

CCGPS Analytic Geometry Day 1: Solving Systems of Equations

Substitution Method:

1. Choose one equation and solve for one variable on one side (either the x or the y)
2. Substitute the solution from step 1 into the second equation and solve for the variable in the equation.
3. Using the value found in step 2, substitute it into the first equation and solve for the second variable.
4. Substitute the values for both variables into both equations to show they are correct.

Elimination Method (Addition/Subtraction Method)

1. Algebraically adjust both equations so that all the variables and constants are lined up and in the same order
2. If needed, multiply one of the equations by a constant so that there is one variable in each equation that has the same coefficient.
3. Subtract one equation from the other.
4. Solve the resulting equation for the one variable.
5. Using the value found in the step 4, substitute it into either equation and solve for the remaining variable.
6. Substitute the values for both variables into the equation not used in step 5 to be sure our solution is correct.

Warm up: Solve the systems.

1. $2x + 3y = 8$
 $y = x - 4$

2. $5x - 2y = -1$
 $10y - 2 = 10x$

3. $6x + 14y = 28$
 $3x + 7y = 11$

Example 1:

$$x = y^2 - 6$$

$$y = x$$

How many solutions are possible? Solve the system:

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Example 2:

$$x^2 + y = 14$$

$$y = x + 2$$

How many solutions are possible? Solve the system:

Example 3:

$$x^2 - 2x + y - 2y = 0$$

$$y + x = 2$$

Example 4:

$$y^2 - 4x = 0$$

$$y - 2x = -4$$

CCGPS Analytic Geometry Day 1: Solving Systems of Equations Homework

Substitution Method:

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Solve each system.

1. $x^2 - y + 5 = 0$

$$x - y + 5 = 0$$

2. $(x + 3)^2 - y = 4$

$$4x + y = 16$$

3. $x^2 + y - 3 = 0$

$$x + y = 1$$

4. $x^2 - 2x + 4y = 3$

$$x - y = 3$$

$$5. 3x + y^2 + 2 = 9$$

$$x = y + 1$$

$$6. x^2 + y = 10$$

$$3x + y = 6$$

$$7. y^2 - y + 4x + 10 = 0$$

$$-2x + 4y = 2$$

Key

Substitution Method:

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Warm up: Solve the systems.

1. $2x + 3y = 8$
 $y = x - 4$

substitution

$$2x + 3(x - 4) = 8$$

$$2x + 3x - 12 = 8$$

$$5x = +20$$

$$\underline{x = 4}$$

$$2x + 3y = 8$$

$$2(4) + 3y = 8$$

$$8 + 3y = 8$$

Example 1:

$$x = y^2 - 6$$

$$y = x$$

$$3y = 0$$

$$\underline{y = 0}$$

$$\boxed{(4, 0)}$$

2. $5x - 2y = -1$
 $10y - 2 = 10x$

$$5x - 2y = -1$$

$$-10x + 10y = 2$$

$$2(5x - 2y = -1)$$

$$-10x + 10y = 2$$

$$10x - 4y = -2$$

$$-10x + 10y = 2$$

$$0 + 6y = 0 \quad \underline{y = 0}$$

$$5x - 2y = -1$$

$$5x - 0 = -1$$

$$x = -1/5$$

$$\boxed{(-1/5, 0)}$$

3. $6x + 14y = 28$
 $-2(3x + 7y = 11)$

$$6x + 14y = 28$$

$$-6x - 14y = -22$$

$$\hline 0 + 0 = 6$$

No solution
 (lines do not intersect)
 parallel lines.

How many solutions are possible? Solve the system:

$$x = x^2 - 6$$

$$0 = x^2 - x - 6$$

a · c	-3 × 2 = -6
-3	2
1	-1
a	b

$$(x - 3)(x + 2) = 0$$

$$x = 3, x = -2$$

$$\downarrow$$

$$y = x$$

2 possible solutions:

$$(3, 3) \text{ and } (-2, -2)$$

$$3 = 9 - 6 \checkmark$$

$$-2 = 4 - 6 \checkmark$$

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Example 2:

$$x^2 + y = 14$$

$$y = x + 2$$

How many solutions are possible? Solve the system:

$$\begin{aligned}x^2 + (x+2) &= 14 \\x^2 + x + 2 - 14 &= 0 \\x^2 + x - 12 &= 0\end{aligned}$$

$$\begin{aligned}(x+4)(x-3) &= 0 \\x+4=0 & \quad | \quad x-3=0 \\ \underline{x=-4} & \quad | \quad \underline{x=3}\end{aligned}$$

$$\begin{aligned}y &= x + 2 \quad \text{plug } x = -4 \\y &= -4 + 2 = -2\end{aligned}$$

$$\boxed{(-4, -2)}$$

$$\begin{aligned}y &= x + 2 \\ \text{plug in } x &= 3 \\y &= 3 + 2 = 5\end{aligned}$$

$$\boxed{(3, 5)}$$

Example 3:

$$x^2 - 2x + y - 2y = 0$$

$$y + x = 2$$

$$y = 2 - x$$

$$x^2 - 2x + (2-x) - 2(2-x) = 0$$

$$x^2 - 2x + 2 - x - 4 + 2x = 0$$

$$x^2 - x - 2 = 0$$

$$\begin{aligned}(x-2)(x+1) &= 0 \\x=2, x &= -1\end{aligned}$$

$$\begin{aligned}y + x &= 2 \\ \text{plug } x &= 2\end{aligned}$$

$$y + 2 = 2$$

$$y = 0$$

$$\boxed{(2, 0)}$$

$$\begin{aligned}y + x &= 2 \\ \text{plug } x &= -1\end{aligned}$$

$$y - 1 = 2$$

$$y = 3$$

$$\boxed{(-1, 3)}$$

Example 4:

$$y^2 - 4x = 0$$

$$y - 2x = -4$$

$$y = -4 + 2x = 2x - 4$$

$$(2x-4)^2 - 4x = 0$$

$$4x^2 - 16x + 16 - 4x = 0$$

$$4x^2 - 20x + 16 = 0$$

$$4(x^2 - 5x + 4) = 0$$

$$4(x-4)(x-1) = 0$$

$$x=4, x=1$$

$$\begin{aligned}y - 2x &= -4 \\ \text{plug } x &= 1\end{aligned}$$

$$y - 2 = -4$$

$$y = -2$$

$$\boxed{(1, -2)}$$

$$\begin{aligned}y - 2x &= -4 \\ \text{plug } x &= 4\end{aligned}$$

$$y - 2(4) = -4$$

$$y - 8 = -4$$

$$y = 4$$

$$\boxed{(4, 4)}$$

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Solve each system.

1. $x^2 - y + 5 = 0$

$x = y - 5$

$x - y + 5 = 0$

$(y - 5)^2 - y + 5 = 0$

$(y^2 - 10y + 25) - y + 5 = 0$

$y^2 - 10y + 25 - y + 5 = 0$

$y^2 - 11y + 30 = 0$

$(y - 6)(y - 5) = 0$
 $y = 5, y = 6$

$x - y + 5 = 0$

plug $y = 5$

$x - 5 + 5 = 0$

$x = 0$

$(0, 5)$

$x - y + 5 = 0$

plug $y = 6$

$x - 6 + 5 = 0$

$x - 1 = 0$

$x = 1$

$(1, 6)$

2. $(x + 3)^2 - y = 4$

$4x + y = 16$

$y = 16 - 4x$

$(x + 3)^2 - (16 - 4x) = 4$

$x^2 + 6x + 9 - 16 + 4x = 4$

$x^2 + 10x - 7 = 4$

$x^2 + 10x - 11 = 0$

$(x + 11)(x - 1) = 0$

$x = -11, x = 1$

$4x + y = 16$

plug $x = 1$

$4 + y = 16$

$y = 12$

$(1, 12)$

$4x + y = 16$

plug $x = -11$

$4(-11) + y = 16$

$-44 + y = 16$

$y = 60$

$(-11, 60)$

3. $x^2 + y - 3 = 0$

$x + y = 1$ $y = 1 - x$

$x^2 + (1 - x) - 3 = 0$

$x^2 + 1 - x - 3 = 0$

$x^2 - x - 2 = 0$

$(x - 2)(x + 1) = 0$
 $x = 2, x = -1$

$x + y = 1$

plug $x = -1$

$-1 + y = 1$

$y = 2$

$(-1, 2)$

$x + y = 1$

plug $x = 2$

$2 + y = 1$

$y = -1$

$(2, -1)$

4. $x^2 - 2x + 4y = 3$

$x - y = 3$

$y = x - 3$

$x^2 - 2x + 4(x - 3) = 3$

$x^2 - 2x + 4x - 12 = 3$

$x^2 + 2x - 15 = 0$

$(x + 5)(x - 3) = 0$

$x = -5, x = 3$

$x - y = 3$

plug $x = 3$

$3 - y = 3$

$0 = y$

$(3, 0)$

$x - y = 3$

plug $x = -5$

$-5 - y = 3$

$-5 - 3 = y$

$-8 = y$

$(-5, -8)$

$$5. 3x + y^2 + 2 = 9$$

$$x = y + 1$$

$$3(y+1) + y^2 + 2 = 9$$

$$3y + 3 + y^2 - 7 = 0$$

$$y^2 + 3y - 4 = 0$$

$$\begin{array}{r|l} 4 & -4 \\ \hline 1 & 3 \end{array} \quad (y+4)(y-1) = 0$$

$$y = 1, y = -4$$

$$x = y + 1$$

$$\text{plug } y = 1$$

$$x = 1 + 1 = 2$$

$$\boxed{(2, 1)}$$

$$x = y + 1$$

$$\text{plug } y = -4$$

$$x = -4 + 1 = -3$$

$$\boxed{(-3, -4)}$$

$$6. x^2 + y = 10$$

$$3x + y = 6$$

$$y = 6 - 3x$$

$$x^2 + (6 - 3x) = 10$$

$$x^2 + 6 - 3x - 10 = 0$$

$$x^2 - 3x - 4 = 0$$

$$\begin{array}{r|l} -4 & 1 \\ \hline 1 & -3 \end{array}$$

$$(x-4)(x+1) = 0$$

$$x = 4, -1$$

$$\underline{\underline{x = 4, -1}}$$

$$3x + y = 6$$

$$\text{plug } x = 4$$

$$3(4) + y = 6$$

$$12 + y = 6$$

$$y = -6$$

$$\boxed{(4, -6)}$$

$$3x + y = 6$$

$$\text{plug } x = -1$$

$$3(-1) + y = 6$$

$$-3 + y = 6$$

$$y = 9$$

$$\boxed{(-1, 9)}$$

$$7. y^2 - y + 4x + 10 = 0$$

$$\begin{array}{r} -2x + 4y = 2 \\ \hline -2 \quad -2 \end{array}$$

$$x - 2y = -1$$

$$x = 2y - 1$$

$$y^2 - y + 4(2y - 1) + 10 = 0$$

$$y^2 - y + 8y - 4 + 10 = 0$$

$$y^2 + 7y + 6 = 0$$

$$(y+6)(y+1) = 0$$

$$y = -6, -1$$

$$x = 2y - 1$$

$$\text{plug } y = -6$$

$$x = 2(-6) - 1$$

$$x = -12 - 1$$

$$x = -13$$

$$\boxed{(-13, -6)}$$

$$x = 2y - 1$$

$$\text{plug } y = -1$$

$$x = 2(-1) - 1$$

$$x = -3$$

$$\boxed{(-3, -1)}$$