decreasing at the rate of 3 units per second, the rate at which y is changing when $y = 2$ is nearest to

(A) -0.6 units/sec

(B) -0.2 units/sec

(C) 0.2 units/sec

[✓ (D) 0.6 units/sec]

(E) 1.0 units/sec

$\frac{dx}{dt} = -3$ units/s

$\frac{dy}{dt} =$ ___

$y = 2$

$xy^2 = 20$

$x(4) = 20$ $x = 5$

*product rule, related rates

$xy^2 = 20$

$\left(\frac{dx}{dt}\right)(y^2) + x\cdot 2y\left(\frac{dy}{dt}\right) = 0$

$(-3)(2)^2 + 5\cdot 2(2)\left(\frac{dy}{dt}\right) = 0$

$-12 + 20\left(\frac{dy}{dt}\right)$

$\frac{dy}{dt} = \frac{12}{20} = \frac{3}{5}$

$= 0.6$ units/sec

Ans $\boxed{D}$

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13. An approximation for $\int_{-1}^{2} e^{\sin(1.5x-1)}\,dx$ using a right-hand Riemann sum with three equal subdivisions is nearest to

(A) 2.5

(B) 3.5

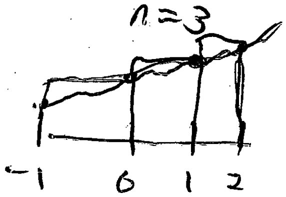
[✓ (C) 4.5]

(D) 5.5

(E) 6.5

$\text{width} = \frac{b-a}{n} = \frac{2-(-1)}{3} = \frac{3}{3} = 1$

$n = 3$



$\int_{-1}^{2} f(x)dx \approx 1(f(0)) + 1\cdot f(1) + 1\cdot f(2)$

$\approx 0.431 + 1.615 + 2.483$

≈ 4.528

$f(x) = e^{\sin(1.5x-1)}$

$f(0) = 0.431$

$f(1) = 1.615$

$f(2) = 2.483$

Ans $\boxed{C}$

14 ~~5.~~ Let f be a functi
f' is shown a
statements

I. f'

II.

and $\frac{dy}{dx} = \frac{\cos x}{x^2+1}$, which of the following statements
:?

mumum.

ection and two relative extrema.

flection and one relative extremum.

flection and two relative extrema.

nflection and two relative extrema.

Ans ☐

15. The graph of the function f is shown at the right. If the function G is defined by $G(x) = \int_{-4}^{x} f(t)\,dt$, for $-4 \le x \le 4$, which of the following statements about G are true?

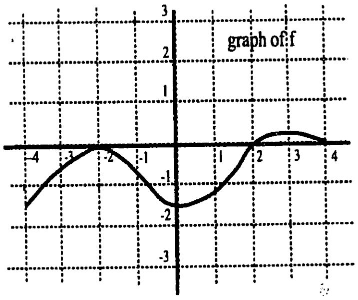
I. G is increasing on $(1, 2)$.

II. G is decreasing on $(-4, -3)$.

III. $G(0) < 0$.

(A) None (B) II only (C) III only (D) II and III only (E) I and II only

graph of the function f

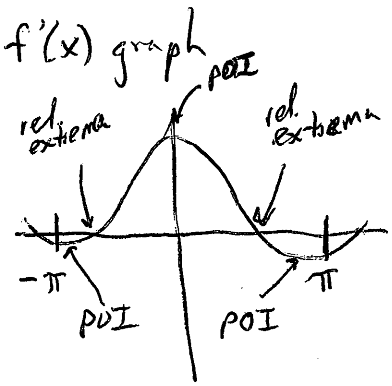


Ans ☐

14. If $f(x)$ is defined on $-\pi \le x \le \pi$ and $\frac{dy}{dx} = \frac{\cos x}{x^2+1}$, which of the following statements about the graph of $y = f(x)$ is true?

(A) The graph has no relative extremum.
(B) The graph has one point of inflection and two relative extrema.
(C) The graph has two points of inflection and one relative extremum.
(D) The graph has two points of inflection and two relative extrema.
[✓ (E)] The graph has three points of inflection and two relative extrema.

(3 max/mins) (2 x-ints)



Ans $\boxed{E}$

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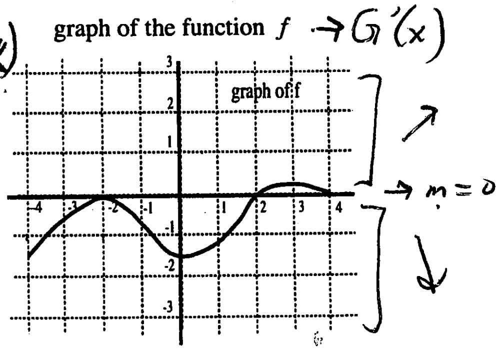
15. The graph of the function f is shown at the right. If the function G is defined by $G(x) = \int_{-4}^{x} f(t)\,dt$, for $-4 \le x \le 4$, which of the following statements about G are true?

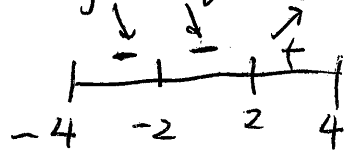
$G'(x) = f(x)$



✗ I. G is increasing on (1, 2).
✓ II. G is decreasing on (−4, −3).
✓ III. G(0) < 0.

(A) None (B) II only (C) III only [✓ (D)] II and III only (E) I and II only

slope sign line of G(x)



$G(0) = \int_{-4}^{0} f(t)dt \approx -1 + -2 \approx -3 < 0$ ✓

Ans $\boxed{D}$

16. The function f is defined on all the reals such that $f(x) = \begin{cases} x^2 + kx - 3 & \text{for } x \le 1 \\ 3x + b & \text{for } x > 1. \end{cases}$

For which of the following values of k and b will the function f be both continuous and differentiable on its entire domain?

(A) $k = -1, b = -3$

(B) $k = 1, b = 3$

(C) $k = 1, b = 4$

[✓ (D)] $k = 1, b = -4$

(E) $k = -1, b = 6$

*set equations of piecewise equal at $x=1$

*set derivatives of piecewise equal at $x=1$

at $x=1$

$x^2+kx-3 = 3x+b$

$1^2+k-3 = 3+b$

$1+1-3 = 3+b$

$-1 = 3+b$

$\boxed{-4=b}$

at $x=1$

$2x+k = 3$

$2+k=3$

$\boxed{k=1}$

Ans

$\boxed{D}$

17. A particle moves along the x-axis with velocity at time t given by: $v(t) = t + 2\sin t$. If the particle is at the origin when $t = 0$, its position at the time when $v = 6$ is $x =$

(A) 17.159 [✓ (B)] 19.159 (C) 23.141 (D) 29.201 (E) 39.309

$x(0) = 0$

$v(t) = t + 2\sin t$

$6 = t + 2\sin t$

$0 = t + 2\sin t - 6$

$t \approx 6.1887$

*final position = initial position + displacement

$$x(b) = x(a) + \int_a^b v(t)\,dt$$

$$x(6.1887) = x(0) + \int_0^{6.1887} t + 2\sin t\,dt$$

$$\boxed{x(6.1887) \approx 19.1589}$$

Ans

$\boxed{B}$