EXAM VI
CALCULUS AB
SECTION I PART A
Time–55 minutes
Number of questions–28

A CALCULATOR MAY NOT BE USED ON THIS PART OF THE EXAMINATION

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the box. Do not spend too much time on any one problem.

In this test:

(1) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(2) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1}x = \arcsin x$).

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1. What is the x-coordinate of the point of inflection on the graph of $y = xe^x$?

(A) −2 (B) −1 (C) 0 (D) 1 (E) 2

*find f''(x), set f''(x)=0, concavity sign line
* product rule

$y = xe^x$
$y' = 1e^x + x \cdot e^x$
$0 = e^x(1+x)$

$y'' = e^x + 1 \cdot e^x + x \cdot e^x$
$0 = 2e^x + xe^x$
$0 = e^x(2+x)$

$x = -2$
$\cap$ $\cup$
$-$ $+$
-2
POI at $x = -2$

Ans $\boxed{A}$

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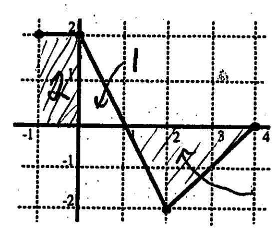
2. The graph of a piecewise-linear function f, for $-1 \le x \le 4$, is shown in the figure. If the function H is defined by

$$H(x) = \int_{-1}^{x} f(t)\,dt, \text{ for } -1 \le x \le 4, \text{ then } H(4) =$$

Graph of f



$\frac{1}{2} \cdot 3 \cdot 2 = \boxed{-3}$

(A) −2 (B) −1 (C) 0 (D) 1 (E) 2

$H(4) = \int_{-1}^{4} f(t)dt = 2 + 1 - 3 = \boxed{0}$

Ans $\boxed{C}$

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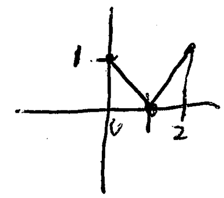
3. $\int_0^2 |x-1|\,dx =$

(A) 0
(B) 1
(C) $\frac{1}{2}$
(D) 2
(E) 3

option 1: sketch graph $y=|x-1|$

$\int_0^2 |x-1|\,dx = \frac{1}{2}+\frac{1}{2} = 1$



or option 2: piecewise function

$y=|x-1| = \begin{cases} x-1, & x>1 \\ -(x-1), & x<1 \end{cases}$

$\int_0^1 -(x-1)\,dx + \int_1^2 x-1\,dx$

$-\frac{x^2}{2}+x\Big]_0^1 \quad \frac{x^2}{2}-x\Big]_1^2 = 2-2-(\frac{1}{2}-1)$

$-\frac{1}{2}+1$

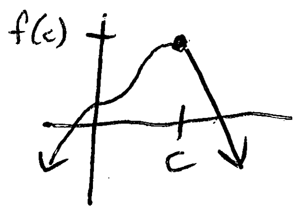
$\frac{1}{2}+\frac{1}{2} = \boxed{1}$

Ans $\boxed{B}$

4. The function f is continuous at the point $(c, f(c))$. Which of the following statements could be false?

✓(A) $\lim_{x\to c} f(x)$ exists ✓(B) $\lim_{x\to c} f(x) = f(c)$ ✓(C) $\lim_{x\to c^-} f(x) = \lim_{x\to c^+} f(x)$

✓(D) $f(c)$ is defined X $\boxed{\text{(E)}}$ $f'(c)$ exists (could be undefined slope)



Ans $\boxed{E}$

5. $\int_0^x 2\sec^2\left(2t+\frac{\pi}{4}\right) dt =$

$2\int\left[\sec\left(2t+\pi/4\right)\right]^2 dt$

$* \int \sec^2 u\,du = \tan u + c$

(A) $2\tan\left(2x+\frac{\pi}{4}\right)$
(B) $2\tan\left(2x+\frac{\pi}{4}\right)-2$
(C) $\tan\left(2x+\frac{\pi}{4}\right)-1$
(D) $2\sec\left(2x+\frac{\pi}{4}\right)\tan\left(2x+\frac{\pi}{4}\right)$
(E) $\sec\left(2x+\frac{\pi}{4}\right)\tan\left(2x+\frac{\pi}{4}\right)$

$u = 2t + \pi/4$

$\frac{du}{dt} = 2$

$dt = \frac{du}{2}$

$2\int \sec^2 u \cdot \frac{du}{2}$

$\int \sec^2 u\,du = \tan u$

$= \tan\left(2t+\pi/4\right)\Big]_0^x$

$= \tan(2x+\pi/4) - \tan(2(0)+\pi/4)$

$= \tan(2x+\pi/4) - \tan(\pi/4)$

Ans $\boxed{C}$

$\boxed{= \tan(2x+\pi/4) - 1}$

6. If $xy + x^2 = 6$, then the value of $\frac{dy}{dx}$ at $x = -1$ is $\rightarrow (-1)y + (-1)^2 = 6 \quad -y + 1 = 6 \quad y = -5$

(A) -7 (B) -2 (C) 0 (D) 1 (E) 3 point $(-1, -5)$

* implicit differentiation
* product rule

$xy + x^2 = 6$

$(1)y + x\left(\frac{dy}{dx}\right) + 2x = 0$

$1(-5) + (-1)\left(\frac{dy}{dx}\right) + 2(-1) = 0$

$-5 - \frac{dy}{dx} - 2 = 0$

$\boxed{-7 = \frac{dy}{dx}}$

Ans $\boxed{A}$

7. $\int_2^3 \frac{x}{x^2 + 1}\, dx =$

(A) $\frac{1}{2}\ln\frac{3}{2}$ (B) $\frac{1}{2}\ln\frac{1}{2}$ (C) $\frac{1}{2}\ln 2$ (D) $2\ln 2$ (E) $\ln 2$

* u-sub

$u = x^2 + 1$

$\frac{du}{dx} = 2x$

$dx = \frac{du}{2x}$

$\int \frac{\cancel{x}}{u} \cdot \frac{du}{2\cancel{x}}$

$\frac{1}{2}\int \frac{1}{u}\, du = \frac{1}{2}\ln|u| \rightarrow \frac{1}{2}\ln|x^2+1|\Big]_2^3$

$= \frac{1}{2}\ln|3^2+1| - \frac{1}{2}\ln|2^2+1|$

$= \frac{1}{2}\ln 10 - \frac{1}{2}\ln 5 = \frac{1}{2}\ln\left(\frac{10}{5}\right) \boxed{= \frac{1}{2}\ln 2}$

Ans $\boxed{C}$

8. Suppose the function f is defined so that $f(0) = 1$ and its derivative, f', is given by $f'(x) = e^{\sin x}$. Which of the following statement are TRUE?

✓ I $f''(0) = 1$

✓ II The line $y = x + 1$ is tangent to the graph of f at $x = 0$.

✓ III If $h(x) = f(x^3 - 1)$, then h is increasing for all real numbers x.

(A) I only (B) II only (C) III only (D) I and II only (E) I, II, III

$f''(x) = e^{\sin x} \cdot \cos x$

✓ $f''(0) = e^{\sin 0} \cdot \cos 0 = \boxed{1}$

$f'(x) = e^{\sin x}$

$f'(0) = e^{\sin 0} = 1$

$f(0) = 1$

tangent line: point: $(0, 1)$ slope: $m = 1$

$y - 1 = 1(x - 0) \rightarrow \boxed{y = x + 1}$

$h(x) = f(x^3 - 1)$

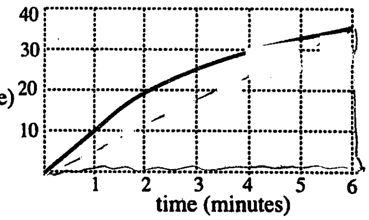
$h'(x) = f'(x^3 - 1) \cdot 3x^2$

positive positive for all x-values

Ans $\boxed{E}$

so $f'(x) > 0$ for all x-values

9. Water flows into a tank at a rate shown in the figure. Of the following, which best approximates the total number of gallons in the tank after 6 minutes?



(A) 75 (B) 95 (C) 115 $\boxed{\text{(D)}}$ 135 (E) 155

trapezoid approx: $\frac{w}{2}[h_1 + 2h_2 + 2h_3 + \dots h_n]$

$$\int_0^6 F(t)\,dt \approx \frac{1}{2}[10 + 2(20) + 2(25) + 2(30) + 2(32) + 35] \approx 129.5$$

Ans $\boxed{D}$

10. What is the instantaneous rate of change at $x = 0$ of the function f given by $f(x) = e^{2x} - 3\sin x$? * find $f'(x)$

(A) −2 $\boxed{\text{(B)}}$ −1 (C) 0 (D) 4 (E) 5

$f'(x) = e^{2x} \cdot 2 - 3\cos x$

$f'(0) = 2e^0 - 3\cos 0 = 2 - 3 = \boxed{-1}$

Ans $\boxed{B}$

11. Suppose f is a function with continuous first and second derivatives on the closed interval $[a, c]$. If the graph of its derivative f' is given in the figure, which of the following is true?

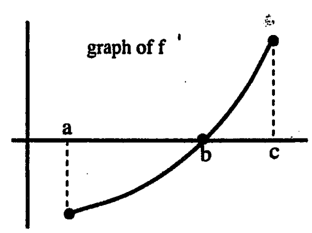
✗ (A) f is increasing on the interval (a, b)

✗ (B) f has a relative maximum at $x = b$.

✗ (C) f has an inflection point at $x = b$.

✗ (D) The graph of f is concave down on the interval (a, b).

✓ $\boxed{\text{(E)}}$ $\int_a^c f'(x)\,dx = f(c) - f(a)$



pos. slope

→ m = 0

neg. slope

↘ − ↗ +

a b c

Ans $\boxed{E}$

12. Suppose $F(x) = \int_0^{x^2} \frac{1}{2+t^3}\,dt$ for all real x, then $F'(-1) =$

(A) 2 (B) 1 (C) $\frac{1}{3}$ (D) -2 (E) $-\frac{2}{3}$

*SFTC:

$\frac{d}{dx}\int_a^{p(x)} f(t)\,dt = f(p(x))\cdot p'(x)$

$F'(x) = \frac{d}{dx}\int_0^{x^2} \frac{1}{2+t^3}\,dt$

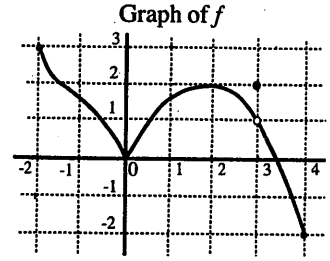
$F'(x) = \frac{1}{2+(x^2)^3}\cdot 2x = \frac{2x}{2+x^6}$

$F'(-1) = \frac{-2}{2+(-1)^6} = \frac{-2}{2+1} = \boxed{\frac{-2}{3}}$

Ans $\boxed{E}$

13. The graph of the function f is shown in the figure. For what values of x, $-2 < x < 4$, is f not differentiable?

Graph of f



(A) 0 only

(B) 0 and 2 only

(C) 2 and 3 only

$\boxed{\text{(D)}}$ 0 and 3 only

(E) 0, 1 and 3 only

not smooth $(x=0)$

not continuous $(x=3)$

* differentiable means slope exists/defined on graph.

Ans $\boxed{D}$

14. A particle moves along the x-axis so that its position at any time $t \geq 0$ is given by $x(t) = \frac{t}{t^2+4}$. The particle is at rest when $t =$

*quotient rule

(A) 0 (B) $\frac{1}{4}$ (C) 1 (D) 2 (E) 4

*particle at rest when $v(t) = x'(t) = 0$

$x'(t) = \frac{(1)(t^2+4)-(t)(2t)}{(t^2+4)^2} = \frac{t^2+4-2t^2}{(t^2+4)^2} = \frac{4-t^2}{(t^2+4)^2}$

$0 = \frac{4-t^2}{(t^2+4)^2}$

$4-t^2 = 0$

$(2-t)(2+t) = 0$

$t = 2, -2$

$t = 2$, $t \geq 0$

Ans $\boxed{D}$

15. Find the maximum value (y-value) of $f(x) = 2x^3 + 3x^2 - 12x + 4$ on the closed interval [0,2].

(A) −3

(B) 2

(C) 4

(D) 8

(E) 24

∗ Apply EVT: test endpts and critical pts.

$f'(x) = 6x^2 + 6x - 12$

$0 = 6(x^2 + x - 2)$

$0 = 6(x+2)(x-1)$

$x = 1, -2$ ← outside interval

$f(0) = 4$

$f(1) = 2 + 3 - 12 + 4 = -3$

$f(2) = 16 + 3(4) - 24 + 4 = 8$ ✓

Ans $\boxed{D}$

16. If $f(x) = \ln(\cos 2x)$, then $f'(x) =$

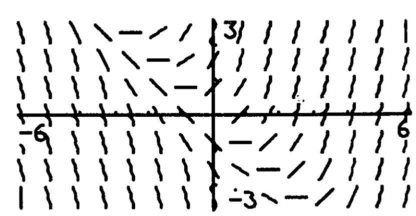
$\frac{d}{dx} \ln u = \frac{u'}{u}$

(A) $-2\tan 2x$ (B) $\cot 2x$ (C) $\tan 2x$ (D) $-2\cot 2x$ (E) $2\tan 2x$

$f'(x) = \frac{-\sin(2x) \cdot 2}{\cos(2x)} = 2 \cdot \frac{-\sin(2x)}{\cos(2x)} = -2\tan(2x)$

Ans $\boxed{A}$

17. The slope field for a differential equation $\frac{dy}{dx} = f(x,y)$ is given in the figure. The slope field corresponds to which of the following differential equations?



(A) $\frac{dy}{dx} = x + y$

(B) $\frac{dy}{dx} = -y$

(C) $\frac{dy}{dx} = y - \frac{1}{2}y^2$

(D) $\frac{dy}{dx} = x^2 + y^2$

(E) $\frac{dy}{dx} = y^2$

Ans $\boxed{A}$

18. The y-intercept of the tangent line to the curve $y = \sqrt{x+3}$ at the point (1, 2) is

(A) $\frac{1}{4}$ (B) $\frac{1}{2}$ (C) $\frac{3}{4}$ (D) $\frac{5}{4}$ (E) $\frac{7}{4}$

$y = (x+3)^{1/2}$

$y' = \frac{1}{2}(x+3)^{-1/2}(1)$

$y' = \frac{1}{2\sqrt{x+3}}$

$y'(1) = \frac{1}{2\sqrt{1+3}} = \frac{1}{2\sqrt{4}} = \frac{1}{4}$

point: $(1,2)$

slope: $m = \frac{1}{4}$

$y - 2 = \frac{1}{4}(x-1)$

$y - 2 = \frac{1}{4}x - \frac{1}{4}$

$y = \frac{1}{4}x - \frac{1}{4} + \frac{8}{4}$

$y = \frac{1}{4}x + \frac{7}{4}$

y-int: $\frac{7}{4}$

Ans $\boxed{E}$

19. The function defined by $f(x) = (x-1)(x+2)^2$ has inflection points at $x =$ * find $f''(x) = 0$

(A) –2 only

(B) –1 only

(C) 0 only

(D) –2 and 0 only

(E) –2 and 1 only

$f'(x) = (1)(x+2)^2 + (x-1)\cdot 2(x+2)'(1)$

$= (x+2)[x+2+2(x-1)]$

$= (x+2)[x+2+2x-2]$

$= (x+2)(3x) = 3x^2+6x$

$f''(x) = 6x+6$

$0 = 6(x+1)$

$x = -1$

$\cap$ $\cup$

$-$ $+$

-1

Ans $\boxed{B}$

20. If $\int_0^b (4bx - 2x^2)\,dx = 36$, then $b =$

(A) –6

(B) –3

(C) 3

(D) 6

(E) 15

$\int 4bx - \int 2x^2$

$\left.\frac{4bx^2}{2} - \frac{2x^3}{3}\right]_0^b = 2b^3 - \frac{2}{3}b^3 = 36$

$\frac{6}{3}b^3 - \frac{2}{3}b^3 = 36$

$\frac{4}{3}b^3 = 36$

$b^3 = 36 \cdot \frac{3}{4}$

$b^3 = 27$

$b = 3$

Ans $\boxed{C}$

21. If $\frac{dy}{dx} = -10y$ and if $y = 50$ when $x = 0$, then $y =$

(A) $50e^{x}$

(B) $50e^{10x}$

(C) $50e^{-10x}$

(D) $50 - 10x$

(E) $50 - 5x^2$

$dy = -10y\,dx$

$\int \frac{dy}{y} = \int -10\,dx$

$\ln|y| = -10x + C$

plug in $(0, 50)$

$\ln|50| = -10(0) + C$, $C = \ln 50$

$e^{\ln|y|} = e^{-10x + \ln 50}$

$|y| = e^{-10x} \cdot e^{\ln 50}$

$|y| = e^{-10x} \cdot 50$

$\boxed{y = 50e^{-10x}}$

Ans $\boxed{C}$

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22. If $f(x) = x^3 - 5x^2 + 3x$, then the graph of f is decreasing and concave down on the interval

$f'(x) < 0$ $f''(x) < 0$

(A) $\left(0, \frac{1}{3}\right)$ (B) $\left(\frac{1}{3}, \frac{2}{3}\right)$ (C) $\left(\frac{1}{3}, \frac{5}{3}\right)$ (D) $\left(\frac{5}{3}, 3\right)$ (E) $(3, \infty)$

$f'(x) = 3x^2 - 10x + 3$

$0 = (3x - 1)(x - 3)$

$x = 1/3,\ x = 3$

$+ \quad - \quad +$

$1/3 \quad 3$

$f''(x) = 6x - 10$

$0 = 2(3x - 5)$

$x = 5/3$

$\cap \quad \cup$

$- \quad +$

$5/3$

Ans $\boxed{C}$

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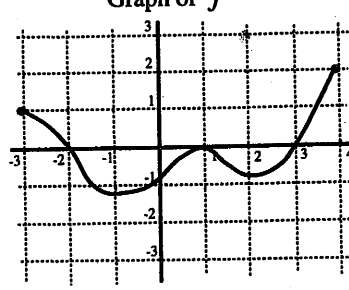
23. The figure shows the graph of f', the derivative of a function f. The domain of f is the closed interval $[-3,4]$. Which of the following is true?

I. f is increasing on the interval $(2,4)$.

II. f has a relative minimum at $x = -2$.

III. The f-graph has an inflection point at $x = 1$.

Graph of f'



pos. slope

neg. slope

slope

-3

(A) I only

(B) II only

(C) III only

(D) I and II only

(E) I, II, III

Ans $\boxed{C}$

24. How many critical values does the function $f(x) = \arctan(2x - x^2)$ have? $\frac{d}{dx}\arctan u = \frac{u'}{1+u^2}$

(A) 0 $f'(x) = \frac{2-2x}{1+(2x-x^2)^2} = \frac{2(1-x)}{1+4x^2-4x^3+x^4}$

(B) 1

(C) 2

(D) 3

(E) 4

$\boxed{x=1}$

Ans $\boxed{B}$

25. Which of the following is continuous at $x = 1$?

✓ I. $f(x) = |x-1|$ $|0| = 0$ ✓

✓ II. $f(x) = e^{x-1}$ $e^0 = 1$ ✓

✗ III. $f(x) = \ln(e^{x-1} - 1)$ $f(1) = \ln(e^0 - 1) = \ln(-1)$ ✗

(A) I only

(B) II only

(C) I and II only

(D) II and III only

(E) I, II, III

Ans $\boxed{C}$

26. The number of motels per mile along a 5 mile stretch of highway approaching a city is modeled by the function $m(x) = 11 - e^{0.2x}$, where x is the distance from the city in miles. The approximate number of motels along that stretch of highway is

(A) 16

(B) 26

(C) 36

(D) 46

(E) 56

$\int_0^5 m(x)\,dx$

$\int_0^5 11 - e^{\frac{1}{5}x}\,dx = 11x - 5e^{\frac{1}{5}x}\Big]_0^5$

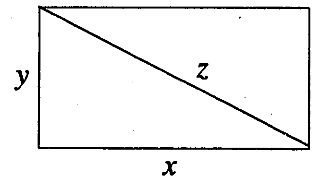
$= 55 - 5e^1 - (0 - 5e^0)$

$= 55 - 5e + 5$

$= 60 - 5e \approx \boxed{46}$

Ans $\boxed{D}$

27. The diagonal z of the rectangle at the right is increasing at the rate of 2 cm/sec and $\frac{dy}{dt} = 3\frac{dx}{dt}$. At what rate is the length x increasing when $x = 3$ cm and $y = 4$ cm ?



(A) 1 cm/sec

(B) $\frac{3}{4}$ cm/sec

(C) $\frac{2}{3}$ cm/sec

(D) $\frac{1}{3}$ cm/sec

(E) $\frac{1}{15}$ cm/sec

$\frac{dz}{dt} = 2\text{cm/s}$ $\quad x = 3$

$z = 5$

$\frac{dy}{dt} = 3\frac{dx}{dt}$ $\quad y = 4$

$x^2 + y^2 = z^2$

$2x\left(\frac{dx}{dt}\right) + 2y\left(\frac{dy}{dt}\right) = 2z\left(\frac{dz}{dt}\right)$

$2(3)\left(\frac{dx}{dt}\right) + 2(4)\left(3\left(\frac{dx}{dt}\right)\right) = 2(5)(2)$

$6\frac{dx}{dt} + 24\left(\frac{dx}{dt}\right) = 20$

$30\frac{dx}{dt} = 20$

$\boxed{\frac{dx}{dt} = \frac{2}{3}}$

Ans $\boxed{C}$

28. If $f(x) = \sin(2x) + \ln(x+1)$, then $f'(0) =$

(A) -1 (B) 0 (C) 1 (D) 2 (E) 3

$f'(x) = \cos(2x)\cdot 2 + \frac{1}{x+1}$

$f'(0) = \cos(0)\cdot 2 + \frac{1}{0+1}$

$= 2 + 1 = \boxed{3}$

Ans $\boxed{E}$

EXAM VI
CALCULUS AB
SECTION I PART B
Time–50 minutes
Number of questions–17

A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS ON THIS PART OF THE EXAMINATION

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the box. Do not spend too much time on any one problem.

In this test:

(1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.

(2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(3) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1}x = \arcsin x$).

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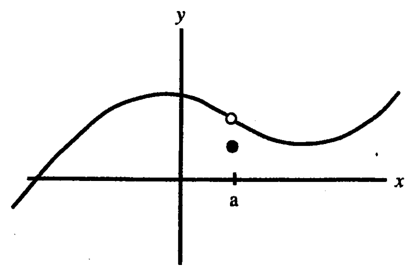
1. The graph of a function f is shown to the right. Which of the following statements about f is false?



✓ (A) f has a relative minimum at $x = a$.

✓ (B) $\lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x)$

✓ (C) $\lim_{x\to a} f(x) \neq f(a)$

✓ (D) $f(a) > 0$

X $\boxed{\text{(E)}}$ $f'(a) < 0$ * slope doesn't exist for f(x) at x=a

Ans $\boxed{E}$

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2. The function f defined by $f(x) = e^{3x} + 6x^2 + 1$ has a horizontal tangent at $x =$

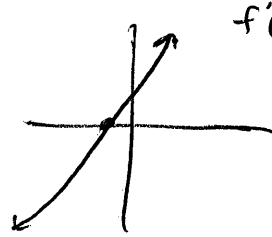
(A) −0.144 (B) −0.150 (C) −0.156 (D) −0.162 (E) −0.168

* set $f'(x) = 0$

$f'(x) = e^{3x} \cdot 3 + 12x$

$0 = 3e^{3x} + 12x$

$\boxed{x \approx -0.156}$



Ans $\boxed{C}$

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3. Boyle's Law states that if the temperature of a gas remains constant, then the pressure P and the volume V of the gas satisfy the equation $PV = c$ where c is a constant. If the volume is decreasing at the rate of 10 in^3 per second, how fast is the pressure increasing when the pressure is 100 lb/in^2 and the volume is 20 in^3?

*product rule
*related rates

(A) $5\ \frac{lb/in^2}{sec}$ (B) $10\ \frac{lb/in^2}{sec}$ (C) $50\ \frac{lb/in^2}{sec}$ (D) $200\ \frac{lb/in^2}{sec}$ (E) $500\ \frac{lb/in^2}{sec}$

$\left(\frac{dP}{dt}\right)V + P\left(\frac{dV}{dt}\right) = 0$

$\frac{dV}{dt} = -10\ in^3/s$ $\quad \frac{dP}{dt} = \_\_$

$P = 100\ lb/in^2$ $\quad V = 20\ in^3$

$\frac{dP}{dt}(20) + (100)(-10) = 0$

$\frac{dP}{dt} = \frac{1000}{20} = 50\ \frac{lb/in^2}{sec}$

Ans $\boxed{C}$

4. The graph of the second derivative of a function g is shown in the figure. Use the graph to determine which of the following are true.

✗ I. The g-graph has points of inflection at $x = 1$ and $x = 3$.

✓ II. The g-graph is concave down on the interval (3, 4).

✓ III. If $g'(0) = 0$, g is increasing at $x = 2$.

(A) I only

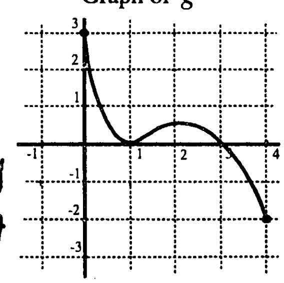
(B) II only

(C) II and III only

(D) I and II only

(E) I, II, III

Graph of g''



concave up

$\to f''(x) = 0$

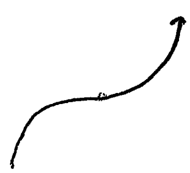
concave down

$f''(x)$ ∪ ∪ ∩

\+ \+ −

0 1 3 4

POI: $x = 3$



Ans $\boxed{C}$

5. A particle moves along a straight line with its position at any time $t \geq 0$ given by

$s(t) = \int_0^t (x^3 - 2x^2 + x)\,dx$, where s is measured in meters and t in seconds. The maximum velocity attained by the particle on the interval $0 \leq t \leq 3$ is

set $v'(t) = 0$, $a(t) = 0$

OR

Graph the $v(t)$ function and look for the maximum of the velocity graph to find ~~rel. max~~ (critical pt)

(A) 0.333 m/sec

(B) 0.148 m/sec

(C) 1 m/sec

(D) 3 m/sec

$\boxed{(E)}$ 12 m/sec

$v(t) = \frac{d}{dt}\int_0^t x^3 - 2x^2 + x\,dx$

$v(t) = t^3 - 2t^2 + t$

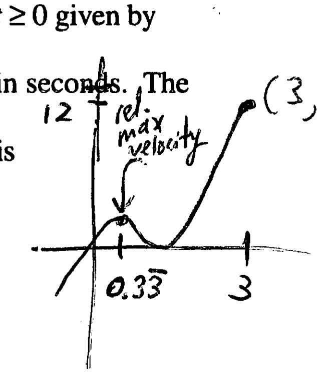
$t = 0.33\overline{3}$

*EVT: test endpts:

$v(0) = 0$

$v(0.3\overline{3}) = 0.148$

*$v(3) = \boxed{12\ m/s}$



Ans $\boxed{E}$

6. If $\frac{dy}{dx} = \sqrt{2x+1}$, then the average rate of change y with respect to x on the closed interval [0, 4] is

(A) 13 (B) $\frac{9}{2}$ (C) $\frac{13}{2}$ (D) $\frac{13}{6}$ (E) $\frac{1}{9}$

* Avg. value : $\frac{1}{b-a}\int_a^b \sqrt{2x+1}\,dx$

$$= \frac{1}{4-0}\int_0^4 \sqrt{2x+1}\,dx \approx \frac{1}{4}(8.6667) = 2.1666 = \frac{13}{6}$$

Ans $\boxed{D}$

7. If f' is a continuous function on the closed interval [0, 2] and $f(0) = f(2)$,

then $\int_0^2 f'(x)\,dx =$

(A) 0
(B) 1
(C) 2
(D) 3
(E) 4

FFTC

$$\int_0^2 f'(x)\,dx = f(2) - f(0)$$

= * since $f(2) = f(0)$,
then $f(2) - f(0) = 0$ ✓

Ans $\boxed{A}$

8. If $k \neq 0$, then $\lim_{x \to k} \frac{x^2 - k^2}{x^2 - kx} =$

$\lim_{x \to k} \frac{k^2 - k^2}{k^2 - k^2} = \frac{0}{0}$ L'Hopital's Rule →

(A) 0
$\boxed{\text{(B)}}$ 2
(C) $2k$
(D) $4k$
(E) nonexistent

$\lim_{x \to k} \frac{2x - 0}{2x - k} = \frac{2k}{2k - k} = \frac{2k}{1k} = \boxed{2}$

Ans $\boxed{B}$

9. Suppose that, during the first year after its hatching, the weight of a duck increases at a rate proportional to its weight. The duckling weighed 2 pounds when it was hatched and 3.5 pounds at age 4 months. How many pounds will the bird weigh at age 6 months?

w $\frac{dw}{dt}$ k

(A) 4.2 lbs
$\boxed{\text{(B)}}$ 4.6 lbs
(C) 4.8 lbs
(D) 5.6 lbs
(E) 6.5 lbs

w = weight

$\frac{dw}{dt} = kw$

$dw = kw\,dt$

$\frac{dw}{w} = k\,dt$

$\int \frac{dw}{w} = \int k\,dt$

$\ln|w| = kt + C$

$w = Ce^{kt}$

$2 = Ce^{k(0)}$

$2 = C$

$w = 2e^{kt}$

$3.5 = 2e^{4k}$

(t, w)
(time, weight)
(0, 2)
(4, 3.5)
(6, __)

$1.75 = e^{4k}$

$\ln 1.75 = \ln e^{4k}$

$\ln 1.75 = 4k$

$\frac{\ln 1.75}{4} = k$

$W = 2e^{\frac{\ln 1.75}{4}t}$

$W = 2e^{\frac{\ln 1.75}{4}(6)}$

$\boxed{W \approx 4.63 \text{ lbs}}$

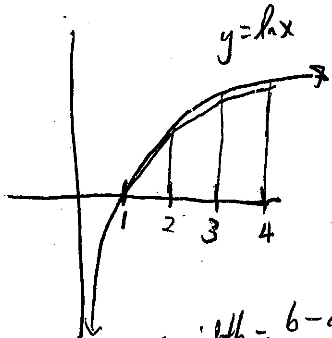
Ans $\boxed{B}$

$\underset{\text{Area}}{\text{Trapezoid}} = \frac{w}{2}[h_1 + 2h_2 + 2h_3 + h_4]$

10. Let R be the region in the first quadrant enclosed by the x-axis and the graph of $y = \ln x$ from $x = 1$ to $x = 4$. If the Trapezoid Rule with 3 subdivisions is used to approximate the area of R, the approximation is

(A) 4.970 (B) 2.510 (C) 2.497 (D) 2.485 (E) 2.473

$f(x) = \ln x$



$$\int_1^4 \ln x\,dx \approx \frac{1}{2}\left[f(1) + 2f(2) + 2f(3) + f(4)\right]$$

$$\approx \frac{1}{2}\left[\ln 1 + 2\ln 2 + 2\ln 3 + \ln 4\right] \approx 2.485$$

$$\text{width} = \frac{b-a}{n} = \frac{4-1}{3} = 1$$

Ans $\boxed{D}$

11. A solid has as its base the region enclosed by the graph of $y = \cos x$ and the x-axis between $x = -\frac{\pi}{2}$ and $x = \frac{\pi}{2}$. If every cross section perpendicular to the x-axis is a square, the volume of the solid is

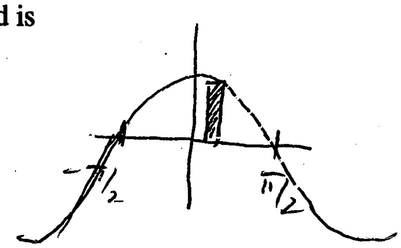
(A) $\frac{\pi}{4}$

(B) $\frac{\pi^2}{4}$

(C) $\frac{\pi}{2}$

(D) $\frac{\pi^2}{2}$

(E) 2



$$\underset{\text{Square}}{\text{Area}} = [\text{base}]^2 = [\cos x]^2$$

$$V = \int_{-\pi/2}^{\pi/2} [\cos x]^2\,dx = \boxed{\frac{\pi}{2}}$$

base = top − bottom

base = $\cos x - 0 = \cos x$

$$V = \int \text{Area of cross section}$$

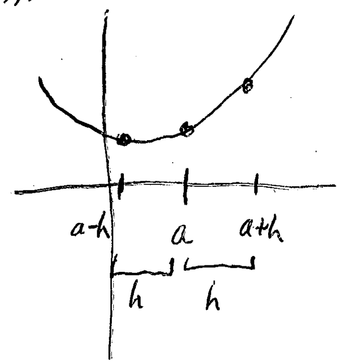
Ans $\boxed{C}$

12. If the function f is differentiable at the point $(a, f(a))$, then which of the following are true?

I. $f'(a) = \lim_{h \to 0} \dfrac{f(a+h) - f(a)}{h}$

II. $f'(a) = \lim_{h \to 0} \dfrac{f(a) - f(a-h)}{h}$

III. $f'(a) = \lim_{h \to 0} \dfrac{f(a+h) - f(a-h)}{2h}$



(A) I only

(B) I and II only

(C) I and III only

(D) II and III only

(E) I, II, III

Ans $\boxed{E}$

13. The level of air pollution at a distance x miles from a tire factory is given by

$$L(x) = e^{-0.1x} + \frac{1}{x^2}.$$

The average level of pollution between 15 and 25 miles from the factory is

(A) 0.144

(B) 0.250

(C) 0.156

(D) 0.162

(E) 0.168

Avg. value theorem : $\frac{1}{b-a}\int_a^b L(x)\,dx$

$\frac{1}{25-15}\int_{15}^{25} e^{-0.1x} + \frac{1}{x^2}\,dx = \frac{1}{10}(1.43711) = \boxed{0.1437}$

Ans $\boxed{A}$

14. Suppose the continuous function f is defined on the closed interval [0, 3] such that its derivative f' is defined by $f'(x) = e^x \sin(x^2) - 1$. Which of the following are true about the graph of f?

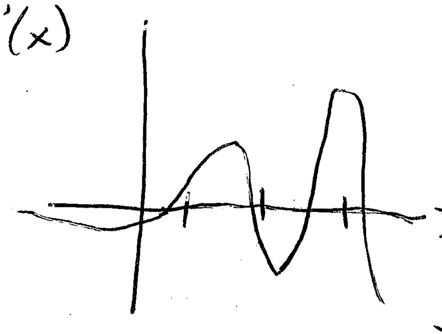
   ✓ I. f has exactly one relative maximum point.
   ✓ II. f has two relative minimum points.
   ✗ III. f has two inflection points. (3 POI's)

   (A) I only
   (B) II only
   (C) III only
   (D) $\boxed{\text{I and II only}}$
   (E) I, II, III

$f'(x)$: ↘ − ↗ + ↘ − ↗ +

0 0.7 1.72 2.52 3

$f'(x)$



$f''(x)$: ∪ + ∩ − ∪ + ∩ −

0 1.38 2.22 2.83 3

Ans $\boxed{D}$

15. If the average value of $y = x^2$ over the interval [1, b] is $\frac{13}{3}$, then the value of b could be

   (A) $\frac{7}{3}$ (B) $\boxed{3}$ (C) $\frac{11}{3}$ (D) 4 (E) $\frac{13}{3}$

$$\text{Avg. value} = \frac{1}{b-a}\int_a^b x^2\,dx$$

$$\frac{13}{3} = \frac{1}{b-1}\int_1^b x^2\,dx$$

$$\frac{13}{3} = \frac{1}{b-1}\cdot\left.\frac{x^3}{3}\right]_1^b$$

$$\frac{13}{3} = \frac{1}{b-1}\left(\frac{b^3}{3} - \frac{1}{3}\right)$$

$$13 = \frac{b^3 - 1}{b-1}$$

$$13b - 13 = b^3 - 1$$

$$0 = b^3 - 13b + 12$$

$$b = 1, \boxed{3}$$

Ans $\boxed{B}$

16. If the function f is defined on the closed interval [0, 3] by $f(x)=\dfrac{2x}{x^2+1}$, which of the following is true?

$$\int_0^3 \frac{2x}{x^2+1}\,dx \qquad u=x^2+1 \quad \frac{du}{dx}=2x \quad dx=\frac{du}{2x}$$

✓ I. $\int_0^3 f(x)\,dx = \ln 10$

II. f has a relative maximum at $x = 1$.

III. $f'(2)=\frac{1}{2}$

$$\int \frac{2x}{u}\cdot\frac{du}{2x} = \int \frac{1}{u}\,du = \ln|u| = \ln|x^2+1|\Big]_0^3 = \ln|10|-\ln|1| = \boxed{\ln|10|}$$

(A) I only
(B) II only
(C) I and II only
(D) II and III only
(E) I, II, III

Ans $\boxed{C}$

17. The area of the region bounded by the graphs of $y=\arctan x$ and $y=4-x^2$ is approximately

(A) 10.80
(B) 10.97
(C) 11.14
(D) 11.31
(E) 11.48

Ans $\boxed{B}$